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A SCHEMATIC MODEL FOR GRAZING HEAVY ION REACTIONS

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A set of approximations are suggested through which it is possible to estimate in a closed form the distribution in energy, angular momenta and mass and charge numbers in grazing heavy ion reactions. Explicit results are calculated for the total energy loss and it is shown that this may amount to hundred's of MeV for very heavy systems.

1. Introduction

Heavy ion reaction offer unique possibilities to study nuclear structure by a probe that is well localized. The localization is due to the short wavelength in the relative motion which allows for a description where the nuclei move on well-defined trajectories. The experiments can thus be analyzed to isolate e. g. grazing collisions where the interplay between single particle, collective surface modes and pairing modes can be studied in the surface region. The reactions in this region are determined partly by the magnitude of the formfactors which (except for Coulomb excitation) fall of exponentially outside the sum of the two nuclear radii and partly by the absorption i. e. the transitions to more complicated states. In the semiclassical approximation one can calculate the probability for the various reactions in perturbation theory analytically and the results in a simple way reveal the observed strong dependence on the Q-values of the reaction and on the angular momentum transfer. Besides giving a physical insight the results can be used also to obtain differential cross sections which are in quantitative agreement with experiments and with detailed quantal descriptions[1]. It is also possible in this way to calculate the absorption i. e. the imaginary part of the optical potential which for low bombarding energies is mainly governed by the transfer reactions[3].

Recently[2] one has attempted to perform realistic coupled channel calculations based on all the important shell model states in the two colliding nuclei. These calculations have given strong support to earlier evaluation of the absorptive potential and the simple treatment of grazing reactions[3]. They contain, however, also a wealth of information about the energy, charge, mass and angular momentum distribution in the two nuclei. The collisions can be followed down to distances of closest approach between the two nuclei which are of the order of the radius of the Coulomb barrier, where the mean energy loss were of the order of magnitude of ten's of MeV in the examples which were studied.

To obtain a simple understanding of such data, there is a need for a simpler picture. The comparison with macroscopic models based on nucleon flow between Fermi gases have not been encouraging[8] mainly because the assumption that the nucleous are transferred without change in velocity, i. e. at optimum Q-value, is not valid.

In a present paper a schematic model for the reaction is attempted which incorporates the Q-value effects and which takes coupled channel effects into account in an average fashion. The fundamental ingredients of the theory are given in section 2 while section 3 contain some applications. In section 4 some generalizations are discussed, which give possible lines for future research.

2. *Model*

In a grazing collision between two spherical heavy ions very many channels are open for transfer. There are six main types of channel: Stripping and pick-up of neutrons or protons and excitation of surface oscillations in target and projectile and each of these elementary reactions can take place between several single particle levels or collective modes. Although there are interesting couplings between the elementary modes the couplings are rather unimportant for general questions of total energy and angular momentum loss in a grazing reaction, because they only mean a redistribution of the strength associated with the transition. For the present purpose we may assume that all the channels are independent.

The solution of the semiclassical coupled equations is especially simple for inelastic processes leading to the excitation of harmonic modes. For a certain mode, j, the probability of exciting it with n_j quanta is given by[4]

$$p_{n_j} = \frac{(p_j^{(1)})^{n_j}}{n_j!} e^{-p_j^{(1)}}, \tag{1}$$

where $p_j^{(1)}$ is equal to the excitation probability in first order perturbation theory i. e.

$$p_j^{(1)} = \left| \frac{1}{i\hbar} \int_{-\infty}^{\infty} f_j(\vec{r})\, e^{\frac{i}{\hbar}\Delta Et}\, dt \right|^2. \tag{2}$$

This probability , which may exceed unity depends on the formfactor f_j for the excitation of the mode as well as on the excitation energy $\Delta E = \hbar\omega_j$.

We shall assume that the expression (1) also holds for the transfer reactions. This can of course not be true because of the Pauli principle. A transfer reaction from a given orbital i in projectile to a given orbital k in target can only occur once. However if $p_{ik}^{(1)}$ is small the probability that the transition takes place ($n_{ik} = 1$) is given by the first order expression (2) and the probability, that the transition does not take place ($n_{ik} = 0$) is $1 - p_{ik}^{(1)}$. This is in agreement with the expression (1) to first order for small $p_{ik}^{(1)}$. For a grazing collision it is in fact rather typical that the transfer between specific orbitals are taking place with small probabilities. Since there are very many such transitions (typically 100) the two colliding nuclei may still suffer a very substantial energy loss.

With the assumption of independence of all the transitions we find the following expression for the probability after the collision that n_{ik} transitions of type (ik) have occurred:

$$p\{n_{ik}\} = \prod_{ik} \frac{(p_{ik}^{(1)})^{n_{ik}}}{n_{ik}!} e^{-p_{ik}^{(1)}} . \tag{3}$$

Rather accurate analytic expressions are available for the quantities p_{ik} in grazing collisions where one may assume the formfactors to have an exponential shape. Neglecting Coulomb excitation which gives a rather small average energy loss one may use a parabolic approximation for the trajectory with

$$r(t) = r_0 + \frac{1}{2}\ddot{r}_0 t^2$$
$$\varphi(t) = \dot{\varphi}_0 t, \tag{4}$$

where r and φ are the polar coordinates in a coordinate system with z-axis along the angular momentum of relative motion. In the schematic model we shall use the expression[3,5]

$$p_{ik}^{(1)} = p_{ik} \exp\left\{-\left(\frac{Q_{ik} - Q_0}{\hbar\omega_0}\right)^2\right\}. \tag{5}$$

