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GENERALIZED ALAGA RULES

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The role played by generalized Alaga rules in the development of nuclear physics is emphasized. The connection between group theory and Alaga rules is outlined. This connection allows one to construct rules for any situation described by a symmetry (or supersymmetry). Some examples are presented.

1. Introduction

In 1955, Alaga, together with Alder, Bohr and Mottelson[1], pointed out that matrix elements of electromagnetic transition operators in deformed nuclei with axial symmetry should obey simple rules. For example, matrix elements of the electric quadrupole operator, $\hat{T}^{(E2)}$, should be governed by the rule

$$\langle KL_1 \| \hat{T}^{(E2)} \| KL_2 \rangle = a \langle L_2, K, 2, 0 | L_1, K \rangle (2L_2 + 1)^{1/2}, \tag{1.1}$$

where the double bar denotes reduced matrix elements with respect to the angular momentum, the coefficient on the rhs is an ordinary Clebsh-Gordan coefficient, K denotes the projection of the angular momentum on the symmetry axis and a is a proportionality constant. The rule (1.1), now called Alaga rule, has formed the basis for the analysis of many experimental data.

Alaga's rule is intimately connected with group theory and thus can be generalized to any situation described by a dynamic symmetry (or supersymmetry). In this article, dedicated to the memory of Gaja Alaga, I briefly review the mathematical framework on which generalized Alaga's rules are based and show examples of application to the study of properties of medium-mass and heavy nuclei. Generalized Alaga's rules have played a major role in the development of nuclear structure physics in the last 15 years and it is thus appropriate to describe them in an article honoring Alaga's contributions to physics.

2. *Generalized Wigner-Eckart theorem*

Generalized Alaga's rules arise from the Wigner-Eckart theorem applied to the general situation described by algebraic structure $G \supset G' \supset \ldots$. Consider for example the case of four algebras

$$\left| \begin{array}{ccccccc} G & \supset & G' & \supset & G'' & \supset & G''' \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \Lambda & & \lambda & & L & & M \end{array} \right\rangle, \tag{2.1}$$

where Λ, λ, L, M denote schematically the quantum numbers labelling the representations. The Wigner-Eckart theorem states that the matrix of any tensor operator $\hat{T}^{\Lambda\lambda LM}$ between states (2.1) are given by[2)]

$$\langle \Lambda_1 \lambda_1 L_1 M_1 | \hat{T}^{\Lambda\lambda LM} | \Lambda_2 \lambda_2 L_2 M_2 \rangle = \langle \Lambda_1 \lambda_1 | \Lambda\lambda\Lambda_2\lambda_2 \rangle \cdot$$

$$\cdot \langle \lambda_1 L_1 | \lambda L \lambda_2 L_2 \rangle \langle L_1 M_1 | LML_2 M_2 \rangle \langle \Lambda_1 \| \| \hat{T}^{\Lambda} \| \| \Lambda_2 \rangle. \tag{2.2}$$

In Eq. (2.2) the coefficients in brackets represent isoscalar factors and the bars denote reduced matrix elements. (One bar is added for each reduction). Eq. (2.2) is the generalization of the familiar Wigner-Eckart theorem for the rotation group

$$\left| \begin{array}{ccc} O(3) & \supset & O(2) \\ \downarrow & & \downarrow \\ L & & M \end{array} \right\rangle, \tag{2.3}$$

which gives

$$\langle L_1 M_1 | \hat{T}^{LM} | L_2 M_2 \rangle = \langle L_1 M_1 | LML_2 M_2 \rangle \langle L_1 \| \hat{T}^{L} \| L_2 \rangle. \tag{2.4}$$

Both Eq. (2.2) and (2.4) have been written in a schematic fashion, by deleting (eventual) multiplicity labels that may appear in the reduction of G and by including all phase factors and eventual factors $\sqrt{2\Lambda + 1}$ in the definition of the isoscalar factors. Furthermore, Racah's factorization lemma has been used in (2.2) by writing the total isoscalar factor as a product of three isoscalar factors for the reduction $G \supset G'$, $G' \supset G''$ and $G'' \supset G'''$.

