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DESCRIPTION OF VIBRATIONAL STATES OF DEFORMED NUCLEI WITHIN THE QUASIPARTICLE-PHONON NUCLEAR MODEL

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The basic equations of the quasiparticle-phonon nuclear model are derived for a finite rank $n_{max} > 1$ separable isoscalar and isovector multipole and spin-multipole particle-hole and particle-particle interactions between quasiparticles. It is shown that allowance for separable $n_{max} > 1$ interactions complicates the RPA calculations but almost does not lead to complications of calculations of the fragmentation of quasiparticle and collective states. It is stated that the QPNM can serve as a basis for calculating many characteristics of deformed nuclei. The QPNM with *p-h* and *p-p* interactions provides a good enough description of low-lying quadrupole, octupole and hexadecapole states in some deformed nuclei, which is exemplified by ^{168}Er.

1. Introduction

Collective degres of freedom of atomic nuclei manifest themselves as vibrational states. All nuclei have low-lying quadrupole and octupole states and high-lying collective states — giant resonances of various types. Excited states of deformed nuclei differ from spherical nuclei by a variety of their properties. Phenomenological and microscopic descriptions of collective states are widely used.

The development of the phenomenological description of rotational and vibrational states and especially investigation of the particle-vibrator coupling have greately been influenced by the works of G. Alaga[9−5]. Widely known are the Alaga rules for transition intensities.

A specific feature of energy spectra of doubly deformed nuclei, except for rotational states, is the existence of low-lying quadrupole and octupole states that are intensively excited in $E2$ and $E3$ transitions. This is assumed[6] to be a manifestation of collective vibrations of the nuclear shape. In systems based on the independent particle model, vibrational states are caused by collective coherent motions of interacting nucleons. The general picture of vibrational excitations depends on the symmetry of the equilibrium ground state. In deformed nuclei the equilibrium shape has axial symmetry. It is invariant under the reflection on the plane perpendicular to the symmetry axis.

As a result of experimental studies of α-, β- and γ-decay and nuclear reactions there were observed quadrupole β- and γ-vibrational and octupole states. All doubly even deformed nuclei were found to have γ-vibrational $K^{\pi}_{\nu} = 2^{+}_{1}$ states intensively excited in $E2$ transitions from the ground states. Many 0^{+} states were observed which have a complicated nature caused by β-vibrations, pairing vibrations, etc. First, octupole collective $K^{\pi}_{\nu} = 2^{-}_{1}$ states were observed in nuclei of the rare-earth region and very low-lying $K^{\pi}_{\nu} = 0^{-}_{1}$ states in the Th and U isotopes. Later, there were observed collective $K^{\pi}_{\nu} = 0^{-}_{1}$ and 1^{-}_{1} states in some nuclei of the rare-earth region and $K^{\pi}_{\nu} = 1^{-}_{1}$ and 2^{-}_{1} states in some nuclei of the actinide region. The experimental data are systematised in Nuclear Data Sheets and in many other papers.

Many investigations presented in Refs. 6—8 and other monographs and papers were devoted to a phenomenological description of vibrational states. Microscopic description of vibrational states is based on the random phase approximation (RPA). Initially[9], the low-lying $K^{\pi}_{\nu} = 0^{-}_{1}$ states in the Th and U istoopes were treated as collective octupole vibrational states. In the RPA, quadrupole[10] and octupole[11] states were described. The experimental data on quadrupole and octupole vibrational states in doubly even deformed nuclei and their comparison with the RPA description are summarised in Ref 12. It is shown in Ref. 13 that the RPA provides a unique description of very collective, weakly collective and two-quasiparticle states and the description of energies, integral and differential characteristics of vibrational states, which is in agreement with the experimental data. The experimental data on nonrotational states are analysed and compared with calculations in Ref. 14 for nuclei of the rare-earth region and in Ref. 15 for nuclei of the actinide region. The coupling of the rotational and vibrational motion is to be taken into account. The discovery of a stable octupole deformation in the Ra isotopes and light Th isotopes made one to analyse once more collective octupole states in deformed nuclei, which was done in Ref. 16.

The above-mentioned experimental and theoretical investigations of phenomenological and microscopic description of vibrational states can be considered as a firts stage in studying low-lying vibrational states in doubly even deformed nuclei.

A modern description of vibrational states of deformed nuclei is made within the phenomenological interacting boson model (IBM)[17–19] and microscopic quasiparticle-phonon nuclear model (QPNM)[20–22]. Calculations in other phenomenological models are restricted by giant resonances. Calculations within such microscopic models as the self-consistent collective coordinate method[23] and multiphonon method[24] are mainly restricted by the description of anharmonicity of two-phonon states.

In the IBM, a small part of the large space of the single-particle nuclear model is extracted — a subspace of collective states. Thus, in the IBM the low-lying vibrational and rotational states are described in terms of s, $J = 0$; d, $J = 2$ and f, $J = 3$ bosons. The boson operators are connected with the pairs of fermion operators. In the IBM, only a part of two-quasiparticle states which enters into s, d and f — bosons is taken into account. In the phenomenological IBM, the number of nucleons (or holes) in unfilled neutron and proton shells is important. The number of valence nucleons is defined of a great extent on the spectra of collective states. In the sd IBM, the first excited $K^{\pi}_{\nu} = 0^+_1$ and 2^+_1 states have the dominating one-boson components. Then, the $K^{\pi} = 0^+$, 2^+ and 4^+ states with the dominating two-boson components follow.

Discrepancies with the experimental data on the states lying above the first 0^+ and 2^+ excited states led to the inclusion[25] of the g boson with $J = 4$ in addition to the s and d bosons. In the sdg IBM, deformed nuclei have states with the dominating one-boson components with $K^{\pi} = 1^+$, 3^+, 4^+ and additional states with $K^{\pi} = 0^+$ and 2^+. In the sdg IBM there are low-lying states with $K^{\pi} = 0^+$, 2^+ and 4^+ with the dominating two-boson components.

To describe collective states with negative parity f bosons with $J = 3$[26] are introduced in addition to the s and d bosons. It has been stated in Ref. 27 that one should introduce additionally the β boson with $J = 1$ and describe octupole states in the spdf IBM. It has been pointed out in Ref. 28 that due to an important role of the g boson, if one does not restrict oneself to 0^+, 2^+ and first octupole states with the corresponding rotational bands, the deformed nuclei should be described within the spdfg IBM. This version of the IBM still waits its consistent formulation.

In the present paper we give the basic assumptions of the QPNM and generalization of the QPNM equations for a finite rank separable interactions between quasiparticles for deformed nuclei. The advantages of the QPNM are demonstrated in describing nonrotational states of ^{168}Er; the comparison with the IBM calculations is made. Specific features of the $E\lambda$ strength distribution among the low-lying states of doubly even deformed nuclei are expounded. The situation with two-phonon collective states in deformed nuclei is described.

2. *Basic assumptions of the quasiparticle-phonon nuclear model*

The Hamiltonian of the quasiparticle-phonon nuclear model (QPNM) consists of an average field of a neutron and a proton systems in the form of an axial-symmetric Saxon-Woods potential, monopole and quadrupole pairing, isoscalar and isovector particle-hole (*p-h*) and particle-particle (*p-p*) multiple and spin-multipole interactions between quasiparticles.

Calculations in the QPNM are made in four states. *The first stage* is calculation of the single-particle energies and wave functions of the Saxon-Woods potential. The parameters of the Saxon-Woods potential are fixed so as to obtain a correct description of the low-lying states in odd-A nuclei taking account of the quasiparticle-phonon interaction. Undoubtedly, one can use another form of the average field potential or to calculate the energies and wave functions of single-particle states within the Hartree-Fock method and to use them in calculations within the QPNM; this arbitrariness is of no fundamental importance. The application of the Hartree-Fock method implies an early stage of parametrisation, i. e. parametrisation of an effective interaction, for instance, in terms of the Skyrme forces. Calculations in the QPNM are performed with the parameters of the Saxon-Woods potential fixed in 1968—1973 and given in Refs. 4, 15, 29. To specify single-particle states S. G. Nilsson introduced asymptotic quantum numbers $Nn_z\Lambda\uparrow$ at $K=\Lambda+1/2$ and $Nn_z\Lambda\downarrow$ at $K=\Lambda-1/2$ denoted by $q\sigma$, $\sigma=\pm 1$.