The coefficient p_{ik} is proportional to the square of the product of the collision time $\tau = 1/\sqrt{\varkappa\ddot{r}_0}$ and the the formfactor f, where $\varkappa$ is the exponential slope of the radial formfactor

$$p_{ik} \approx \frac{2\pi}{\hbar^2 \varkappa \ddot{r}_0} (f(r_0))^2 \sim e^{-2\varkappa r_0}, \tag{6}$$

which is related to the binding energy of the particle. The energy dependence (cf. Eq. (2)) i. e. the dependence on the Q-value, Q_{ik} of the transition is contained in the adiabatic cut-off function, which is a Gaussian centered around an optimum Q-value, Q_0, and with a width $\hbar\omega_0$ determined from the collision time τ

$$\omega_0 = 1/\tau = \sqrt{\varkappa\ddot{r}_0}. \tag{7}$$

We have here as well as in the coefficient p_{ik} neglected the dependence on the transferred angular momenta. It may be included as discussed in section 4 below.

We shall thus assume that the optimum Q-value is given by

$$Q_0 = \begin{cases} \dfrac{(Z_a - Z_A)\, e^2}{r_0} & \text{for proton stripping} \\ \dfrac{(Z_A - Z_a)\, e^2}{r_0} & \text{for proton pick-up} \\ 0 & \text{for neutron transfer} \end{cases} \tag{8}$$

where Z_a and Z_A are the charge numbers of projectile and target, respectively. In the following we shall only consider transfer reactions and study the total energy loss. First we consider only nucleon stripping. In this case the total energy loss to intrinsic excitation is

$$E_{\{n_{ik}\}} = \sum_{ik} n_{ik}^{NS} (\varepsilon_k - \varepsilon_i' - (\varepsilon_F - \varepsilon_F')), \tag{9}$$

where we specified the orbital i from which the nucleon is taken in the projectile and the orbital k in the target to which it is transferred. The corresponding single particle energies are ε'. and ε_k while ε_F' and ε_F denote the Fermi levels.

The distribution function in the total energy loss is (with $\bar{\varepsilon} = \varepsilon - \varepsilon_F$)

$$P(E) = \sum\nolimits_{\{n_{ik}\}} \delta (E - \sum n_{ik}^{NS} (\bar{\varepsilon}_k - \bar{\varepsilon}_i'))$$

$$\times \prod\nolimits_{ik} \frac{(p_{ik}^{NS})^{n_{ik}^{NS}}}{n_i^N} \mathrm{e}^{-p_{ik}^{NS}}, \tag{10}$$

where the summation is taken over all combinations of the numbers n_{ik}. In order to evaluate $P(E)$ we introduce the characteristic function

$$Z(\beta) = \int_{-\infty}^{\infty} \mathrm{d}E\, \mathrm{e}^{\mathrm{i}E\beta} P(E) = \prod\nolimits_{ik} \sum\nolimits_{n_{ik}^{NS}} \frac{(p_{ik}^{NS}\, \mathrm{e}^{\mathrm{i}(\bar{\varepsilon}_k - \bar{\varepsilon}_i')})^{n_{ik}^{NS}}}{n_{ik}^{NS}!} \mathrm{e}^{-p_{ik}^{(NS)}} = \tag{11}$$

$$= \exp \{\sum\nolimits_{ik} p_{ik}^{NS} (\mathrm{e}^{\mathrm{i}(\bar{\varepsilon}_k - \bar{\varepsilon}_i')\beta} - 1)\}.$$

This quantity can be evaluated directly from a knowledge of the single particle levels and the first order expression for the nucleon stripping probabilities p_{ik}^{NS}. In order to obtain simple expressions we shall instead assume that the single particle strength in projectile and target are spread uniformly with a constant level density around the Fermi surfaces. The increase in density with energy due to the degeneracy we only take into account by letting the average proton or neutron level densities ϱ_a and ϱ_A depend on the mass numbers We thus find

$$\ln Z(\beta) = \int_{-\infty}^{\varepsilon_F'} \varrho_a \, d\varepsilon' \int_{\varepsilon_F}^{\infty} \varrho_A d\varepsilon \, (e^{i(\bar{\varepsilon}-\bar{\varepsilon}')\beta} - 1) \, p_0^{NS} \exp\left(-\left(\frac{\varepsilon' - \varepsilon - Q_0^{NS}}{\hbar\omega_0}\right)^2\right) \quad (12)$$

where we used that the Q-value is $Q_{ik}^{NS} = -\varepsilon_k + \varepsilon_i' = \varepsilon' - \varepsilon$. Because of the adiabatic cut-off function we may extend the integrations to infinity. We have introduced an average first order stripping probability p_0^{NS} and assuming also constant level densities we find for proton (π) stripping

$$\ln Z(\beta) = \varrho_a^\pi \varrho_A^\pi p_0^\pi \int_0^\infty d\delta \int_{-\delta/2}^{\delta/2} d\sigma \, e^{-\left(\frac{\delta+Q}{\hbar\omega_0}\right)^2} (e^{+i\delta\beta} - 1) =$$