If, in addition to being a tensor operator with respect to G, the operator $\hat{T}$ is a generator of G, the Wigner-Eckart theorem simplifies further, since then

$$\langle \Lambda_1 \| \| \hat{T}^{\Lambda} \| \| \Lambda_2 \rangle = f(\Lambda, \Lambda_1)\, \delta_{\Lambda_1 \Lambda_2}. \tag{2.5}$$

Combining (2.2), (2.4) and (2.5), one obtains

$$\langle \Lambda_1 \lambda_1 L_1 \| \hat{T}^{\Lambda\lambda L} \| \Lambda_2 \lambda_2 L_2 \rangle = \langle \Lambda_1 \lambda_1 | \Lambda\lambda \Lambda_2 \lambda_2 \rangle \langle \lambda_1 L_1 | \lambda L \lambda_2 L_2 \rangle \times$$
$$\times f(\Lambda, \Lambda_1)\, \delta_{\Lambda_1 \Lambda_2}. \tag{2.6}$$

Here the double bar denotes matrix elements reduced only with respect to the angular momentum, $O(3) \supset O(2)$. Eq. 2.6) is the form usually employed in describing Alaga's rules. The isoscalar factors are quite often denoted by the alternative symbol

$$\left(\begin{matrix} \Lambda_2 & \Lambda \\ \lambda_2 & \lambda \end{matrix} \middle| \begin{matrix} \Lambda_1 \\ \lambda_1 \end{matrix} \right) \equiv \langle \Lambda_1 \lambda_1 | \Lambda \lambda \Lambda_2 \lambda_2 \rangle. \tag{2.7}$$

3. *Examples of generalized Alaga rules in even-even nuclei*

3.1. *Electromagnetic transitions rates in γ-unstable nuclei*

The wave functions of γ-unstable even-even nuclei can be written as[3)]

$$\left| \begin{matrix} O(6) & \supset & O(5) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ (\sigma_1, \sigma_2, \sigma_3) & & (\tau_1, \tau_2), \nu_\Delta & & L & & M \end{matrix} \right\rangle. \tag{3.1}$$

The multiplicity index ν_Δ can be, for purposes of the discussion here, omitted. The transition operator inducing electric quadrupole transitions can be written as a tensor operator $\hat{T}^{\Lambda\lambda LM}$ with $\Lambda \equiv (1, 0, 0)$, $\lambda \equiv (1, 0)$, $L = 2$, $M = \pm 2, \pm 1, 0$. It is a generator of $O(6)$. This information allows one to write down all reduced matrix elements of $\hat{T}$, as

$$\langle (\sigma_1', \sigma_2', \sigma_3'), (\tau_1', \tau_2'), L' \| \hat{T}^{(1,0,0),(1,0),2} \| (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), L \rangle =$$
$$= \left(\begin{matrix} (\sigma_1, \sigma_2, \sigma_3) & (1,0,0) \\ (\tau_1, \tau_2) & (1,0) \end{matrix} \middle| \begin{matrix} (\sigma_1', \sigma_2', \sigma_3') \\ (\tau_1', \tau_2') \end{matrix} \right) \left(\begin{matrix} (\tau_1, \tau_2) & (1,0) \\ L & 2 \end{matrix} \middle| \begin{matrix} (\tau_1', \tau_2') \\ L' \end{matrix} \right) \times$$
$$\times f(\sigma_1, \sigma_2, \sigma_3)\, \delta_{(\sigma_1, \sigma_2, \sigma_3),\, (\sigma_1', \sigma_2', \sigma_3')}. \tag{3.2}$$