The second stage is the canonical Bogolubov transformation

$$a_{q\sigma}=u_q\alpha_{q\sigma}+\sigma v_q\,\alpha^+_{q-\sigma} \tag{1}$$

under which one passes from particle operators $a^+_{q\sigma}$ and $a_{q\sigma}$ to the quasiparticle operators $\alpha^+_{q\sigma}$ and $\alpha_{q\sigma}$, and the calculation in the model of independent quasiparticles[13]. Taking simultaneously into account monopole pairing with the constants G_τ and quadrupole pairing with the constant G^{20}_τ and under the condition of exclusion of 0^+ spurious states, the following equations have been derived in Ref. 30:

$$1=\frac{G_\tau}{2}\sum_q^\tau\frac{C_\tau+f^{20}(qq)\,C_{2\tau}}{C_\tau\varepsilon_q}, \tag{2}$$

$$1=G^{20}\left\{\sum_q^\tau\frac{f^{20}(qq)C_\tau}{2\,C_{2\tau}\varepsilon_q}+\sum_{qq'}^\tau\frac{f^{20}(qq')(v^{(+)}_{qq'})^2}{\varepsilon_q}\right\}, \tag{2'}$$

$$N_\tau=\sum_q^\tau\left[1-\frac{\xi(q)}{\varepsilon_q}\right]. \tag{2''}$$

Neglecting nondiagonal matrix elements $f^{20}(qq'')$ in Eq. (2′), one arrives at the equations derived earlier in Ref. 31. Here

$$\begin{aligned}
&\varepsilon_q=[\Delta_q^2+\xi^2(q)]^{1/2}, && \xi(q)=E(q)-\lambda_\tau,\\
&\Delta_q=C_\tau+f^{20}(qq)\,C_{2\tau}, && C_\tau=G_\tau\sum_q^\tau u_q v_q,\\
&C_{2\tau}=G^{20}\sum_q^\tau f^{20}(qq)\,u_q v_q, && \\
&u^{(\pm)}_{qq'}=u_qv_{q'}\pm u_{q'}v_q, && v^{(\pm)}_{qq'}=u_qu_{q'}\pm v_qv_{q'}.
\end{aligned} \tag{3}$$

Then $E(q)$ is the single-particle energy, $\sum\limits_{qq'}^{\tau}$ means summation over single-particle levels of a neutron at $\tau=n$ and a proton at $\tau=p$ systems, λ_τ are chemical potentials, G_τ are the monopole pairing constants, and $G^{\lambda\mu}$ are the p-p interaction constants of multipolarity λ with projection μ. The constants G_τ and G^{20} are determined from the pairing energies and energies of two-quasiparticle states with $K>4$[32]. The energies of two-quasiparticle states are calculated taking the blocking effect into account.

The effective interactions between quasiparticles are expressed as the series of multipoles and spin-multipoles. The effective interactions seem to compensate equations rejected within the HFB method. They are also related to nucleon-

nucleon interactions in the nuclear matter and some of their terms correspond to the exchange by one or two mesons. For the calculations within the QPNM *it is essential* that the interaction between quasiparticles is represented in a separable (factorised) form. Separable potentials are widely used in describing nucleon-nucleon interactions and in studying three-body nuclear systems and light nuclei. Separable representations of rank $n_{max} < 5$ of the Paris and Bonn potentials provide a satisfactory approximation for these potentials. Separable potentials are used in the cases where the results of calculations are more sensitive to the form of radial dependence of forces in comparison with the calculations of the properties of complex nuclei within the QPNM. It is to be noted that the matrix elements of effective interactions are used in the calculations. The single-particle wave functions truncate a small part of interactions. One can construct separable interactions whose matrix elements are similar to those of more complex forces. It may be assumed that appropriately chosen interactions between quasiparticles in a separable form do not limit the accuracy of calculations. Therefore, the use of separable interactions of finite rank in calculating the characteristics of complex nuclei is justified.

We introduce, as in Ref. 33 for spherical nuclei, separable interactions of finite rank for deformed nuclei. Expand over multipoles the central spin-independent interaction and write it down in the second quantised form

$$\sum_{\substack{q_1 q_2 q_2' q_1' \\ \sigma_1 \sigma_2 \sigma_2' \sigma_1'}} \langle q_1\sigma_1, q_2\sigma_2 | \, [R^{\lambda\mu}(\tau_1\tau_2) + (\vec{\tau}^{(1)}\vec{\tau}^{(2)})\, \overline{R}^{\lambda\mu}(\tau_1\tau_2)] \cdot$$

$$\sum_{\sigma} Y_{\lambda\sigma\mu}(\Theta_1\varphi_1) \cdot Y_{\lambda-\sigma\mu}(\Theta_2\varphi_2)\, | q_2'\sigma_2', q_1'\sigma_1' \rangle\, a^+_{q_1\sigma_1} a^+_{q_2 q_2} a_{q_2'\sigma_2'} a_{q_1'\sigma_1'}.$$

If one takes a separable interaction of rank $n_{max} > 1$ for p-h and p-p interactions in the form

$$R^{\lambda\mu}(\tau_1\tau_2) = \varkappa_0^{\lambda\mu} \sum_{n=1}^{n_{max}} R_n^{\lambda\mu}(\tau_1)\, R_n^{\lambda\mu}(\tau_2),$$

$$\overline{R}^{\lambda\mu}(\tau_1\tau_2) = \varkappa_1^{\lambda\mu} \sum_{n=1}^{n_{max}} \overline{R}_n^{\lambda\mu}(\tau_1)\, \overline{R}_n^{\lambda\mu}(\tau_2),$$

$$R^{\lambda\mu}(\tau_1\tau_2) = G_0^{\lambda\mu} \sum_{n=1}^{n_{max}} \widetilde{R}_n^{\lambda\mu}(\tau_1)\, \widetilde{R}_n^{\lambda\mu}(\tau_2) \tag{4}$$

$$R^{\lambda\mu}(\tau_1\tau_2) = G_1^{\lambda\mu} \sum_{n=1}^{n_{max}} \widetilde{\overline{R}}_n^{\lambda\mu}(\tau_1)\, \widetilde{\overline{R}}_n^{\lambda\mu}(\tau_2),$$

the expansion over multipoles takes the form

$$\sum_{\lambda\mu} \sum_{n=1}^{n_{max}} \Bigg\{ \sum_{\tau\varrho=\pm 1} (\varkappa_0^{\lambda\mu} + \varrho\varkappa_1^{\lambda\mu}) \sum_{\sigma} M_{\lambda\sigma\mu n}(\tau)\, M_{\lambda-\sigma\mu n}(\varrho\tau) +$$

$$+ \sum_{\tau\sigma} (G_0^{\lambda\mu} + G_1^{\lambda\mu})\, P^+_{\lambda\sigma\mu n}(\tau)\, P_{\lambda\sigma\mu n}(\tau) + \varkappa_1^{\lambda\mu} \sum_{\sigma} (M^{CH}_{\lambda\sigma\mu n})^+\, M^{CH}_{\lambda\sigma\mu n} +$$

$$+ G_1^{\lambda\mu} \sum_{\sigma} (P^{CH}_{\lambda\sigma\mu n})^+\, P^{CH}_{\lambda\sigma\mu n} + G_1^{\lambda\mu} \sum_{\tau\sigma} P^+_{\lambda\sigma\mu n}(\tau)\, P_{\lambda\sigma\mu n}(-\tau) \Bigg\}.$$

The first term allows for p-h and the second for p-p interactions, the third and fourth terms for charge-exchange p-h and p-p interactions and the last term for the exchange by two nucleons. Here $\varkappa_0^{\lambda\mu}$, $\varkappa_1^{\lambda\mu}$ and $G_0^{\lambda\mu}$, $G_1^{\lambda\mu}$ are the isoscalar and isovector constants of p-h and p-p interactions; then, we use $G^{\lambda\mu} = G_0^{\lambda\mu} + G_1^{\lambda\mu}$.

Introduction of a separable interaction of a finite rank $n_{max} > 1$ in comparison with $n_{max} = 1$ leads to additional summation over n. Introduction of a separable interaction of rank n_{max} is meaningful if n_{max} is much less than the rank of the determinant of the RPA secular equation for a non-separable interaction. There is a certain arbitrariness in choosing radial dependence of separable interactions. The existence of collective vibrational quadrupole and octupole state indicates a maximum on the nuclear surface. Therefore, $R^{\lambda\mu}(\tau) = \tau^\lambda$ or $R^{\lambda\mu}(\tau) = \partial V(\tau)/\partial\tau$ is used where, $V(\tau)$ is the central part of the Woods-Saxon potential. The calculations[34,35] were made with $R^{\lambda\mu}(\tau) = \partial V(\tau)/\partial\tau$.

Then, the RPA phonons $Q^+_{\lambda\mu i\sigma}$ and $Q_{\lambda\mu i\sigma}$ are introduced where

$$Q^+_{\lambda\mu i\sigma} = \frac{1}{2}\sum_{qq'}[\psi^{\lambda\mu i}_{qq'}\, A^+(qq';\mu\sigma) - \varphi^{\lambda\mu i}_{qq'}\, A(qq',\mu-\sigma)], \tag{5}$$

$$A^+(qq';\mu\sigma) = \sum_{\sigma'}\delta_{\sigma'(K-K'),\sigma\mu}\,\sigma'\, a^+_{q\sigma'}a^+_{q'-\sigma'} \text{ or } \sum_{\sigma'}\delta_{\sigma'(K+K'),\sigma\mu}\, a^+_{q\sigma'}a^+_{q'\sigma'}, \tag{6}$$

where $\sigma = \pm 1$, $i = 1, 2, 3, \ldots$ is the root number of the RPA secular equation. In the QPNM, one-phonon states are used as a basis. Therefore, *the third stage* are calculations of the one-phonon basis. At this stage all the QPNM constants are fixed.