$$= \frac{\sqrt{\pi}}{2} (\hbar\omega_0)^2 \varrho_a^\pi \varrho_A^\pi p_0^\pi \left[\bar{Q}_P \operatorname{erfc}(\bar{Q}_P)\right. \quad (13)$$

$$\left. - \left(\bar{Q}_P - i\frac{\bar{\beta}}{2}\right) e^{-i\bar{Q}_P\bar{\beta} - \frac{1}{4}\bar{\beta}^2} \operatorname{erfc}\left(\bar{Q}_P - i\frac{\bar{\beta}}{2}\right)\right],$$

where

$$\bar{Q}_P = Q_P/\hbar\omega_0 = (Q_0^\pi + \varepsilon_F^\pi - \varepsilon_F^{\pi'})/\hbar\omega_0, \quad (14)$$

and

$$\bar{\beta} = \beta \cdot \hbar\omega_0, \quad (15)$$

while erfc is the complementary error function. Including also pick-up reactions we find the intrinsic energy

$$E = \sum_{ik} n_{ik}^{NS} (\bar{\varepsilon}_k - \bar{\varepsilon}_i') + \sum_{jl} n_{jl}^{NP} (\bar{\varepsilon}_j' - \bar{\varepsilon}_l) \quad (16)$$

and the calculation of Z leads to

$$Z(\beta) = \exp\{\sum_{ik} p_{ik}^{NS} (e^{i(\bar{\varepsilon}_k - \bar{\varepsilon}_i')\beta} - 1) + \sum_{jl} p_{jl}^{NS} (e^{i(\bar{\varepsilon}_j - \bar{\varepsilon}_l')\beta} - 1)\}. \quad (17)$$

The optimum Q-value for pick-up is $-Q_0^{NS}$ while $p_0^{NS} = p_0^{NP}$. A similar calculation as above leads to

$$\ln Z(\beta) = k_p e^{-\bar{Q}_P^2} [G_1\left(\bar{Q}_P - \frac{i}{2}\bar{\beta}\right) - G_1(\bar{Q}_P)$$

$$+ G_1\left(-\bar{Q}_P - \frac{i}{2}\bar{\beta}\right) - G_1(-\bar{Q}_P)] \quad (18)$$

$$+ k_N \cdot e^{\overline{Q}_N} [G_1 \left(\overline{Q}_N - \frac{i}{2} \overline{\beta}\right) - G_1 (\overline{Q}_N)$$

$$+ G_1 \left(- \overline{Q}_N - \frac{i}{2} \overline{\beta}\right) - G_1 (-\overline{Q}_N)].$$

We have here included also neutron (ν) transfer and introduced the quantities

$$k_P = \frac{\sqrt{\pi}}{2} \varrho_a^\pi \varrho_A^\pi p_0^\pi (\hbar\omega_0)^2, \tag{19}$$

$$k_N = \frac{\sqrt{\pi}}{2} \varrho_a^\nu \varrho_A^\nu p_0^\nu (\hbar\omega_0)^2,$$

and

$$\overline{Q}_N = \frac{Q_N}{\hbar\omega_0} = \frac{\varepsilon_F^\nu - (\varepsilon_F')^\nu}{\hbar\omega_0}. \tag{20}$$

For convenience we have also introduced the function $G_n(z)$ defined by

$$G_n(z) = e^{z^2} i^n \operatorname{erfc}(z), \tag{21}$$

where $i^n \operatorname{erfc}(z)$ is the n-fold integral of the error function. It satisfies the recursion relation[6)]

$$G_n(z) = -\frac{z}{n} G_{n-1}(z) + \frac{1}{2n} G_{n-2}(z) \tag{22}$$

with

$$G_0(z) = e^{z^2} \operatorname{erfc}(z) \tag{23}$$

and

$$G_{-1}(z) = \frac{2}{\sqrt{\pi}}.$$

One also finds

$$\frac{d}{dz} G_n(z) = -2(n+1) G_{n+1}(z). \tag{24}$$

The average first order transfer probabilities p_0^π and p_0^ν can be estimated from (6) neglecting the Q-value dependence. One finds

$$p_0 = \left| \sqrt{\frac{2\pi}{\varkappa \bar{r}_0 \hbar^2}} f(R) e^{-(r_0 - R)\varkappa} \right|^2 \tag{25}$$

where we assumed an exponential formfactor with slope $\varkappa_\nu$ or $\varkappa_\pi$ for neutrons or protons. We thus find

$$p_0 = \frac{2\pi}{(\hbar\,\omega_0)^2}\,(f\,(R))^2\,\mathrm{e}^{-\frac{r-R}{a}} \tag{26}$$

where $a = (2\varkappa)^{-1}$ is the diffuseness of the neutron or proton densities in the nuclear surface. The formfactors $f\,(R)$ at the sum of the nuclear radii

$$R = 1.25\,(A_a^{1/3} + A_A^{1/3})\ \mathrm{fm} \tag{27}$$

are similar for neutrons and protons being on the average about 3 MeV[7].