Particularly important in these nuclei are the transitions among members of the ground state band having $(\sigma_1, \sigma_2, \sigma_3) = (N, 0, 0)$, where N is the total boson number. For these transitions one has

$$\langle (N,0,0),(\tau+1,0),L' \| \hat{T}^{(1,0,0),(1,0),2} \| (N,0,0),(\tau,0,L\rangle =$$

$$= \left(\begin{matrix} (N,0,0) \\ (\tau,0) \end{matrix} \; \begin{matrix} (1,0,0) \\ (1,0) \end{matrix} \middle| \begin{matrix} (N,0,0) \\ (\tau+1,0) \end{matrix} \right) \left(\begin{matrix} (\tau,0) \\ L \end{matrix} \; \begin{matrix} (1,0) \\ 2 \end{matrix} \middle| \begin{matrix} (\tau+1,0) \\ L' \end{matrix} \right) \alpha_2, \qquad (3.3)$$

where α_2 is a constant equal to $f(\sigma_1, \sigma_2, \sigma_3)$. Inserting the appropriate values of the coefficients in (3.3) one obtains the Alaga rules for decays of states in the ground state band. Table 1 shows some of these rules. The rules are written in terms of $B(E2)$ values defined as

$$B(E2; L_2 \to L_1) = \frac{1}{2L_2+1} |\langle L_1 \| \hat{T}^{(E2)} \| L_2 \rangle|^2. \qquad (3.4)$$

TABLE 1.

$B(E2); [N], \sigma = N. \tau+1, \tilde{\nu}'_\Delta = 0, L' = 2\tau+2 \to [N], \sigma = N, \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau) =$

$= \alpha_2^2 \frac{\tau+1}{2\tau+5} (N-\tau)(N+\tau+4),$

$B(E2; [N], \sigma = N, \tau+1, \tilde{\nu}'_\Delta = 0, L' = 2\tau \to [N], \sigma = N, \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau) =$

$= \alpha_2^2 \frac{4\tau+2}{(2\tau+5)(4\tau-1)} (N-\tau)(N+\tau+4),$

$B(E2; [N], \sigma = N, \tau+1, \tilde{\nu}'_\Delta = 0, L' = 2\tau-1 \to [N], \sigma = N, \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau) =$

$= \alpha_2^2 \frac{(2\tau-2)(4\tau+1)}{(2\tau+5)(2\tau-1)(4\tau-1)} (N-\tau)(N+\tau+4),$

$B(E2; [N], \sigma = N, \tau+1, \tilde{\nu}'_\Delta = 0, L' = 2\tau-1 \to [N], \sigma = N \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau-2) =$

$= \alpha_2^2 \frac{3(2\tau+1)}{(2\tau+1)(\tau-1)(4\tau-1)} (N-\tau)(N+\tau+4),$

$B(E2; [N], \sigma = N, \tau+1, \tilde{\nu}'_\Delta = 0, L' = 2\tau \to [N], \sigma = N, \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau-2) =$

$= \alpha_2^2 \frac{(\tau-1)(4\tau+3)}{(2\tau+5)(4\tau-1)} (N-\tau)(N+\tau+4),$

$B(E2; [N], \sigma = N, \tau+1, \tilde{\nu}'_\Delta = 0, L = 2\tau-1 \to [N], \sigma = N, \tau, \tilde{\nu}'_\Delta = 0, L = 2\tau-3) =$

$= \alpha_2^2 \frac{\tau(\tau-2)(2\tau+1)}{(\tau-1)(2\tau-1)(2\tau+5)} (N-\tau)(N+\tau+4).$

Alaga rules for O (6) nuclei. States are denoted as in Ref. 3.

Table 2 shows an example of application of these rules to the study of $E2$ transition rates in the Xe isotopes.

TABLE 2.