The QPNM Hamiltonian is transformed to

$$H_{QPNM} = \sum_{q\sigma}\varepsilon_q\, a^+_{q\sigma}\, a_{q\sigma} + H_v + H_{vq}. \tag{7}$$

The first two terms in (7) describe quasiparticles and phonons, and H_{vq} describes quasiparticle-phonon interactions. *The fourth stage* is the allowance for a quasiparticle-phonon interaction. The wave functions of excited states are expressed as a series over the number of phonon operators; in odd nuclei each term is multiplied by a quasiparticle operator. The approximation consists in the cut-off of this series. The cut-off of the series in the number of phonons is the approximation similar to the cut-off of the chain of equations in the HFB approximation. At present, our expansion is limited to two phonons. To elucidate the influence of many-phonon terms of the wave functions on the calculated effects is as difficult as to evaluate the role of chains of equations of the many-body problem neglected in the HFB approximation. It is stated in both the cases that approximate equations describe correctly the properties of nuclear excitations and the terms neglected are partially taken into account by using constants fixed from the experimental data. In the calculations the Pauli principle is taken into account by using the exact commutation relations between the phonon and quasiparticle operators. To calculate the characteristics of highly excited states, the strength function method is used. By using a version of the strength function method developed in Ref. 20

one can directly calculate the reduced transition probabilities, spectroscopic factors, transition densities, cross sections and other nuclear characteristics without solving the relevant secular equations (see Ref. 21).

3. The QPNM Hamiltonian and phonon basis

The starting Hamiltonian for deformed nuclei is taken in the form

$$H = \sum_{\tau}\{\sum_{q\sigma}^{\tau}(E'(q) - \lambda_{\tau})\, a^{+}_{q\sigma}a_{q\sigma} - G_{\tau}\sum_{qq'}^{\tau} a^{+}_{q+}a^{+}_{q-}a_{q'-}a_{q'+} -$$

$$-\frac{1}{2}\sum_{n\lambda\mu\sigma}[\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\mu}+\varrho\varkappa_1^{\lambda\mu})\, M_{\lambda\sigma\mu n}(\tau)\, M_{\lambda-\sigma\mu n}(\varrho\tau) + G^{\lambda\mu} P^{+}_{\lambda\sigma\mu n}(\tau)\, P_{\lambda\sigma\mu n}(\tau)] - \quad (8)$$

$$-\frac{1}{2}\sum_{nLK\sigma}\sum_{\lambda=L,L\pm1}[(\varkappa_0^{\lambda LK}+\varrho\varkappa_1^{\lambda LK})\, S^{\lambda}_{L\sigma Kn}(\tau)\, S^{\lambda}_{L-\sigma Kn}(\varrho\tau) +$$

$$+ G^{\lambda LK}(P^{\lambda}_{L\sigma Kn}(\tau))^{+} P^{\lambda}_{L\sigma Kn}(\tau)]\}.$$

The last sum corresponds to the spin-multipole interaction. After transformations the QPNM Hamiltonian takes the form (7)

$$H_{QPNM} = \sum_{q\sigma}\varepsilon_q\, a^{+}_{q\sigma}a_{q\sigma} + H_v + H_{vq}.$$

Here

$$H_v = H_v^{00} + \sum_{\lambda} H^{\lambda 0}_{Ev} + \sum_{\substack{\lambda\mu\\ \mu\neq 0}} H^{\lambda\mu}_{Ev} + \sum_{LK} H^{LK}_{sv}, \quad (9)$$

$$H_v^{00} = -\sum_{ii'} W^{00}_{ii'}\, Q^{+}_{20i}\, Q_{20i'}, \quad (10)$$

$$W^{00}_{ii'} = \frac{1}{2}\sum_{\tau} G_{\tau}\sum_{qq'}^{\tau}[(u_q^2 - v_q^2)(u_{q'}^2 - v_{q'}^2)\, g^{20i}_{qq}\, g^{20i'}_{q'q'} + w^{20i}_{qq}\, w^{20i'}_{q'q'}], \quad (10')$$

$$H^{\lambda 0}_{Ev} = -\sum_{ii'} W^{\lambda 0}_{ii'}\, Q^{+}_{\lambda 0i}\, Q_{\lambda 0i'}, \quad (11)$$

$$W^{\lambda 0}_{ii'} = \sum_{n\tau}\{(\varkappa_0^{\lambda 0}+\varrho\varkappa_1^{\lambda 0})\, D^{\lambda 0i}_{n\tau}\, D^{\lambda 0i'}_{n\varrho\tau} + G^{\lambda 0}(D^{\lambda 0i}_{qn\tau} D^{\lambda 0i'}_{qn\tau} + D^{\lambda 0i}_{wn\tau} D^{\lambda 0i'}_{wn\tau}) -$$

$$-\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\lambda 0}+\varrho\varkappa_1^{\lambda\lambda 0})\, D^{\lambda\lambda 0i}_{n\tau}\, D^{\lambda\lambda 0i'}_{n\varrho\tau} + G^{\lambda\lambda 0}(D^{\lambda\lambda 0i}_{qn\tau}\, D^{\lambda\lambda 0i'}_{qn\tau} + \quad (11')$$

$$+ D^{\lambda\lambda 0i}_{wn\tau}\, D^{\lambda\lambda 0i'}_{wn\tau})\},$$

$$H^{\lambda\mu}_{Ev} = -\sum_{ii'\sigma} W^{\lambda\mu}_{ii'}\, Q^{+}_{\lambda\mu i\sigma}\, Q_{\lambda\mu i'\sigma}, \quad (12)$$

$$W^{\lambda\mu}_{ii'} = \frac{1}{4}\sum_{n\tau}\{\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\mu}+\varrho\varkappa_1^{\lambda\mu})\, D^{\lambda\mu i}_{n\tau}\, D^{\lambda\mu i}_{n\varrho\tau} + G^{\lambda\mu}(D^{\lambda\mu i}_{qn\tau}\, D^{\lambda\mu i'}_{qn\tau} + D^{\lambda\mu i}_{wn\tau}\, D^{\lambda\mu i'}_{wn\tau}) - \quad (12')$$

$$-\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\lambda\mu}+\varrho\varkappa_1^{\lambda\lambda\mu})\, D^{\lambda\lambda\mu i}_{n\tau}\, D^{\lambda\lambda\mu i'}_{n\varrho\tau} + G^{\lambda\lambda\mu}(D^{\lambda\lambda\mu i}_{gn\tau}\, D^{\lambda\lambda\mu i'}_{gn\tau} + D^{\lambda\lambda\mu i}_{wn\tau}\, D^{\lambda\lambda\mu i'}_{wn\tau})\}.$$

Eqs. (11′) and (12) include spin-multipole terms of the electric type with $\lambda = L$. In most of the cases they are neglected

$$H_{sv}^{LK} = -\sum_{ii'} W_{sii'}^{LK} \; Q_{LKi\sigma}^{+} Q_{LKi'\sigma}, \tag{13}$$

$$W_{sii'}^{LK} = \frac{1}{4}\sum_{n\tau}\sum_{\lambda=L\pm1}\{\sum_{\varrho=\pm1}(\varkappa_0^{\lambda LK} + \varrho\varkappa_1^{\lambda LK}) D_{n\tau}^{\lambda LKi} D_{n\varrho\tau}^{\lambda LKi'} + \tag{13'}$$
$$+ G^{\lambda LK}(D_{qn\tau}^{\lambda LKi} D_{qn\tau}^{\lambda LKi'} + D_{wn\tau}^{\lambda LKi} D_{wn\tau}^{\lambda LKi'})\}.$$

$$H_{vq} = H_{Evq} + H_{svq}, \tag{14}$$

$$H_{Evq} = H_{vq}^{00} + \sum_{\lambda} H_{vq}^{\lambda 0} + \sum_{\substack{\lambda\mu \\ \mu\neq 0}} H_{vq}^{\lambda\mu}, \tag{15}$$

$$H_{vq}^{00} = -\sum_{\tau i} G_\tau \sum_{qq'}^{\tau} (u_q^2 - v_q^2)\, u_{q'} v_{q'} \{(\psi_{qq'}^{20i} Q_{20i}^{+} + \varphi_{qq'}^{20i} Q_{20i}) \sum_{\sigma} a_{q\sigma}^{+} a_{q\sigma} + \text{h.c.}\} \tag{16}$$

