YU ISSN 0015—3206
FZKAAA 22 (1) 101 (1990)

FEATURES OF THE α-CLUSTER TYPE NUCLEI IN THE FRAMEWORK OF THE RESTRICTED DYNAMICS MODEL

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Received 10 October 1989

UDC 539.142

Original scientific paper

The binding energies, excited level schemes, nucleon and charge distributions, formfactors and *E2* transition probabilities of the α-cluster type nuclei have been studied within the framework of the Strictly Restricted Dynamics Model, based on the operational decomposition of the microscopic nuclear Hamiltonian. The reasonable agreement of the theoretical results with the experimental data has been achieved.

1. The Restricted Dynamics Nuclear Models

In this paper we describe some results of the application of the Restricted Dynamics Models (RDM) to the nuclear structure. RDM, introduced and developed in the last decade, are based on the idea, proposed in Ref. 1, suggesting to derive the nuclear models of various degrees of complexity using the decomposition

$$H = H_{coll} + H_{acoll} + H' \tag{1}$$

of the microscopic nuclear Hamiltonian H with specific terms H_{coll} and H_{acoll} involved, taking into account the collective and anticollective effects. This operational decomposition for given H of the form

$$H = H_k + H_e + H_p \tag{2}$$

with the kinetic H_k, Coulomb H_e and potential H_p energy terms, produces the RDM Hamiltonian

$$H_{RD} = H_{coll} + H_{acoll} \tag{3}$$

in which the term H' of H, present in (1), is neglected (for details and references see Refs. 1—3). The Schrödinger equation for H_{RD} gives the states Ψ_{RD} and the spectrum E_{RD} of H_{RD}. Our aim is to discuss the suitability of such an approach to describe the features of the α-cluster type nuclei with the mass numbers $A = 4, 8, \ldots, 40$ and the number of protons Z equal to the number of neutrons N, i. e. with $Z = N$.

Before proceeding with a more detailed description of H_{RD} we note that the definite H is given only when the set of NN-interaction potentials $V_s(r_{ij})$ ($s = 1, 2, \ldots$) is chosen for central H_c, vectorial H_v and tensorial H_t terms of

$$H_p = H_c + H_v + H_t. \tag{4}$$

Since V_s are unknown, we shall use in applications the effective NN-potentials $V_s(r_{ij}, \xi)$ depending on the set of shape parameters ξ. We shall regard ξ as the parameters adjustable to the experimental data and shall find their values via the comparison of the spectrum E_{RD} with the experimentally observed low-lying states energies.

The complexity of the RDM Hamiltonian H_{RD} can be controlled. It depends on the symmetry requirements put on the terms H_{coll} and H_{acoll} of H_{RD}. In the most general case H_{coll} is the scalar term of H[1] with respect to the orthogonal group O_{A-1}. The anticollective term H_{acoll} is defined as the term obtained from H when two-body matrix elements $\langle \gamma lm | V_s | \gamma' lm \rangle$ of the potential, depending on the angular momentum l of the relative motion of a pair of nucleons, are substituted by matrix elements averaged with respect to l and its z-projection m in the following way:

$$\langle \gamma lm | V_s | \gamma' lm \rangle \to \delta(\gamma\gamma') \sum_{lm} \frac{1}{d_\gamma} \langle \gamma lm | V_s | \gamma lm \rangle, \tag{5}$$

where γ is the quantum numbers of the radial relative motion function used and d_γ is the number of values which lm takes for given γ. Note that in the case of non-central V_s, (5) vanishes. Consequently, only the central term H_c of H_p contributes to H_{acoll}, while all the terms of H_p contribute to H_{coll}.