$B(E2)$ ratios	Nucleus				
	^{124}Xe	^{126}Xe	^{128}Xe	^{130}Xe	O (6)
$\frac{2_2 \to 0_1}{2_1 \to 2_1}$	3.9	1.4	1.2	0.6	0
$\frac{3_1 \to 4_1}{3_1 \to 2_2}$	46	47	37	25	40
$\frac{3_2 \to 2_1}{3_1 \to 2_2}$	1.6	1.1	1	1.4	0
$\frac{4_2 \to 3_1}{4_2 \to 2_2}$	—	—	—	—	0
$\frac{4_2 \to 4_1}{4_2 \to 2_2}$	91	42	133	107	91
$\frac{4_2 \to 2_1}{4_2 \to 2_1}$	0.4	1.0	1.7	3.2	0
$\frac{5_1 \to 4_2}{5_1 \to 3_1}$	106	127	88	—	46
$\frac{5_1 \to 6_1}{5_1 \to 3_1}$	—	—	204	—	45
$\frac{5_1 \to 4_1}{5_1 \to 3_1}$	3.8	4.9	3.7	—	0
$\frac{0_2 \to 2_1}{0_2 \to 2_2}$	1	9	14	26	0

Experimental examples of Alaga rules in O (6) nuclei. All $B(E2)$ ratios are in units of 10^{-2}.

4. *Generalized reduction theorem*

The usefulness of Alaga's rules increases with the complexity of the situation one wants to describe. A particular complex situation is that encountered in odd-even and odd-odd nuclei. In odd-even nuclei, the collective degrees of freedom (described by bosons) are coupled to the single particle degrees of fredom (described by fermions). Alaga's rules can be generalized to this complex situation by making use of the generalized reduction theorem for coupled systems. Consider two systems a and b described by the coupled algebraic structure

$$\left| \begin{array}{cccccc} G_a \otimes & G_b \supset & G \supset & G' \supset & G'' \supset & G''' \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \Lambda_a & \Lambda_b & \Lambda & \lambda & L & M \end{array} \right\rangle . \tag{4.1}$$

Consider an operator $\hat{T}$ which is the sum of tensor operators of rank $\Lambda\lambda LM$ acting on a and b,

$$\hat{T}^{\Lambda\lambda LM} = \hat{T}_a^{\Lambda\lambda LM} + \hat{T}_b^{\Lambda\lambda LM}. \tag{4.2}$$

The matrix elements of (4.2) can be obtained by expanding the wave functions (4.1) into product functions of system a and b by means of appropriate isoscalar factors and then using the Wigner-Eckart theorem for system a and b. This yields

$$\langle(\Lambda_{1a}\otimes\Lambda_{1b})\,\Lambda_1\lambda_1L_1M_1|\,(\hat{T}_a^{\Lambda\lambda LM}+\hat{T}_b^{\Lambda\lambda LM})\,|(\Lambda_{2a}\otimes\Lambda_{2b})\,\Lambda_2\lambda_2L_2M_2\rangle =$$

$$= \sum_{\substack{\lambda_{1a}\lambda_{1b}\ \lambda_{2a}\lambda_{2b}\\ L_{1a}L_{1b}L_{2a}L_{2b}\\ M_{1a}M_{1b}M_{2a}M_{2b}}} \langle\Lambda_1\lambda_1|\,\Lambda_{1a}\lambda_{1a}\Lambda_{1b}\lambda_{1b}\rangle\,\langle\lambda_1\Lambda_1|\,\lambda_{1a}L_{1a}\lambda_{1b}L_{1b}\rangle$$

$$\langle L_1M_1|\,L_{1a}M_{1a}L_{1b}M_{1b}\rangle\,\langle\Lambda_2\lambda_2|\Lambda_{2a}\lambda_{2a}\Lambda_{2b}\lambda_{2b}\rangle$$

$$\langle\lambda_2L_2|\,\lambda_{2a}L_{2a}\lambda_{2b}L_{2b}\rangle\,\langle L_2M_2|\,L_{2a}M_{2a}L_{2b}M_{2b}\rangle$$

$$\times\,[\langle\Lambda_{1a}\lambda_{1a}L_{1a}M_{1a}|\,\hat{T}_a^{\Lambda\lambda LM}\,|\Lambda_{2a}\lambda_{2a}L_{2a}M_{2a}\rangle\,\delta_{\Lambda_{1b}\Lambda_{2b}}\delta_{\lambda_{1b}\lambda_{2b}}\delta_{L_{1b}L_{2b}}\delta_{M_{1b}M_{2b}}+$$