$$H_{vq}^{\lambda 0} = -\sum_{\tau i}\sum_{qq'}^{\tau} V_{\tau}^{\lambda 0i}(qq')\{(Q_{\lambda 0i}^{+} + Q_{\lambda 0i})\, B(qq', \mu = 0) + \text{h.c.}\} \tag{17}$$

$$V_{\tau}^{\lambda 0i}(qq') = \frac{1}{2}\sum_{n}\{f_n^{\lambda 0}(qq')\,[\sum_{\varrho=\pm1}(\varkappa_0^{\lambda 0} + \varrho\varkappa_1^{\lambda 0})\, v_{qq'}^{(-)} D_{n\varrho\tau}^{\lambda 0i} - G^{\lambda 0} u_{qq'}^{(+)} D_{qn\tau}^{\lambda 0i} +$$
$$+ \sum_{\varrho=\pm1}(\varkappa_0^{\lambda\lambda 0} + \varrho\varkappa_1^{\lambda\lambda 0})\, v_{qq'}^{(+)} D_{n\varrho\tau}^{\lambda\lambda 0i}]\}, \tag{17'}$$

$$H_{vq}^{\lambda\mu} = -\frac{1}{2}\sum_{ni\tau\sigma}\sum_{qq'}^{\tau}\{[V_{n\tau}^{\lambda\mu i}(qq') f_n^{\lambda\mu}(qq') + V_{n\tau}^{\lambda\lambda\mu i}(qq') f_n^{\lambda\lambda\mu i}(qq')] \cdot$$
$$\cdot\, Q_{\lambda\mu i\sigma}^{+} B(qq'; \mu - \sigma) + \text{h.c.}\}, \tag{18}$$

$$V_{n\tau}^{\lambda\mu i}(qq') = \frac{1}{2}\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\mu} + \varrho\varkappa_1^{\lambda\mu})\, v_{qq'}^{(-)} D_{n\varrho\tau}^{\lambda\mu i} - \frac{1}{2} G^{\lambda\mu} u_{qq'}^{(+)} D_{\varrho n\tau}^{\lambda\mu i}, \tag{18'}$$

$$V_{n\tau}^{\lambda\lambda\mu i}(qq') = \frac{1}{2}\sum_{\varrho=\pm1}(\varkappa_0^{\lambda\lambda\mu} + \varrho\varkappa_1^{\lambda\lambda\mu})\, v_{qq'}^{(+)} D_{n\varrho\tau}^{\lambda\lambda\mu i}, \tag{18''}$$

$$H_{svq} = \sum_{LK} H_{svq}^{LK}, \tag{19}$$

$$H_{svq}^{LK} = -\frac{1}{4}\sum_{\tau i}\sum_{qq'}^{\tau} V_{s\tau}^{LKi}(qq')\,[(Q_{LKi\sigma}^{+} + Q_{LKi-\sigma})\, B(qq'; K\sigma) + \text{h.c.}], \tag{20}$$

$$V_{s\tau}^{LKi}(qq') = \sum_{\lambda=L\pm1}\sum_{n\varrho=\pm1} f_n^{\lambda LK}(qq')\,(\varkappa_0^{\lambda LK} + \varrho\varkappa_1^{\lambda LK})\, v_{qq'}^{(+)} D_{n\varrho\tau}^{\lambda LKi}. \tag{20'}$$

$$\left.\begin{aligned} D_{n\tau}^{\lambda\mu i} &= \sum_{qq'}^{\tau} f_n^{\lambda\mu}(qq')\, u_{qq'}^{(+)} g_{qq'}^{\lambda\mu i}, & D_{n\tau}^{\lambda LKi} &= \sum_{qq'}^{\tau} f_n^{\lambda LK}(qq')\, u_{qq'}^{(-)} g_{qq'}^{LKi}, \\ D_{gn\tau}^{\lambda\mu i} &= \sum_{qq'}^{\tau} f_n^{\lambda\mu}(qq')\, v_{qq'}^{(-)} g_{qq'}^{\lambda\mu i}, & D_{gn\tau}^{\lambda LKi} &= \sum_{qq'}^{\tau} f_n^{\lambda LK}(qq')\, v_{qq'}^{(-)} g_{qq'}^{LKi}, \\ D_{wn\tau}^{\lambda\mu i} &= \sum_{qq'}^{\tau} f_n^{\lambda\mu}(qq')\, v_{qq'}^{(+)} w_{qq'}^{\lambda\mu}, & D_{wn\tau}^{\lambda LKi} &= \sum_{qq'}^{\tau} f_n^{\lambda LK}(qq')\, v_{qq'}^{(+)} w_{qq'}^{LKi}, \end{aligned}\right\} \tag{21}$$

$f_n^{\lambda\mu}(qq')$ and $f_n^{\lambda LK}(qq')$ are the single-particle matrix elements of the multipole and spin-multipole operators; their explicit form is given in Ref. 21

$$g_{qq'}^{\lambda\mu i} = \psi_{qq'}^{\lambda\mu i} + \varphi_{qq'}^{\lambda\mu i}, \qquad w_{qq'}^{\lambda\mu i} = \psi_{qq'}^{\lambda\mu i} - \varphi_{qq'}^{\lambda\mu i},$$

$$B(qq';\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K-K'),\sigma\mu}\, \sigma' a_{q\sigma'}^{+}\, a_{q'\sigma'} \quad \text{or} \quad \sum_{\sigma'} \delta_{\sigma'(K+K'),\sigma\mu}\, \sigma' a_{q\sigma'}^{+}\, a_{q'-\sigma'}, \tag{22}$$

$$\mathscr{B}(qq';\mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K-K'),\sigma\mu}\, \sigma' a_{q\sigma'}^{+}\, a_{q'\sigma'} \quad \text{or} \quad \sum_{\sigma'} \delta_{\sigma'(K+K'),\sigma\mu}\, a_{q\sigma'}^{+}\, a_{q'\sigma'}.$$

Note that for the RPA solutions, in averaging over the phonon vacuum the following relation is fulfilled:

$$\langle\{\sum_{q\sigma} \varepsilon_q a_{q\sigma}^{+} a_{q\sigma} + H_v\}\, Q_{\lambda\mu i\sigma}^{+}\, Q_{\lambda\mu i'\sigma}^{+}\rangle = 0.$$

Therefore, in H_{QPNM} there are no terms proportional to $Q_{\lambda\mu i\sigma}^{+} Q_{\lambda\mu i'\sigma}^{+}$ and $Q_{\lambda\mu i\sigma} Q_{\lambda\mu i\sigma}$.

We get the RPA equations for the energies $\omega_{\lambda\mu i}$ and wave functions of one-phonon states of the electric type

$$Q_{\lambda\mu i\sigma}^{+}\, \Psi_0 = 0, \tag{23}$$

where Ψ_0 is the ground state wave function of a doubly even nucleus which is determined as a phonon vacuum. Normalisation (23) has the form

$$\frac{1}{2} \sum_{qq'} [(\psi_{qq'}^{\lambda\mu i})^2 - (\varphi_{qq'}^{\lambda\mu i})^2] = \frac{1}{2} \sum_{qq'} g_{qq'}^{\lambda\mu i} w_{qq'}^{\lambda\mu i} = 1. \tag{23'}$$

To describe one-phonon states of the electric type with $K^{\pi} \neq 0^{+}$ we use the following part of the Hamiltonian (7)

$$\sum_{q\sigma} \varepsilon_q\, a_{q\sigma}^{+}\, a_{q\sigma} + \sum_{\lambda\mu} H_{Ev}^{\lambda\mu}. \tag{24}$$

Neglecting spin-multiple terms we assume $\varkappa_0^{\lambda\lambda\mu} = \varkappa_1^{\lambda\lambda\mu} = G^{\lambda\lambda\mu} = 0$.