The quantities k_N and k_P can therefore be estimated by

$$k = \pi^{3/2}\,\varrho_a\varrho_A\,(f\,(R))^2\,\exp\left(\frac{r_0 - R}{a}\right). \tag{28}$$

3. *Applications*

From the result (18) of the previous section we can calculate the energy distributions as the Fourier transform of $Z\,(\beta)$ i. e.

$$P\,(E) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\beta E}\exp\,(\ln Z\,(\beta))\,\mathrm{d}\beta. \tag{29}$$

Before calculating this quantity we note a few general properties of the characteristic function. We note first that

$$\int_{-\infty}^{\infty} P\,(E)\,\mathrm{d}E = \exp\,(\ln Z\,(0)) = 1 \tag{30}$$

as is seen directly from (18).

The mean energy and the spread can be obtained directly from $\ln Z$ since

$$\langle E\rangle = \int_{-\infty}^{\infty} P\,(E)\,E\,\mathrm{d}E = \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty}\frac{\mathrm{d}}{\mathrm{d}\beta}\,(\mathrm{e}^{-\mathrm{i}\beta E})\,Z\,(\beta)\,\mathrm{d}E\,\mathrm{d}\beta = \frac{1}{\mathrm{i}}\frac{\mathrm{d}}{\mathrm{d}\beta}\,(\ln Z)|_{\beta=0}, \tag{31}$$

and

$$\sigma_E^2 = \langle E^2\rangle - \langle E\rangle^2 = -\frac{\mathrm{d}^2}{\mathrm{d}\beta^2}\ln Z\,(\beta)|_{\beta=0}. \tag{32}$$

One may easily calculate these quantities for the case considered in the previous section making use of (24). One finds the mean energy

$$\langle E\rangle = 2\hbar\omega_0\{k_P(\mathrm{i}^2\,\mathrm{erfc}\,(\bar{Q}_P) + \mathrm{i}^2\,\mathrm{erfc}\,(-\bar{Q}_P)) + k_N(\mathrm{i}^2\,\mathrm{erfc}\,(\bar{Q}_N)$$

$$+ \mathrm{i}^2\,\mathrm{erfc}\,(-\bar{Q}_N))\} = \hbar\omega_0\,\{k_P(2\bar{Q}_P^2 + 1) + k_N(2\bar{Q}_N^2 + 1)\} \qquad (33)$$

and spread

$$\sigma_E = 6\,(\hbar\omega_0)^2\,\{k_P(\mathrm{i}^3\,\mathrm{erfc}\,(\bar{Q}_P) + \mathrm{i}^3\,\mathrm{erfc}\,(-\bar{Q}_P)) +$$

$$+ k_N(\mathrm{i}^3\,\mathrm{erfc}\,(\bar{Q}_N) + \mathrm{i}^3\,\mathrm{erfc}\,(-\bar{Q}_N))\}. \qquad (34)$$

It is interesting to notice that the ratio of $\sigma_E^2/\langle E\rangle$ is rather independent of the distance of closest approach (i. e. k_N and k_P) but mainly depends on $\hbar\omega_0$ and on the parameters $\bar{Q}_P$ and $\bar{Q}_N$. The functions $2\bar{Q}^2 + 1$ and $6\,(\mathrm{i}^3\,\mathrm{erfc}\,\bar{Q} + \mathrm{i}^3\,\mathrm{erfc}\,(-\bar{Q}))$ are illustrated in Fig. 1.

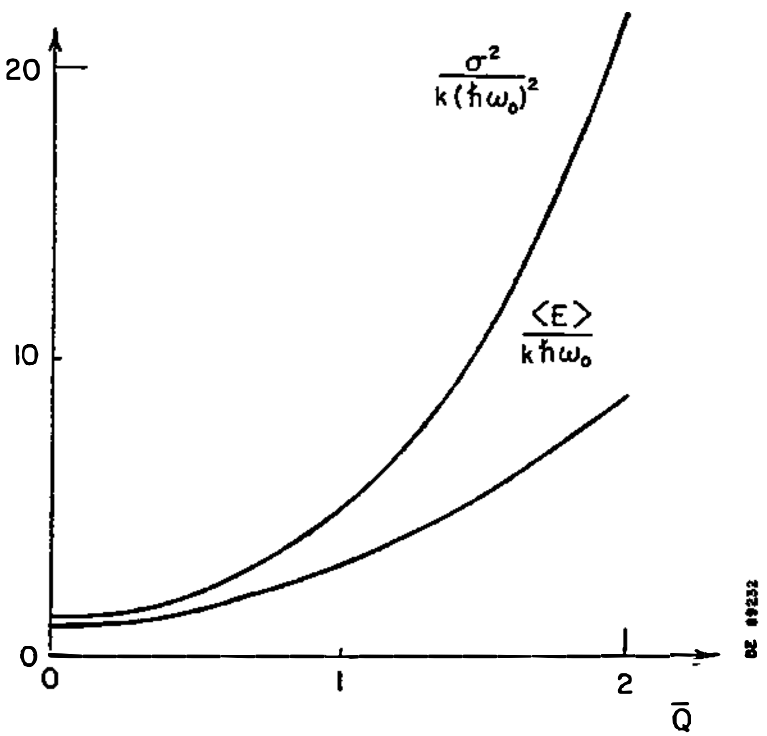

Fig. 1. The functions $2\bar{Q}^2 + 1$ and $6\,(i^3\,\mathrm{erfc}\,(\bar{Q}) + i^3\,\mathrm{erfc}\,(-\bar{Q}))$ which determine the dependence of $\langle E\rangle$ and σ^2 on the quantity $\bar{Q} = (Q_0 + \varepsilon_P - \varepsilon_P')/\hbar\omega_0$ where Q_0 is the optimum Q-value for stripping reaction.