$$+\,\langle\Lambda_{1b}\lambda_{1b}L_{1b}M_{1b}|\,\hat{T}_b^{\Lambda\lambda LM}|\Lambda_{2b}\lambda_{2b}L_{2b}M_{2b}\rangle\,\delta_{\Lambda_{1a}\Lambda_{2a}}\delta_{\lambda_{1a}\lambda_{2a}}\delta_{L_{1a}L_{2a}}\delta_{M_{1a}M_{2a}}]. \tag{4.3}$$

The matrix elements of $\hat{T}_a$ and $\hat{T}_b$ are then given by Eq. (2.2). The products of soscalar factors summed over the indices can be rewritten, if one wishes, in terms of generalized $6j$ symbols yielding the generalized reduction theorem.

5. *Examples of generalized Alaga rules in odd-even nuclei*

5.1. *Electromagnetic transition rates in γ-unstable nuclei with an odd particle in $j = 3/2$*

Gamma-unstable odd-even nuclei with an odd particle in $j = 3/2$ can be described by wave functions of the type[4)]

$$\left|\begin{array}{cccccc} O^B(6) \otimes & SU^F(4) \supset & Spin^{BF}(6) \supset & Spin^{BF}(5) \supset & Spin^{BF}(3) \supset & Spin^{BF}(2) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (\Sigma, 0, 0) & \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) & (\sigma_1, \sigma_2, \sigma_3) & (\tau_1, \tau_2) & \nu_\Delta, J & M_J \end{array}\right\rangle \tag{5.1}$$

while the transition operator for $E2$ transitions can be written as

$$\hat{T}^{(1,0,0),(1,0),2,M} = \hat{T}_B^{(1,0,0),(1,0),2,M} + \hat{T}_F^{(1,0,0),(1,0),2,M}. \tag{5.2}$$

The matrix elements of the operator (5.2) in the basis states (5.1) can be evaluated using the generalized reduction theorem described in the previous section. They yield generalized Alaga rules for odd-even nuclei. Table 3 shows some of these rules for transitions between low-lying states. Table 4 shows a comparison between experimental and calculated $B(E2)$ values in the odd-even nucleus ^{197}Au.

TABLE 3.

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{5}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1+\frac{1}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{\tau_1+\frac{1}{2}}{2\tau_1+4}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{3}{2} \rightarrow N+\frac{1}{2}, \tau_1\, 2\tau_1+\frac{1}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{3}{(2\tau_1+4)(4\tau_1+1)}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{3}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1-\frac{1}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{\left(\tau_1-\frac{1}{2}\right)(4\tau_1+5)}{(2\tau_1+4)(4\tau_1+1)}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{1}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1+\frac{1}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{16\left(\tau_1-\frac{1}{2}\right)\left(\tau_1+\frac{3}{2}\right)^2}{(2\tau_1)(2\tau_1+4)^2(4\tau_1+1)}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{1}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1-\frac{1}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{50(2\tau_1+1)}{(2\tau_1+4)^2(4\tau_1-1)(4\tau_1+1)}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1+1, 2\tau_1+\frac{1}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1-\frac{3}{2}\right) =$$

$$= a_2^2\left(N-\tau_1+\frac{1}{2}\right)\left(N+\tau_1+\frac{9}{2}\right)\frac{\left(\tau_1-\frac{3}{2}\right)(2\tau_1+2)^2(4\tau_1+3)}{(2\tau_1)(2\tau_1+4)^2(4\tau_1-1)}$$

$$B\left(E2;\ N+\frac{1}{2}, \tau_1, 2\tau_1+\frac{1}{2} \rightarrow N+\frac{1}{2}, \tau_1, 2\tau_1-\frac{1}{2}\right) =$$

$$= a_2^2\left(\frac{2N+5}{2\tau_1+4}\right)^2\frac{2\left(\tau_1-\frac{1}{2}\right)(4\tau_1+3)}{(4\tau_1-1)(4\tau_1+1)}$$