We get the average value of (24) over the state (23) and using the variational principle we get the following equations

$$D_{n\tau}^{\lambda\mu i} = \sum_{n'=1}^{n_{max}} \{\sum_{\varrho=\pm 1} (\varkappa_0^{\lambda\mu} + \varrho\varkappa_1^{\lambda\mu}) X_{nn'}^{\lambda\mu i}(\tau) D_{n'\varrho\tau}^{\lambda\mu i} + G^{\lambda\mu} [X_{nn'}^{\lambda\mu i}(\tau) D_{gn'\tau}^{\lambda\mu i} + X_{nn'}^{\lambda\mu i\omega}(\tau) D_{\omega n'\tau}^{\lambda\mu i}]\}, \tag{25}$$

$$D_{gn\tau}^{\lambda\mu i} = \sum_{n'=1}^{n_{max}} \{\sum_{\varrho=\pm 1} (\varkappa_0^{\lambda\mu} + \varrho\varkappa_1^{\lambda\mu}) X_{nn'}^{\lambda\mu i\varepsilon}(\tau) D_{n'\varrho\tau}^{\lambda\mu i} + G^{\lambda\mu} [X_{nn'}^{\lambda\mu i\varepsilon -}(\tau) D_{gn'\tau}^{\lambda\mu i} + X_{nn'}^{\lambda\mu iv}(\tau) D_{\omega n'\tau}^{\lambda\mu i}]\}, \tag{26}$$

$$D_{\omega n\tau}^{\lambda\mu i} = \sum_{n'=1}^{n_{max}} \{\sum_{\varrho=\pm 1} (\varkappa_0^{\lambda\mu} + \varrho\varkappa_1^{\lambda\mu}) X_{nn'}^{\lambda\mu i\omega +}(\tau) D_{n'\varrho\tau}^{\lambda\mu i} + G^{\lambda\mu} [X_{nn'}^{\lambda\mu iv}(\tau) D_{gn'\tau}^{\lambda\mu i} + X_{nn'}^{\lambda\mu i\varepsilon +}(\tau) D_{\omega n'\tau}^{\lambda\mu i}]\}, \tag{27}$$

where

$$X_{nn'}^{\lambda\mu i}(\tau) = \sum_{qq'}^{\tau} \frac{f_n^{\lambda\mu}(qq') f_{n'}^{\lambda\mu}(qq') (u_{qq'}^{(+)})^2 \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{nn'}^{\lambda\mu i\varepsilon}(\tau) = \sum_{qq'}^{\tau} \frac{f_n^{\lambda\mu}(qq') f_{n'}^{\lambda\mu}(qq') u_{qq'}^{(+)} v_{qq'}^{(-)} \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{nn'}^{\lambda\mu i\omega}(\tau) = \sum_{qq'}^{\tau} \frac{f_n^{\lambda\mu}(qq') f_{n'}^{\lambda\mu}(qq') u_{qq'}^{(+)} v_{qq'}^{(+)} \omega_{\lambda\mu i}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{nn'}^{\lambda\mu i\varepsilon\pm} = \sum_{qq'}^{\tau} \frac{f_n^{\lambda\mu}(qq') f_{n'}^{\lambda\mu}(qq') (v_{qq'}^{(\pm)})^2 \varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$X_{nn'}^{\lambda\mu iv}(\tau) = \sum_{qq'}^{\tau} \frac{f_n^{\lambda\mu}(qq') f_{n'}^{\lambda\mu}(qq') v_{qq'}^{(+)} v_{qq'}^{(-)} \omega_{\lambda\mu i}}{\varepsilon_{qq'}^2 - \omega_{\lambda\mu i}^2},$$

$$\varepsilon_{qq'} = \varepsilon_q + \varepsilon_{q'}.$$

From Eqs. (25), (26) and (27) at $\tau = n$ and $\tau = p$ one can derive the secular equation for the energies of one-phonon states $\omega_{\lambda\mu i}$ as an equality to zero of the determinant of rank $6 \cdot n_{max}$. The use of separable interactions of rank n_{max} increases the rank of the determinant n_{max} times in comparison with simple separable interactions. With spin-multipole interactions with $\lambda = L$ taken into account the rank of the determinant is $12 \cdot n_{max}$. If only multipole p-h interactions are taken into account, the rank of the determinant is $2 \cdot n_{max}$. In calculating the phonon amplitudes $\psi_{qq'}^{\lambda\mu i}$ and $\varphi_{qq'}^{\lambda\mu i}$ we use the condition (23').

The secular equation for describing 0^+ states with a simple separable interaction ($n_{max} = 1$) has been obtained in Ref. 30 as an equality to zero of the determinant of the 10th order. With the inclusion of a separable interaction of rank $n_{max} > 1$, the rank of the determinant for the energies ω_{20i} of one-phonon 0^+ states is $4 + 6 \cdot n_{max}$.

The RPA equations for states of the magnetic type and for charge-exchange states were obtained in a similar way. The RPA equations for charge-exchange multipole and spin-multipole states for simple $n_{max} = 1$ p-h interactions are given in Ref. 36. For p-h and p-p Gamov-Teller interactions they were obtained in Ref. 37.

If the RPA secular equations for states of the electric and magnetic type are solved and the energies $\omega_{\lambda\mu i}$ and phonon amplitudes $\psi^{\lambda\mu i}_{qq'}$ and $\varphi^{\lambda\mu i}_{qq'}$ are found, the Hamiltonian (7) turns out to be unambiguously determined. It has no any free parameters and no unfixed constants.

The total space of p-h and p-p two-quasiparticle states is substitued by the space of one-phonon states. For each value of K^π the number of one-phonon states equals the number of two-quasiparticle states. In deformed nuclei, the phonon basis can be formed of phonons of the electric type and only $K^\pi = 1^+$ states can be described by phonons of the magnetic type.

4. *The QPNM equations for doubly even deformed nuclei*

We give formulae for describing nonrotational states with $K^\pi \neq 0^+$ of doubly even deformed nuclei in the QPNM with p-h and p-p interactions the wave functions of which have one- and two-phonon terms

$$\Psi_\nu (K_0^{\pi_0} \sigma_0) = \{\sum_{i_0} R^\nu_{i_0} Q^+_{g_0\sigma_0} + \sum_{g_1 g_2 \sigma_1 \sigma_2} \frac{(1 + \delta_{g_1 g_2})^{1/2}\, \delta_{\sigma_1\mu_1 + \sigma_2\mu_2, \sigma_0 K_0}}{2\,[1 + \delta_{K_0,0}(1 - \delta_{\mu_1,0})]^{1/2}} \cdot$$

$$\cdot P^\nu_{g_1 g_2} Q^+_{g_1\sigma_1} Q^+_{g_2\sigma_2}\} \Psi_0, \tag{28}$$

where $g = \lambda\mu i$, $\mu_0 \equiv K_0$. Its normalisation condition has the form

$$\sum_{i_0} (R^\nu_{i_0})^2 + \sum_{g_1 \geqslant g_2} (P^\nu_{g_1 g_2})^2\, [1 + \mathcal{K}^{K_0}(g_1 g_2)] = 1. \tag{29}$$

To take the Pauli principle into account in two-phonon terms of the wave function (28) we introduce the function

$$\mathcal{K}^{K_0}(g_2, \lambda_1\mu_1 i' | g_1 g_2) = \frac{1}{1 + \delta_{g_1 g_2}} \sum_{\sigma_1 \sigma_2} \delta_{\sigma_1\mu_1 + \sigma_2\mu_2, \sigma_0 K_0}$$

$$\langle Q_{g_2\sigma_2} [[Q_{\lambda_1\mu_1 i'\sigma_1}, Q^+_{g_1\sigma_1}], Q^+_{g_2\sigma_2}] \rangle, \tag{30}$$

at $i' = i$ it is denoted by $\mathcal{K}^{K_0}(g_1 g_2)$; the explicit form is given in Refs. 21, 22

By using the variational principle we get the following equations that can be used, with allowance made for (29), to find the energies η_γ and functions $R^\gamma_{i_0}$ and $P^\gamma_{g_1 g_2}$:

$$(\omega_{g_0} - \eta_\gamma) R^\gamma_{i_0} - \sum_{g_1 \geqslant g_2} (1 + \delta_{g_1 g_2})^{-1/2} [1 + \delta_{K_0,0}(1 - \delta_{\mu_1,0})]^{-1/2} \cdot$$

$$\cdot P^\gamma_{g_1 g_2} U^{g_0}_{g_1 g_2} [1 + \mathscr{K}^{K_0}(g_1 g_2)] = 0, \tag{31}$$

$$[\omega_{g_1} + \omega_{g_2} + \Delta\omega(g_1 g_2) - \eta_\gamma] P^\gamma_{g_1 g_2} -$$

$$- \sum_{i_0} (1 + \delta_{g_1 g_2})^{-1/2} [1 + \delta_{K_0,0}(1 - \delta_{\mu_1,0})]^{-1/2} R^\gamma_{i_0} U^{g_0}_{g_1 g_2} = 0, \tag{31'}$$

where

$$\Delta\omega(g_1 g_2) = \sum_{i'} \{\mathscr{K}^{K_0}(g_2, \lambda_1 \mu_1 i' | g_1, g_2) W^{\lambda_1 \mu_1}_{i_1 i'} +$$

$$+ \mathscr{K}^{K_0}(\lambda_2 \mu_2 i', g_1 | g_1, g_2) W^{\lambda_2 \mu_2}_{i_2 i'}, \tag{32}$$

$$U^{g_0}_{g_1 g_2} [1 + \mathscr{K}^{K_0}(g_1 g_2)] = - \frac{1}{2} \sum_{\sigma_1 \sigma_2} \delta_{\sigma_1 \mu_1 + \sigma_2 \mu_2, \sigma_0 K_0} \cdot$$

$$\cdot \{\langle Q_{g_2 \sigma_2} H_{vq} Q^+_{g_1 \sigma_1} Q^+_{g_2 \sigma_2} \rangle + \langle Q_{g_2 \sigma_2} Q_{g_1 \sigma_1} H_{vq} Q^+_{g_0 \sigma_0} \rangle\}. \tag{33}$$

The function $W^{\lambda\mu}_{ii'}$ is given by formula (12); the function $U^{g_0}_{g_1 g_2}$ includes the functions $V^{\lambda 0 i}_\tau (qq')$ and $V^{\lambda\mu i}_{n\tau}(qq')$, determined by formulae (17′) and (18′). The explicit form of $U^{g_0}_{g_1 g_2}$ in the case of a simple ($n_{max} = 1$) interaction is given in Ref. 21.