We note that for the collision of identical nuclei $\bar{Q}_P = \bar{Q}_N = 0$. We therefore find in this case

$$\langle E \rangle = \hbar\omega_0 \, (k_N + k_P), \tag{35}$$

and

$$\sigma^2 = \frac{2}{\sqrt{\pi}} (\hbar\omega_0)^2 \, (k_N + k_P). \tag{36}$$

The ratio

$$\frac{\sigma^2}{\langle E \rangle} = \frac{2}{\sqrt{\pi}} \hbar\omega_0 \tag{37}$$

is in this case quite independent of the distance of closest approach. Since $\hbar\omega_0$ is typically of the order 3—4 MeV (cf. e. g. (47) below) we see already from (37) that the energy distribution is quite broad even up to energy loss of 100 MeV.

The energy distribution which we obtain by inserting (18) in (29) is singular because $Z(\beta)$ for large values of $\bar{\beta}$ is nonvanishing. In fact we find

$$G_n(z) \approx \frac{2}{\sqrt{\pi}} \frac{1}{(2z)^{n+1}} \text{ for } |z| \to \infty \text{ and } |\arg z| < \frac{3\pi}{4}, \tag{38}$$

i. e.

$$\ln Z(\infty) = -k_P \left(G_1(\bar{Q}_P) + G_1(-\bar{Q}_P)\right) \mathrm{e}^{-Q_P^2}$$
$$- \, k_N \left(G_1(\bar{Q}_N) + G_1(-\bar{Q}_N)\right) \mathrm{e}^{-Q_N^2}. \tag{39}$$

We may (for $E < 0$) complete the path of integration of large distances in the positive $\bar{\beta}$ half plane and conclude that (fortunately!)

$$P(E < 0) = 0. \tag{40}$$

For $E = 0$ the integral diverges. We may in fact extract a δ-function by adding and subtracting a term proportional to $Z(\infty)$ in the integral i. e.

$$P(E) = \frac{1}{2\pi\hbar\omega_0} \int_{-\infty}^{\infty} \mathrm{d}\bar{\beta} \, \mathrm{e}^{-\mathrm{i}\bar{\beta}\frac{E}{\hbar\omega_0}} \left(\exp(\ln Z(\beta)) - \exp \ln Z(\infty)\right)$$
$$+ \frac{1}{\hbar\omega_0} \, \delta\left(\frac{E}{\hbar\omega_0}\right) Z(\infty). \tag{41}$$

The δ-function indicates the probability that after the collision one is still in the ground state i. e.

$$P_0 = Z(\infty). \tag{42}$$

For the above example of symmetric systems we find

$$P_0 = e^{-\frac{2}{\sqrt{\pi}}(k_P + k_N)} = e^{-\frac{2}{\sqrt{\pi}}\frac{\langle E \rangle}{\hbar\omega_0}} \tag{43}$$

If the average energy loss is comparable to $\hbar\omega_0$ the energy distribution is very wide

$$\frac{\sigma}{\langle E \rangle} \approx \sqrt{\frac{\hbar\omega_0}{\langle E \rangle}}. \tag{44}$$

If however $k_N + k_P$ becomes large the main contribution to the integral (39) comes from small values of $\overline{\beta}$ and the integral can be evaluated to give

$$P(E) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{E - \langle E \rangle}{2\sigma^2}\right)}. \tag{45}$$

The expressions which we have derived above are only correct for grazing collisions where the distance of closest approach r_0 is larger than the sum R of the nuclear radii. In this grazing region we may estimate r_0 by the expression[4)]

$$\ddot{r}_0 = \frac{2E - E_B}{m_0 r_0} \tag{46}$$

valid for the Coulomb scattering. We therefore find

$$(\hbar\omega_0)^2 \approx 30 \cdot \frac{Z_a Z_A (A_a + A_A)}{A_a A_A (A_a^{1/3} + A_A^{1/3})} \left(2\frac{E}{E_B} - 1\right). \tag{47}$$

It should be noted however that close to orbiting $\hbar\omega_0$ rather suddenly goes to zero.

There is for low bombarding energies a natural boundary for the grazing collisions because there exists, even for very heavy systems a Coulomb barrier where orbiting occurs. This means that for important parameters less than a critical value the trajectory of relative motion of the two nuclei surpasses the barrier and moves into regions of substantial more overlap. In this region the formfactors go through a maximum and the collision time becomes essentially longer. The attraction between the nuclear surfaces which create the Coulomb barrier also supply a driving force for the nuclear deformations which, because they are damped, give rise to a new damping mechanism for the relative motion.

In this region beyond the Coulomb barrier a macroscopic description of the reaction seems more appropriate. The final outcome of the collision is determined not only by the energy (and angular momentum) dissipation but also by the question of whether the attraction between the nuclear surfaces can hold the nuclei together when they deform under the centrifugal forces.

In any case one must conclude from the above discussion of the reaction mechanism that there is a rather sharp distinction between grazing collisions on the one hand, where energy dissipations may be described by the above formulae and deep inelastic or fusion or fusion-fission reactions on the other hand.