$$B\left(E2;\ N+\frac{1}{2},\tau_1,2\tau_1+\frac{1}{2}\rightarrow N+\frac{1}{2},\tau_1,2\tau_1-\frac{3}{2}\right)=$$

$$=\alpha_2^2\left(\frac{2N+5}{2\tau_1+4}\right)^2\frac{4\left(\tau_1-\frac{1}{2}\right)\left(\tau_1-\frac{3}{2}\right)}{(2\tau_1)(2\tau_1+2)(4\tau_1-1)}$$

$$B\left(E2;\ N+\frac{1}{2},\tau_1,2\tau_1-\frac{1}{2}\rightarrow N+\frac{1}{2},\tau_1,2\tau_1-\frac{3}{2}\right)=$$

$$=\alpha_2^2\left(\frac{2N+5}{2\tau_1+4}\right)^2\frac{12\left(\tau_1-\frac{3}{2}\right)}{(2\tau_1)(2\tau_1+2)(4\tau_1-3)(4\tau_1-1)}$$

Alaga rules for $\mathrm{Spin}^{BF}(6)$ nuclei. States are denoted as in Ref. 4.

TABLE 4.

E_i (keV)	$(\sigma_1, \tau_1, J)_i$ →	E_f (keV)	$(\sigma_1, \tau_1, J)_f$	$B(E2)$ (e^2b^2) Exp	$B(E2)$ (e^2b^2) $\mathrm{Spin}^{BF}(6)$
		$^{197}_{79}\mathrm{Au}_{118}$			
77	$\frac{11}{2}, \frac{3}{2}, 1/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	0.260 ± 0.014	0.231
279	$\frac{11}{2}, \frac{3}{2}, 5/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	0.209 ± 0.005	0.231
547	$\frac{11}{2}, \frac{3}{2}, 7/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	0.226 ± 0.009	0.231
269	$\frac{11}{2}, \frac{5}{2}, 3/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	0.083 ± 0.006	0
503	$\frac{11}{2}, \frac{5}{2}, 5/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	≤ 0.003	0
737	$\frac{11}{2}, \frac{5}{2}, 7/2$	0	$\frac{11}{2}, \frac{1}{2}, 3/2$	≤ 0.004	0
855	$\frac{11}{2}, \frac{5}{2}, 9/2$	279	$\frac{11}{2}, \frac{3}{2}, 5/2$	0.258 ± 0.038	0.228
1231	$\frac{11}{2}, \frac{5}{2}, 11/2$	547	$\frac{11}{2}, \frac{3}{2}, 7/2$	0.269 ± 0.060	0.290

An experimental example of Alaga rules in $\mathrm{Spin}^{BF}(6)$ nuclei.

6. Conclusions

Generalized Alaga rules arise from the Wigner-Eckart and reduction theorems applied to an algebra G. These theorems allow one to write the reduced matrix elements of a tensor operator (in particular of a generator of G) in terms of isoscalar factors of the appropriate algebras. Apart from phases and $\sqrt{2L+1}$ factors the isoscalar factors are nothing but the Clebsh-Gordan coefficients coupling two representations of G. They are readily available either in analytic or in numerical form. The use of generalized Alaga rules simplifies considerably the analysis of experimental data, especially in complex situations, such as those encountered in the spectroscopy of medium-mass and heavy odd-even and odd-odd nuclei. With the help of Alaga's rules it has been possible to unreveal the complex patterns displayed by these nuclei and understand in greater detail their properties. In this respect Alaga's rules have played a major role in the development of nuclear structure physics that has occurred in the last 15 years.

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Istaknuta je uloga, koju su poopćena Alagina pravila imala u razvoju nuklearne fizike. Potcrtana je veza između grupne teorije i Alaginih pravila. Ta veza omogućuje postavljanje pravila za bilo koju situaciju opisanu simetrijom ili supersimetrijom.