Finding $P^\gamma_{g_1 g_2}$ from Eq. (31′) and substituting it into (31) we get a secular equation of the form

$$\det \left\| (\omega_{g_0} - \eta_\gamma) \delta_{i_0 i_0'} - \sum_{g_1 \geqslant g_2} \frac{1 + \mathscr{K}^{K_0}(g_1 g_2)}{(1 + \delta_{g_1 g_2}) [1 + \delta_{K_0 0}(1 - \delta_{\mu_1,0})]} \cdot \right.$$

$$\left. \cdot \frac{U^{\lambda_0 \mu_0 i_0}_{g_1 g_2} U^{\lambda_0 \mu_0 i_0'}_{g_1 g_2}}{\omega_{g_1} + \omega_{g_2} + \Delta\omega(g_1 g_2) - \eta_\gamma} \right\| = 0. \tag{34}$$

The rank of this determinant equals the number of one-phonon terms in the wave function (28). Inclusion of the Pauli principle in the two-phonon terms (28) generates in (34) the factor $1 + \mathscr{K}^{K_0}(g_1 g_2)$ and the shift $\Delta\omega(g_1 g_2)$ of the two-phonon pole. Equations for 0^+ states have the same form.

The form of Eqs. (31), (31′) and (34) is the same in the cases of $n_{max} > 1$ and $n_{max} = 1$, if p-h and p-p or only p-h interactions are taken into account. It is essential that a transition from simple with $n_{max} = 1$ to finite rank with $n_{max} > 1$ separable interactions does not increase the rank of the determinant (34). It complicates the functions $U^{g_0}_{g_1g_2}$ and $\Delta\omega\,(g_1g_2)$. This complication of the functions turns out to be inessential in computer calculations.

It should be noted that the inclusion of finite rank separable interactions does not lead to a somewhat essential complication of equations for calculating the fragmentation of quasiparticle and collective motions. This means that the QPNM may further serve as a basis for calculations of many characteristics of deformed nuclei.

5. *Vibrational states in ^{168}Er and their description of the QPNM and IBM*

The efficiency of describing nonrotational states in the QPNM will be exemplified by ^{168}Er. Let us compare the description of nonrotational states within the QPNM and IBM. The choice of ^{168}Er is caused by the rich experimental data[38–42] and numerous calculations[14,18,22–26,28,34,35,41,43–46]. The comparison of the calculated results for nonrotational states in ^{168}Er with experimental data is given in the table. The experimental data for the (n, γ) reactions are taken from Ref. 38; and for the $B\,(E2)$, $B\,(E3)$ and $B\,(E4)$ values, from Ref. 41. The contribution of two-quasiparticle components to the wave function normalisation, extracted from the (d, p), (t, d) and $(\vec{t}, \alpha)$ reactions, is taken from Ref. 39. In some cases, the contribution (in per cent) of two-quasiparticle components to the wave function has been determined experimentally[39]; in other cases, a two-quasiparticle component is pointed out through which a reaction proceeds. The results of calculations within the QPNM are given in Ref. 35, the results of calculations of quadrupole states in the sd IBM are taken from Ref 41; of octupole states within the sdf IBM, from Ref. 26 and of 0^+_2 and 0^+_3 states and hexadecapole states in the sdg IBM, from Ref. 25.

The calculations in the QPNM have been performed in Ref. 35 with single-particle wave functions of the Woods-Saxon potential for the zone $A = 165$ with monopole and quadrupole pairing, isoscalar and isovector multipole p-h and p-p interactions without taking the Coriolis coupling into account. In the table the $B\,(E\lambda)$ values are given in the single-particle units. The calculated structure is given as a contribution (in per cent) of one-phonon $\lambda\mu i$ and two-phonon $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ components to the wave function normalisation. The Pauli principle is taken into account in the contribution of two-phonon components. Then, we give (in per cent) the largest two-quasineutron nn and two-quasiproton pp components of the wave functions of one-phonon states $\lambda\mu i$.

Now we proceed to the discussion of the results given in the table. In the sd IBM, a good description has been obtained[18] of the rotational bands constructed on the ground, β- and γ-vibrational states and $E2$ transitions between them. This is by far the best result of the IBM.

In ^{168}Er five states with $K^\pi = 2^+$ were observed. The 2^+_1 and 2^+_4 states are strongly excited in the $(\vec{t}, \alpha)$ reaction. According to the calculations, each wave function of the first five $K^\pi = 2^+$ states has the dominating one-phonon component. The first 2^+_1 state is strongly collective and the others are weakly collective. The calculations are in reasonable agreement with experimental data on the structure of the first five 2^+ states. Thus, the largest part of the configuration pp $411\downarrow$ +

TABLE 1.

	Experiment		Calc. in IBM	Calc. in QPNM	
K^{π}_{ν}	η_ν MeV $[B(E\lambda)_{s.p.u}]$	Structure, %	η_ν MeV $[B(E\lambda)_{s.p.u.}]$	η_8, MeV $[B(E\lambda)_{s.p.u}]$	Structure, %
1	2	3	4	5	6
2^+_1	0.821	$\vec{(t, a)}$: pp 413↓—411↓ 50	0.821	0.8	221:96
	[4.7]	pp 411↑+411↓ 37	[4.6]	[4.6]	221: pp 413↓—411↓ 26
		log ft = 5.2			pp 411↑+411↓ 30
					nn 523↓—521↓ 18
					nn 521↑+521↓ 23
4^-_1	1.094	(dp): nn 633↑+521↓ 70	—	1.0	541:99
	—	$\vec{(ta)}$: pp 411↓+523↑ 25	—	—	541: nn 633↑+521↓ 80
					pp 411↓+523↑ 18
0^+_1	1.217	$\vec{(ta)}$: nn 633↑—633↑	1.217	1.4	201:77
	[<0.1]	60—80	[0.05]	[0.3]	201: nn 633↑—633↑ 30
					pp 411↓—411↓ 6
1^-_1	1.358	(dt): nn 633↑—512↑ 80	1.358	1.4	311:98
	[3.92]	(dp):	[5.5]	[4.6]	311: nn 633↑—512↑ 72
					nn 633↑—523↓ 4
0^+_2	1.422	(td): nn 633↑—633↑ 20	1.422	1.6	202:78 201:10
	—		—	[0.1]	202: nn 633↑—633↑ 20
					pp 411↓—411↓ 2
3^-_1	1.542	(dp): nn 633↑—521↓ 90	—	1.6	331:98
	[0.25]	$\vec{(ta)}$: pp 523↑—411↓ 4	—	[0.14]	331: nn 633↑—521↓ 95
					pp 523↑—411↓ 2
2^-_1	1.569		1.57	1.5	321:94 {201, 321}: 3
	[4.94]		[8.0]	[4.6]	321: nn 633↑—521↑ 25
					pp 523↑—411↑ 29
3^+_1	1.653	(aa′): is large for 4^+3_1	1.75	1.5	431:99
	—		[50.8]	[0.8]	431: nn 512↑+521↓ 98
6^-_1	1.773	(dp):			
		(td): nn 633↑+512↑ 90	—	1.8	nn 633↑+512↑ 100
			—	—	
0^-_1	1.786		1.786	1.9	301:98
	[1.96]		[4.6]	3.0	301: nn 642↑—512↑ 25
					nn 514↓—633↑ 7
3^-_2	1.828	(dp): nn 633↑—521↓ 10	—	2.1	332:75 333:20
	[0.60]		—	[0.60]	332: nn 521↓+642↑ 80
					nn 633↑—521↓ 2
0^+_3	1.833	$\vec{(ta)}$: pp 411↓—411↓ 25	1.8	1.9	203:80 204:9
	—		—	[0.01]	203: nn 633↑—633↑ 10
					pp 411↓—411↓ 28
2^+_2	1.848	log ft = 6.1	—	1.8	222:98
	—		—	[0.01]	222: nn 512↑—521↓ 97
4^-_2	1.905	$\vec{(ta)}$: pp 411↓+523↑~60	—	1.6	541:99
	—	(dp): nn 633↑+521↓~30	—	—	541: pp 411↓+523↑ 80
					nn 633↑+521↓ 18
2^+_3	1.930	log ft = 6.2	—	1.9	223:94
	—		—	[0.2]	223: nn 523↓—521↓ 3
					pp 411↑+411↓ 13
					nn 521↑+521↓ 60