The meaning of the first term in (3) is easy to explain using the matrix representation. We suppose that the operators of group O_{A-1} act on the Jacobi vectors $\vec{\xi}_i$ with i running values $i = 1, 2, \ldots, A-1$ (we use $\vec{\xi}_i$ instead of single-particle

vectors $\vec{x_i}$, with $i = 1, 2, \ldots, A$, in order to take into account explicitly the translational invariancy of the nuclear Hamiltonian H). Let ω denote the irreducible representation (IR) of the group O_{A-1} and ν is the basis of ω. Then, similarly to (5), the O_{A-1}-scalar term of H in the matrix representation has expression

$$\langle \Gamma\omega |\, H_{coll} \,|\Gamma'\omega\rangle = \sum_{\nu} \frac{1}{d_\omega} \langle \Gamma\omega\nu |\, H \,|\Gamma'\omega\nu\rangle, \tag{6}$$

where Γ denotes some set of the additional quantum numbers and d_ω gives the dimension of ω. Using the density matrix technique, the expression (6) can be presented in the form

$$\langle \Gamma\omega |\, H_{coll} \,|\Gamma'\omega\rangle =$$

$$= \sum_{\substack{vlm \\ v'l'm'}} Q_{vlm,v'l'm'}(\Gamma\omega, \Gamma'\omega) \langle vlm |\, h_{A-1,A} \,|v'l'm'\rangle, \tag{7}$$

where Q is the density matrix components, depending on the basis used, and the last factor under the sum in (7) is the two-body interaction $h_{A-1,A}$ in the matrix representation. The detailed discussion of the expression (7) can be found in Ref. 3.

Various types of simplified versions of H_{RD} have been introduced (see diagram on p. 148 in Ref.. 2 as well as Fig. 3) covering and generalizing numerous phenomenological nuclear models, such as the Bohr-Mottelson Model, Interacting Boson and Elliott's Models, Symplectic Model etc. In this paper we shall discuss the results obtained in the approach based on the symmetry of the unitary groups U_{A-1} and U_3 embedded into the unitary group $U_{3(A-1)}$ via the chain $U_{3(A-1)} \supset \supset U_3 \times U_{A-1}$.

We shall use simplified version of the restricted dynamics Hamiltonian H_{RD} (denoting it as H_0) consisting of U_{A-1} - and U_3-scalar terms $H_{0\,coll}$ and $H_{0\,acoll}$

$$H_0 = H_{0\,coll} + H_{0\,acoll}, \tag{8}$$

which can be obtained using the decompositions

$$H_{coll} = H_{0\,coll} + H'_{coll} \tag{9}$$

and

$$H_{acoll} = H_{0\,acoll} + H'_{acoll}, \tag{10}$$

(for details — see Ref. 1).

$H_{0\,coll}$ in matrix representation is given by the expression

$$\langle {}^{\Gamma E}_{\mu_3} |\, H_{0\,coll} \,|{}^{\Gamma' E}_{\mu'_3}\rangle = \sum_{\mu} \frac{1}{d_E} \langle {}^{\Gamma E\mu}_{\mu_3} |\, H \,|{}^{\Gamma' E\mu}_{\mu_3}\rangle, \tag{11}$$

where E denote both U_3- and U_{A-1}-irreducible representations, μ_3 - the basis of the U_3 irreducible representation E, μ - the basis and d_E is the dimension of

the U_{A-1} irreducible representation E. We shall use for the matrix representation (11) the $3(A-1)$-dimensional isotropic harmonic oscillator functions depending on the oscillator frequency ν_A.

For the matrix representation of $H_{0\,acoll}$ we shall take the interaction given by (5) with γ substituted by the isotropic harmonic oscillator quantum number ε, giving the number of quanta for the relative radial motion of nucleons. This type of interaction projects from the matrix of total Hamiltonian H the matrix of $H_{0\,acoll}$, defined as U_3-scalar part of H, i. e. gives the matrix of the first term in (10). In another equivalent formulation, analogous to (11),

$$\langle \Gamma E \mu |\, H_{0\,acoll}\, | \Gamma' E \mu' \rangle = \sum_{\mu_3} \frac{1}{d_{E_3}} \langle {}^{\Gamma E \mu}_{\mu_3} |\, H\, |^{\Gamma' E \mu'}_{\mu_3} \rangle, \tag{12}$$

where d_{E_3} is the dimension of the U_3 irreducible representation E.