The method that we have presented should be generalized to give the energy distribution as a function of time during the collision, so that one may calculate the energy (and angular momentum) distribution at the Coulomb barrier. This is an important element in determining the fusion cross section. In fact for a trajectory which would surpass the Coulomb barrier if the energy of relative motion would be that of the entrance channel, may only lead to fussion with a small probability especially in heavy nuclei the energy loss in the surface region is substantial.

The important parameters k_N and k_P for the energy distribution were estimated in the previous section (cf. (28)). For grazing (quasielastic) reactions the distance of closest approach r_0 should be larger than the radius of the Coulomb barrier for the effective potential in the radial motion

$$U(r) = \frac{Z_a Z_A e^2}{r} + U^N(r) + \frac{l(l+1)\hbar^2}{2m_0 r^2}. \tag{48}$$

We may estimate this distance r_g by[4)]

$$r_g = 1.07\,(A_a^{1/3} + A_A^{1/3}) + 2.72 - a \ln\left(1 + \frac{2(E - E_B)}{E_B}\right) \text{fm}, \tag{49}$$

where a is the diffuseness (≈ 0.6 fm) of the nuclear potential U^N. It is seen that this quantity is always larger than R (cf. (27)) and especially so for light nuclei.

On the basis of the above discussion we may make a few conclusions on the magnitude of the quantities we have evaluated.

First we notice that the average probability p_0 (cf. (26)) is usually a number smaller than or of the order of magnitude unity. This is because $\hbar\omega_0$ (from (47)) is typically 3—4 MeV, $f(R)$ is typically 3 MeV, and r_0 is larger than r_g which especially in light nuclei is larger than R. This conclusion on the magnitude of p_0 lends general confidence to one of the fundamental approximations (3) that we have introduced.

Secondly we may estimate k_N and k_P (cf. Eq. (28)) according to

$$k_P = \frac{Z_a^{1/2} Z_A^{1/2} A_a^{1/3} A_A^{1/3}}{40} \exp\left(-\frac{r_0 - R}{a_P}\right) \tag{50}$$

and

$$k_N = \frac{N_a^{1/2} N_A^{1/2} A_a^{1/3} A_A^{1/3}}{40} \exp\left(-\frac{r_0 - R}{a_N}\right)$$

with[5)]

$$1/a_P = 1.65 + 0.1\,(A_a^{-1/3} + A_A^{-1/3})$$
$$1/a_N = 1.16 + 0.5\,(A_a^{-1/3} + A_A^{-1/3}). \tag{51}$$

These estimates are rather uncertain mainly because of the difficulties in estimating the effective level densities. We may however make a few general conclusions.

For heavy nuclei, where a_P is much smaller than a_N, proton transfer usually contributes much less than neutron transfer. This is however not true for close collisions, where $r_0 \approx r_g$ may approach the sum of the nuclear radii R. In this case the exponential factors are of minor significance and the contributions from neutrons and protons become comparable. For light nuclei k_N and k_P are always of similar magnitude because a_P and a_N are similar.

The absolute magnitude of k_N and k_P can become quite large. For symmetric systems at the distance r_g we find

$$k_P(r_g) = \frac{Z \cdot A^{2/3}}{3300} e^{0.42A^{1/3} - 2.7A^{-1/3}} \tag{52}$$

which is approximately 30 for Pb + Pb. Similarly

$$k_N(r_g) = \frac{N \cdot A^{2/3}}{660} e^{0.42A^{1/3} - 2.7A^{-1/3}} \tag{53}$$

which is approximately 50 for Pb + Pb. From this we conclude that in a grazing collision of Pb + Pb one may lose $\langle E \rangle \approx \hbar\omega_0 \cdot 80 = 300$ MeV.

Although this result is quite uncertain it shows that substantial energy loss may occur in the grazing region. It also shows that for very heavy ion collisions grazing distances are only reached if the bombarding energy is essentially higher than the Coulomb barrier.

We finally mention that the result (42) can be reformulated in terms of an absorptive potential. One finds[3)]

$$W(r_0) = -\frac{\hbar\omega_0}{2\sqrt{\pi}} \ln Z(\infty)$$

where $\ln Z(\infty)$ is given by (39). The exponential character of $W(r)$ in the surface region enters through the r_0 dependence of the quantities k_P and k_N given in (50).

4. Generalizations

It is quite simple to generalize the distribution in total energy loss to a distribution in all the observables i. e. the energies E_a and E_A of projectile and target, their angular momenta M_a and M_A perpendicular to the scattering plane and the change in neutron and proton numbers $\Delta N_a = -\Delta N_A$ and $\Delta Z_a = -\Delta Z_A$. We introduce the characteristic function

$$Z(\beta_a, \beta_A, \zeta_a, \zeta_A, \gamma_N, \gamma_Z) =$$

$$= \int_{-\infty}^{\infty} \mathrm{d}E_a \, \mathrm{d}E_A \, \mathrm{d}M_a \, \mathrm{d}M_A \, \mathrm{d}\Delta N \, \mathrm{d}\Delta Z \, P(E_a E_A M_a M_A \Delta N \Delta Z) \tag{54}$$

$$\exp(\mathrm{i}\Delta N \gamma_N + \mathrm{i}\Delta Z \gamma_Z + \mathrm{i}M_a \zeta_a + \mathrm{i}M_A \zeta_A + \mathrm{i}E_a \beta_a + \mathrm{i}E_A \beta_A)$$