1	2	3	4	5	6
1^-_2	1.936 —	$\overrightarrow{(t\alpha)}$: is small	2.3 [1.1]	1.9 [0.35]	312:96 {221, 311}: 1 312: nn 633↑—523↓ 85
3^-_3	1.99 [0.42]	$\overrightarrow{(t\alpha)}$: pp 523↑—411↓ 75 (dp): nn 633↑—521↓ 10	— —	2.2 [0.3]	333:72 332:22 333: pp 523↑—411↓ 76 pp 514↑—512↓ 9 nn 633↑—521↓ 1
4^+_1	2.055 [0.6]		2.03 [0.6]	2.1 [2.0]	441:88 {201, 441}: 4 441: nn 514↓+521↓ 15 nn 512↑+512↓ 14 pp 523↑+541↓ 6
4^-_3	2.059	(dp): } nn 633↑+510↑ 33 (td): }	— —	— —	
7^-_1	2.122 —	(dp): nn 633↑+541↑ 95	— —	2.2 —	pp 523↑+404↓ 100
1^+_1	2.133 —		— —	— —	
3^+_2	2.186 —		— —	2.0 [0.003]	432:98 432: nn 523↓+521↓ 99
2^+_4	2.193 —	$\overrightarrow{(t\alpha)}$: pp 411↑+411↓ 20—30 log ft = 4.8	— —	2.2 [0.06]	224:98 224: nn 521↑+521↓ 4 pp 411↑+411↓ 28 nn 523↓—521↑ 60
2^-_2	2.230 —		2.6 [0.3]	2.1 [0.2]	322:96
4^+_2	2.238 —		— —	2.5 [0.1]	442:53 443:38 442: nn 514↓+521↓ 62 512↑+512↓ 32
3^-_4	2.262 [4.68]		2.3 [5.8]	2.4 [2.0]	334:91 333:1{221, 311}: 2 334: pp 514↑—411↑ 51 624↑—521↑ 12
5^+_1	2.267 —		— —	2.2 —	nn 523↑+512↓ 100
5^+_2	2.298 —		— —	— —	
3^-_5	2.323 [1.53]		— —	— —	
5^-_1	2.365 —		— —	— —	
2^+_5	2.425 —	$\overrightarrow{(t\alpha)}$: pp 411↑+411↓ log ft = 4.6	— —	2.5 [0.2]	225:97 225: nn 633↑—651↑ 36 nn 521↑+521↓ 6 pp 411↑+411↓ 15 nn 523↓—521↓ 15

Nonrotational states in ^{168}Er.

$+$ 411↑ is concentrated in the 2^+_1 and 2^+_4 states; and a small part, in the 2^+_3 and 2^+_5 states. In the β-decay of ^{168}Ho $K^\pi = 3^-$ p 523↑ — n521↓ exhibits the configuration nn 523↓ — 521↓ which is observed in all the five 2^+ states; its largest part is concentrated in the 2^+_4 state. According to Ref. 42 the γ-vibrational state 2^+_1 contains a large admixture of the hexadecapole component $\lambda\mu = 42$, which manifests itself in large values of $B(E4)$ in excitation of the level with $I^\pi K_\gamma = 4^+2_1$. In Ref. 46, in the QPNM such a large admixture can be described only if the multipole forces with $\lambda\mu = 22$ and 42 are taken into account simultaneously. A correct $B(E4)$ value has been obtained in Ref. 25 in the sdg IBM.

The 4^-_1 and 4^-_2 states with energies 1.094 and 1.905 MeV in ^{168}Er have earlier been treated[13,14] as two-quasiparticle ones. According to the experimental data[39] these states are not pure two quasiparticle and have the structure given in the table. In Ref. 32 there were introduced multipole forces $\lambda\mu = 54$ with the isoscalar constant similar to the constants used for describing octupole and hexadecapole one-phonon states. As a resuts, the structure of these states has been obtained which is in agreement with the experimental data. Such a mixing indicates the necessity of studying the role of interactions with high multipolarity $\lambda\mu$.

According to the experimental data the first octupole 0^-_1, 1^-_1 and 2^-_1 states in ^{168}Er are collective. There are six $K^\pi = 3^-$ states that behave unusually. The fourth 3^-_4 state is the most collective one; the strength concentrated in it is 3.5 times larger than in the first three states. The 3^-_1, 3^-_2 and 3^-_3 states are also collective as their $B(E3)$ values are comparatively large and the wave function of the 3^-_3 state contains components pp 523↑ — 411↓ and nn 633↑ — 521↓.

In the calculations of the octupole states, in the sdf IBM[41] the $B(E3)$ value was normalised to the 3^-1_1 state. As a result, for the $K^\pi_\gamma = 3^-_1$ state the calculated $B(E3)$ value turned out to be about 500 times larger than the experimental one. One can hardly describe within the IBM concentration of most of the $E\lambda$ strength not on the first K^π_1 state. This is confirmed by calculations[26] of octupole states in the sdf IBM. In these calculations, the first three $K^\pi = 3^-$ states are rejected and the fourth 3^-_4 state is thought to be the first one in which most of the $E3$ strength is concentrated. However, the rejected first three states 3^-_1, 3^-_2 and 3^-_3 are not two-quasiparticle ones. According to the calculations[35], the $B(E3)$ values for 3^-_1, 3^-_2 and 3^-_3 (30—60) times exceed the values of the corresponding two-quasiparticle states. Why these states are not attributed to collective ones?

The energies, $B(E3)$ values and the structure of octupole states calculated in the QPNM[35] are in reasonable agreement with experimental data, all having dominating one-phonon components. The fourth 3^-_4 state is the most collective of all the $K^\pi = 3^-$ states. The amount of the $E3$ strength in the 3^-_4 states is twice larger than in the 3^-_1, 3^-_2 and 3^-_3 states. The total octupole $E3$ strength in the states of ^{168}Er with an energy up to 2.5 MeV is equal, according to the experimental data[41], to 20 s. p. u. and according to the calculations[35], to 20.3 s. p. u.

According to the experimental data[47], there are collective hexadecapole states with $K^\pi = 3^+$ in the Er, Yb and Hf isotopes and with $K^\pi = 4^+$ in the Hf, W and Os isotopes. The calculations[46] within the QPNM correctly reproduce these experimental data. According to the calculations[35] within the QPNM, in ^{168}Er the first 3^+_1 and 4^+_1 states are collective and the second 3^+_2 and 4^+_2 states are weakly collective. For the 4^+_1 state the calculated $B(E4)$ value exceeds the experimental one. In the sdg IBM[25], a correct $B(E4)$ value for the 4^+_1 state and

$B(E4) = 50.8$ s. p. u. for the 4^+3_1 state with energy 1.736 MeV are obtained. By the QPNM calculations[35], $B(E4) = 0.8$ s. p. u. for the 4^+3_1 state. Hence, it is seen that the difference between the $B(E4)$ values in the IBM snd QPNM is 100 times. It would be useful to check such a large difference experimentally.

In ^{168}Er there are two-quasiparticle states that are correctly described in the QPNM. It is seen from the table that more experimental data on nonrotational states became available during the last decade. Not only the number of levels has increased but, which is more important, their quantitative characteristics. There appeared possibilities for check and comparison of different models.

The sdg IBM provides a correct description of the energies, $B(E2)$ and $B(E4)$ value and relative intensities of (p, t) reactions for the 0^+_1, 0^+_2, 2^+_1 and 4^+_1 states and probably for the 3^+_1, 2^+_2 and 0^+_3 states. It is doubtful whether other states with positive parity are correctly described. The sdf IBM provides the description of the energies and $B(E3)$ values of the 0^-_1, 1^-_1, 2^-_1 and 3^-_4 states. The problem of describing other states with negative parity is still open.

Within the QPNM one can correctly describe the energies, $B(E\lambda)$ values and the structure of almost all the states. The advantage of the QPNM in describing uniquely strongly and weakly collective and two-quasiparticle states is demonstrated in the table.

6. *The Eλ-strength distribution*

The standard distribution of the $E2$ and $E3$ strength is the following: β- and γ-vibrational and first octupole states are collective. Above the excitation energy there are no collective states up to the isoscalar giant quadrupole and low-lying and high-lying octupole resonances. Such a standard distribution of the quadrupole and octupole strength underlies all phenomenological models of describing low-lying vibrational states including the IBM.

Based on the experimental data[41,48] on inelastic scattering of α-particles and the calculations[35] a qualitatively new statement has been made within the QPNM about the difference of the $E\lambda$-strength distribution from the standard one. There are cases when the most collective is not the first but a higher-lying state with a given value of K^π, or the largest part of the $E\lambda$-strength is concentrated not on the first state but in the energy interval 2—3 MeV.

The $E2$ strength distribution in ^{172}Yb, differs from the standard one. This is exhibited by the fact that apart from the first 2^+_1, the second 2^+_2 state is collective the according to Ref. 48 the $E2$ strength in the energy interval 2—3 MeV is 1.7 times larger than in the first 2^+_1 state. The distribution of the $E2$ strength is qualitatively correctly described in the QPNM[35] and is not described in the IBM.