We shall refer to the model, based on the Schrödinger equation for H_0, as to the Strictly Restricted Dynamics Model (SRDM). The Hamiltonians $H_{0\,coll}$ and $H_{0\,acoll}$ consist of the following explicitly written terms[1,4,5]:

$$H_{0\,coll} = H_{0k\,coll} + H_{0e\,coll} + H_{0c\,coll} + H_{0v\,coll} + H_{0t\,coll}, \tag{13}$$

$$H_{0\,acoll} = H_{0k\,acoll} + H_{0e\,acoll} + H_{0c\,acoll}, \tag{14}$$

where k, e, c, v and t refer, correspondingly, to the kinetic, Coulomb, central, vectorial and tensorial terms.

Due to the high symmetry the SRDM Hamiltonian H_0 conserves both U_{A-1}- and U_3-irreducible representations $E \equiv [E_1, E_2, E_3]$ (see (11) and (12)), as well as SU_3-irreducible representations (λ, μ) with $\lambda = E_1 - E_2$, $\mu = E_2 - E_3$, the total isospin T and its z-projection $M_T = (N-Z)/2$. Term $H_{0\,coll}$ of H_0 also conserves the space symmetry f given by the Young patterns $f = [4\ldots4\,3\ldots3\,2\ldots2\,1\ldots1]$; in some states of $H_{0\,acoll}$ the symmetry f is also conserved. All the results, discussed in this paper, have been obtained in the diagonal with respect to f approach and for the irreducible states $\omega \equiv (\omega_1, \omega_2, \omega_3)$ of the orthogonal group O_{A-1} repeating the labelling of U_{A-1} states, i. e. for $\omega_1 = E_1$, $\omega_2 = E_2$, $\omega_3 = E_3$ (see for details §15 in Ref. 3). In many cases of low-lying states of even-even nuclei H_0 also conserves the total spin S and the orbital angular momentum L, while in the odd nuclei case the vectorial and tensorial terms of H_0 couple S with L to the total angular momentum J and its z-projection M_J. We shall distinguish the multiplicity of L in (λ, μ) by means of the Elliott's quantum number K.

Summarizing we conclude that, after diagonalization with respect to K, the eigenvalues E_0 of H_0 depend on $N_0\,(\lambda, \mu) f\, T\, M_T\, S\, L\, J$, where $N_0 = E_1 + E_2 + E_3$. The states

$$\Psi \equiv |E_0 N_0\,(\lambda, \mu) f\, T\, M_T\, S\, L\, J\, M_J\rangle \tag{15}$$

of H_0 have the positive parity, if N_0 is even, and the negative parity, if N_0 is odd. For most of even-even nuclei discussed in this paper, when H_0 conserves L, K takes only one value for given (λ, μ) and L. In such cases (15) is additionally characterized by K.

At this point we end the short review on RDM. In next sections we shall discuss SRDM applications to the α-cluster type even-even nuclei with mass numbers $A = 4, 8, \ldots, 40$, using the wave function localization correlated with data on nuclear radii. In Sect. 2—4 we shall compare theoretical results with experimental data on binding energies, level schemes, reduced transition probabilities $B(E2)$, the nucleon and charge density distributions as well as formfactors. The discussion of the results as well as the prospects for future research within the framework of restricted dynamics models are discussed in the last section.

2. *The binding energies and level schemes*

In the light α-cluster type nuclei region the calculations, based on the density matrix technique, have been carried out with the SRDM Hamiltonian (8), depending on the effective multipole-Gauss type NN-potential

$$V(r_{ij}) = V_0 (2r_{ij}^2/r_p^2)^{q/2} \exp(-2\sigma r_{ij}^2/r_p^2), \tag{16}$$

common for both collective and anticollective terms. In (16) $\sigma = 0, \pm 1$, q takes some integer values (we shall use $q = 0, \pm 2, \pm 4, +6, +8$) and r_p^2 is the scale parameter of $V(r_{ij})$. The matrix elements of $V(r_{ij})$ on the basis of the isotropic harmonic oscillator functions depend on ν_A and r_p^2 in the form $\nu_A r_p^2$; this dependence can be easily regained if r_p^2 in (16) is substituted by $\nu_A r_p^2$.