where

$$\begin{aligned} E_a &= \Sigma_{ik} n_{ik}^{NS} (\varepsilon_F' - \varepsilon_i') + \Sigma_{jl} n_{jl}^{NP} (\varepsilon_j' - \varepsilon_F') \\ E_A &= \Sigma_{ik} n_{ik}^{NS} (\varepsilon_k - \varepsilon_F) + \Sigma_{jl} n_{jl}^{NP} (\varepsilon_F - \varepsilon_l) \end{aligned} \tag{55}$$

$$\begin{aligned} M_a &= \Sigma_{jl} n_{jl}^{NP} m_j' - \Sigma_{ik} n_{ik}^{NS} m_i' \\ M_A &= \Sigma_{ik} n_{ik}^{NS} m_k - \Sigma_{jl} n_{jl}^{NP} m_l \end{aligned} \tag{56}$$

$$\begin{aligned} \Delta N = \Delta N_a &= \Sigma_{jl} n_{jl}^{\nu P} - \Sigma_{ik} n_{ik}^{\nu S} \\ \Delta Z = \Delta Z_a &= \Sigma_{jl} n_{jl}^{\pi P} - \Sigma_{ik} n_{ik}^{\pi S}. \end{aligned} \tag{57}$$

We have here neglected inelastic reactions and have denoted the angular momentum projections of the single particle orbitals in projectile by m_j' and m_i' and in target by m_k and m_l.

With this notation we readily find in complete analogy to the derivation of (11)

$$\ln Z(\beta_a \beta_A \zeta_a \zeta_A \gamma_N \gamma_Z) =$$

$$\begin{aligned} &= \Sigma_{ik} p_{ik}^{\nu S} (\exp(-\mathrm{i}\gamma_N - \mathrm{i}m_i' \zeta_a + \mathrm{i}m_k \zeta_A - \mathrm{i}\bar{\varepsilon}_i' \beta_a + \mathrm{i}\bar{\varepsilon}_k \beta_A) - 1) \\ &+ \Sigma_{jl} p_{jl}^{\nu P} (\exp(+\mathrm{i}\gamma_N + \mathrm{i}m_j' \zeta_a - \mathrm{i}m_l \zeta_A + \mathrm{i}\bar{\varepsilon}_j' \beta_a - \mathrm{i}\bar{\varepsilon}_l \beta_A) - 1) \\ &+ \Sigma_{ik} p_{ik}^{\pi S} (\exp(-\mathrm{i}\gamma_P - \mathrm{i}m_i' \zeta_a + \mathrm{i}m_k \zeta_A - \mathrm{i}\bar{\varepsilon}_i' \beta_a + \mathrm{i}\bar{\varepsilon}_k \beta_A) - 1) \\ &+ \Sigma_{jl} p_{jl}^{\pi P} (\exp(+\mathrm{i}\gamma_P + \mathrm{i}m_j' \zeta_a - \mathrm{i}m_l \zeta_A + \mathrm{i}\bar{\varepsilon}_j' \beta_a - \mathrm{i}\bar{\varepsilon}_l \beta_A) - 1). \end{aligned} \tag{58}$$

We notice that the inclusive probability integrated over all values of e. g. M_a is obtained by setting the corresponding variable (ζ_a) equal to zero. The distribution in total energy is obtained by setting $\beta_a = \beta_A = \beta$ and we thus directly recover the result (17).

The generalization with respect to angular momentum may be especially interesting since the probabilities p depend on m_k and m_l if the bombarding energy is not very low.

We shall here however only be concerned with the distribution $P(E, \Delta N, \Delta Z)$ which is the distribution in total energy loss for a specific charge $(Z_a + \Delta Z)$ and mass $(A_a + \Delta Z + \Delta N)$ number of the scattered particle. We find

$$P(E, \varDelta N, \varDelta Z) = \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \mathrm{d}\gamma_Z \int_{-\pi}^{\pi} \mathrm{d}\gamma_N \int_{-\infty}^{\infty} \mathrm{d}\beta\, \mathrm{e}^{-i\varDelta N_{\gamma}N - i\varDelta Z_{\gamma}Z - i\beta E}\, Z(\gamma_N \gamma_Z \beta), \tag{59}$$

where

$$\begin{aligned} \ln Z(\gamma_N \gamma_Z \beta) = {} & \sum_{ik} p_{ik}^{\nu S} (\exp(-\mathrm{i}\gamma_N + \mathrm{i}(\bar{\varepsilon}_k - \bar{\varepsilon}_i')\beta) - 1) \\ & + \sum_{jl} p_{jl}^{\nu P} (\exp(\mathrm{i}\gamma_N + \mathrm{i}(\bar{\varepsilon}_j' - \bar{\varepsilon}_l)\beta) - 1) \\ & + \sum_{ik} p_{ik}^{\nu P} (\exp(-\mathrm{i}\gamma_Z + \mathrm{i}(\bar{\varepsilon}_k - \bar{\varepsilon}_i')\beta) - 1) \\ & + \sum_{jl} p_{jl}^{\pi P} (\exp(\mathrm{i}\gamma_Z + \mathrm{i}(\bar{\varepsilon}_j' - \bar{\varepsilon}_l)\beta) - 1) \\ & = (F^{\nu S}(\beta)\, \mathrm{e}^{-\mathrm{i}\gamma_N} - q^{\nu S}) + (F^{\nu P}(\beta)\, \mathrm{e}^{\mathrm{i}\gamma_N} - q^{\nu P}) \\ & + (F^{\pi S}(\beta)\, \mathrm{e}^{-\mathrm{i}\gamma_Z} - q^{\pi S}) + (F^{\pi P}(\beta)\, \mathrm{e}^{\mathrm{i}\gamma_Z} - q^{\pi P}). \end{aligned} \tag{60}$$