The $E3$ strength-distribution in ^{168}Er, ^{172}Yb and ^{178}Hf differs from the standard one. The concentration of the $E3$ strength on the fourth $K^\pi_\nu = 3^-_4$ but not on the first state in ^{168}Er has already been mentioned. According to the calculations[35], in ^{172}Yb the $E3$ strength in the interval 2—3 MeV somewhat exceeds the $E3$ strength for the 0^-_1, 1^-_1, 2^-_1 and 3^-_1 states lying below 1 MeV. In ^{178}Hf the $E3$ strength for the $K^\pi = 1^-$ states lying in the interval 2—3 MeV is by an order of magnitude larger than the $E3$ strength of the first two 1^-_1 and 1^-_2 states.

It is interesting to check these specific features of the $E3$ strength distribution experimentally.

The distribution of the $E\lambda$-strength when most of the strength of low-lying states belongs not to the first states can hardly be described, if at all, within the sdg IBM and sdf IBM.

7. *On energy centroids of two-phonon collective states*

Based on the calculations in the QPNM it has been stated[22] that there are no collective two-phonon states in deformed nuclei. Two-phonon is the state in which the contribution of the two-phonon component to the wave function normalisation exceeds 50%. The existence of two-phonon states is still being discussed in papers for instance, in Refs. 23—25, 35, 49. Up to now there are no reliable experimental data on collective two-phonon states in deformed nuclei.

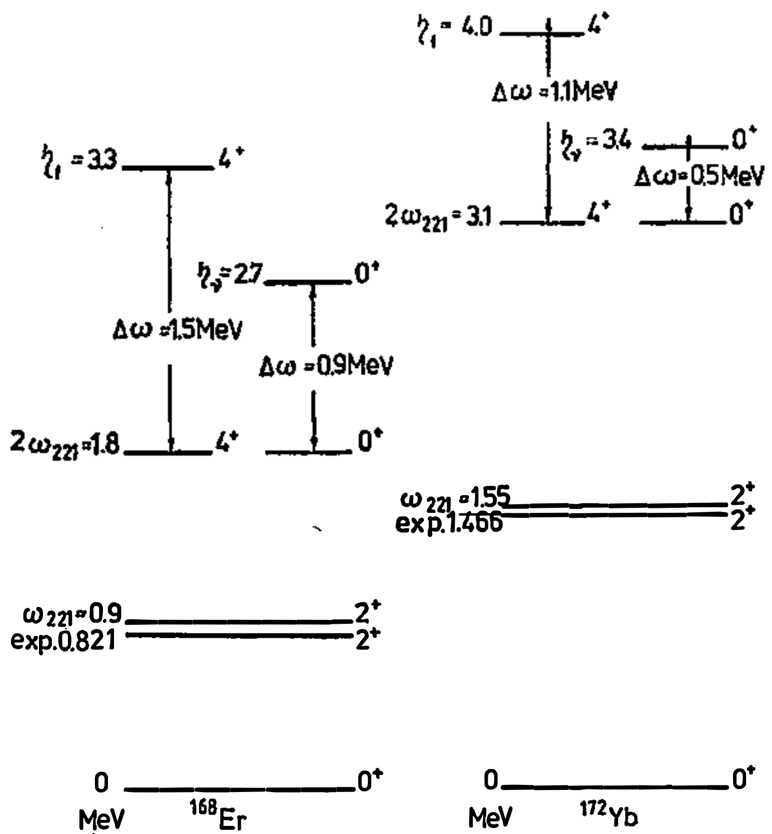

Fig. 1. Energy centroids of the two-phonon γ-vibrational states in ^{168}Er and ^{172}Yb.

Increase in the energy centroids of the two-phonon states $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ with respect to the sum of energies of vibrational states with the dominating one-phonon components of their wave functions is caused by two reasons. The first is the anharmonicity of vibrations since the energies of two-phonon states are larger than the sum of energies of two one-phonon states. The second is the shift of the two-phonon pole $\Delta\omega\,(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ due to the allowance for the Pauli principle in the two-phonon terms of the wave function (28). These reasons for the increase in energy centroids are demonstated in Fig. 1 for γ-vibrational two-phonon states with $K^\pi = 4^+$ and 0^+ in ^{168}Er and ^{172}Yb calculated in the QPNM[35]. The calculations[35] with the p-h and p-p interactions included provided the energies and $B\,(E2)$-values for the 2_1^+ and first octupole states which are in agreement with experimental data. The inclusion of p-p interactions led to decrease in collectivity of the 2_1^+ and first octupole states and thus to decrease in the shifts $\Delta\omega\,(g_1 g_2)$ in comparison with the calculations[22] in which p-p interactions were not taken into account. In ^{168}Er there is the most low-lying collective γ-vibrational state, and therefore, for 4^+ {221, 221} the shift $\Delta\omega$ is maximal and equals 1.5 MeV. In ^{172}Yb the γ-vibrational state is less collective and the shift $\Delta\omega$ (221, 221) equals 1.1 MeV for $K^\pi = 4^+$ and 0.5 MeV for $K^\pi = 0^+$ states. In all the calculated nuclei the shifts $\Delta\omega$ do not exceeds 1.5 MeV; they take the values from 0.1 up to 1.5 MeV.

The energy centroid of the 0^+ {221, 221} state in ^{168}Er equals[35] 2.7 MeV; according to the calculations in Ref. 23 it equals 2.9 MeV and in Ref. 24 it equals 2.8 MeV. The energy centroids of the 0^+ {221, 221} states calculated in the QPNM are approximately the same as those calculated in Ref. 24 by the multiphonon method. The discrepancy takes place for the 4^+ {221, 221} states. It is unclear from the discrepancy what is more important: a large number of degrees of freedom as a large number of one-phonon and two-phonon states in the QPNM or one degree of freedom γ-vibrational phonon and the wave function with multiphonon configurations in the multiphonon method. The multiphonon terms of the wave function do not usually lead to a strong shift of the root from the corresponding pole. If the root η_ν is very much lowered with respect to the pole $\omega_{g_1} + \omega_{g_2} + \Delta\omega\,(g_1 g_2)$, this two-phonon state turns out to be strongly fragmented. Therefore, one can hardly expect additional large shifts of energy centroids of the two-phonon states without strong fragmentation when multiphonon terms are added in the wave function (28).

If the state is very collective and its energy is not large, the shift of $\Delta\omega$ is large. If the energy of a state with the dominating one-phonon component is not small and the collectivity is not strong, the shift of $\Delta\omega$ is small. In all the cases the energy centroids of the lowest collective two-phonon states are equal to 2.5—4.0 MeV. At these energies the strength of two-phonon states should be distributed over many levels. Therefore, the statement that collective two-phonon states cannot exist in well deformed nuclei made earlier in Ref. 22 is still valid.

According to the phenomenological models expounded in Refs. 6—8, various modifications of the IBM and the methods [23,24], deformed nuclei should contain collective two-phonon states. In recent years, much attention has been concentrated on the level $K^\pi = 4^+$ with the energy 2.055 MeV in ^{168}Er. According to the calculations in the QPNM, this level has the dominating hexadecapole one-phonon component, and according to Refs. 18, 23—25, 43, 44 it is the two-phonon one with a large anharmonicity. A large anharmonicity of the two-phonon state is described from quite the opposite positions in Ref. 24 and Ref. 25. In Ref. 25 the γ-vibrational state has the dominating one-boson component and the anharmonicity is due to the interaction between ideal bosons. In Ref. 24 the Tamm-Dancoff phonons are used and the anharmonicity is due to the difference between the Tamm-Dancoff phonons and ideal bosons.

The absence of two-phonon collective states in deformed nuclei will require fundamental modification of the IBM and restrict the region of applicability of phenomenological models to the first quadrupole and octupole states. Therefore, the existence or nonexistence of collective two-phonon states in deformed nuclei is the problem of basic scientific interest to be solved experimentally.

8. Conclusion

For a deeper insight into the structure of deformed nuclei and overall experimental study of excited states with an energy of 2—3 MeV is of great interest. First, it would be desirable to perform detailed measurements as in ^{168}Er for many other deformed nuclei. We hope that states with energies of 2—3 MeV will be studied experimentally by many-detector systems at the new generation of accelerators with large energy resolution.

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OPIS VIBRACIONIH STANJA DEFORMIRANIH JEZGARA U OKVIRU KVAZIČESTIČNO-FONONSKOG MODELA JEZGRE

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Dobivene su osnovne jednadžbe kvazičestično-fononskog modela jezgre za različite interakcije između kvazičestica. Taj model s p-h i p-p interakcijama dovoljno dobro opisuje niskoležeća stanja deformirane jezgre, što je pokazano na primjeru ^{168}Er.