We have here introduced the notation (cf. (13) and (18))

$$\begin{aligned} F^{\pi S}(\beta) &= k_P\, G_1 \left(\bar{Q}_P - \frac{\mathrm{i}}{2}\bar{\beta}\right) \mathrm{e}^{-\bar{Q}_P^2} \\ q^{\pi S} &= F^{\pi S}(0) = k_P\, G_1 (\bar{Q}_P)\, \mathrm{e}^{-\bar{Q}_P^2} \\ F^{\pi P}(\beta) &= k_P\, G_1 \left(-\bar{Q}_P - \frac{\mathrm{i}}{2}\beta\right) \mathrm{e}^{-\bar{Q}_P^2} \\ q^{\pi P} &= F^{\pi P}(0) = k_P\, G_1 (-\bar{Q}_P)\, \mathrm{e}^{-\bar{Q}_P^2} \end{aligned} \tag{61}$$

and similar for neutrons. Note that for $\gamma_N = \gamma_Z = 0$ we recover the earlier expression for $\ln Z(\beta)$.

If we introduce the quantities $z = \mathrm{e}^{\mathrm{i}\gamma_N}$ and $z' = \mathrm{e}^{\mathrm{i}\gamma_S}$ we may write (60) as

$$\begin{aligned} P(E, \varDelta N, \varDelta Z) = {} & \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\bar{\beta}\, \mathrm{e}^{-\mathrm{i}\bar{\beta}E}\, \mathrm{e}^{-(q^{\nu S} + q^{\pi S} + q^{\nu P} + p^{\pi P})} \\ & \times \frac{1}{2\pi\mathrm{i}} \int_0 \frac{\mathrm{d}z}{z} z^{-\varDelta N}\, \mathrm{e}^{(F^{\nu S}(\beta) z^{-1})}\, \mathrm{e}^{(F^{\nu P}(\beta) z)} \\ & \times \frac{1}{2\pi\mathrm{i}} \int_0 \frac{\mathrm{d}z}{z} z^{-\varDelta Z}\, \mathrm{e}^{(F^{\pi S}(\beta) z^{-1})}\, \mathrm{e}^{(F^{\pi P}(\beta) z)} \end{aligned} \tag{62}$$

where the contour integrals are performed along the unit circle. Utillizing the expansion

$$e^{F_+ z + F_- z^{-1}} = \sum_{k=-\infty}^{\infty} \left(\frac{F_+}{F_-}\right)^{k/2} I_{|k|}(2\sqrt{F_+F_-}) \cdot z^k \tag{63}$$

in terms of modified Bessel functions, we may evaluate the contour integrals to find

$$P(E, \Delta N, \Delta Z) = e^{-q^{\pi S} - q^{\pi P} - q^{\nu S} - q^{\nu P}}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\beta\, e^{-i\beta E} \left(\frac{F^{\pi P}}{F^{\pi S}}\right)^{\Delta Z/2} \left(\frac{F^{\nu P}}{F^{\nu S}}\right)^{\Delta N/2} \tag{64}$$

$$I_{|\Delta Z|}(2\sqrt{F^{\pi P}F^{\pi S}})\, I_{|\Delta N|}(2\sqrt{F^{\nu P}F^{\nu S}}).$$

With the explicit expressions for F these energy distributions can readily be evaluated.

We note that integrating over E we find

$$P(\Delta N, \Delta Z) = P(\Delta N) \cdot P(\Delta Z) \tag{65}$$

with

$$P(\Delta N) = \left(\frac{q^{\nu P}}{q^{\nu S}}\right)^{\frac{\Delta N}{2}} I_{|\Delta N|}(2\sqrt{q^{\nu P} q^{\nu S}})\, e^{-q^{\nu S} - q^{\nu P}}$$

and similar for $P(\Delta Z)$. These Bessel distributions which are characteristic for random walk with drift can also be derived in simpler ways. Through the present treatment one may however also obtain simple results from (64) for the mean energy and its spread for each mass and charge partition.

5. Conclusion

The present paper contains a number of preliminary results that may become a useful tool to understand the mean features of grazing heavy ion reactions that do not depend critically on the special spectroscopic properties of the two colliding nuclei. The result have still not been confronted with experimental data. They were developed in order to understand recent coupled channel calculations and have been favorably compared to these.

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SHEMATSKI MODEL ZA TEŠKOIONSKE REAKCIJE DODIRA

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Predlaže se niz aproksimacija, pomoću kojih je moguće procijeniti oblik distribucije energije, angularnog momenta mase i naboja u teškoionskim reakcijama dodira. Izračunat je ukupni gubitak energije i pokazano je da to može iznositi i do stotinjak MeV za vrlo teške sisteme.