For the oscillator frequency ν_A we shall use the expression (see (7) in Ref. 5)

$$\nu_A = \frac{5(N_0 + 3(A-1)/2)}{3r_0^2 A^{5/3}}, \tag{17}$$

$$r_0^2 = a + b/A + c/A^2, \tag{18}$$

where a, b, c are the constants fitted to the experimental data on the nuclei radii (for the numerical values of a, b, c — see Table 1).

TABLE 1.

Nuclei	$[f_0]$	$[E_1, E_2, E_3]$	(λ, μ)	K	ν_A [fm]$^{-2}$
^{4}He	$[4^1]$	[0, 0, 0]	(0, 0)	0	0.43763686
^{8}Be	$[4^2]$	[4, 0, 0]	(4, 0)	0	0.40446468
^{12}C	$[4^3]$	[4, 4, 0]	(0, 4)	0	0.35043776
^{16}O	$[4^4]$	[4, 4, 4]	(0, 0)	0	0.30893287
^{20}Ne	$[4^5]$	[12, 4, 4]	(8, 0)	0	0.30213598
^{24}Mg	$[4^6]$	[16, 8, 4]	(8, 4)	0, 2, 4	0.28935870
^{28}Si	$[4^7]$	[20, 8, 8]	(12, 0)	0	0.27545501
^{32}S	$[4^8]$	[20, 16, 8]	(4, 8)	0, 2, 4	0.26200882
^{36}Ar	$[4^9]$	[20, 20, 12]	(0, 8)	0	0.24952015
^{40}Ca	$[4^{10}]$	[20, 20, 20]	(0, 0)	0	0.23809668

Characteristics of α-cluster type nuclei.

Note: The oscillator frequency ν_A values are calculated from the expressions (17), (18) with the following values of the constants a, b, c (in [fm]^2) : $a = 1.71856$, $b = 2.459589$, $c = -10.119970$. (see. Ref 8).

The low-lying states of the α-cluster type nuclei with $A = 4, 8, \ldots, 40$ have the space symmetry $f_0 = [4 \ldots 4]$ and other characteristics listed in Table 1. In the case of the space symmetry f_0 the spin and isospin of nucleus have the values $S = 0$ and $T = 0$, therefore the matrix elements of the terms $H_{0v\,coll}$ and $H_{0t\,coll}$ in (13) vanish. In the states, characterized by the Elliott's quantum number K, the only possible non-diagonal elements of the nonvanishing terms of H_0, explicitly given in (13) and (14), are those with respect to K.

Let us discuss the notations we shall use for the matrix elements.

1) For the collective kinetic energy term (see (1.16) in Ref. 4 or (5) in Ref. 5):

$$E_{0k\,coll}(K'L, KL) = \delta(K'K)\, E_{kin}/2, \tag{19}$$

where (see (1.17) in Ref. 4 and (6) in Ref. 5)

$$E_{kin}\,[\mathrm{MeV}] = 20.7357\,(N_0 + 3\,(A-1)/2)\,\nu_A; \tag{20}$$

2) for the collective Coulomb energy term (see (2.4) in Ref. 6 or (57) in Ref. 7):

$$E_{0c\,coll}(K'L, KL) = \frac{e^2}{8}(A - 2M_T)\,(A - 2M_T - 2)\,\mathcal{C}^{coll}_{cW}(K'L, KL), \tag{21}$$

where e — elementary charge ($e^2 = 1.43986$ MeV · fm) and $\mathcal{C}^{coll}_{cW}(K'L, KL)$ — the matrix elements of the Coulomb interaction given below;

3) for the collective central NN-interaction energy, assuming the radial dependence (16) common for Wigner (W), Majorana (M), Bartlett (B) and Heisenberg (H) terms:

$$E_{0c\,coll}(K'L, KL) = [A_0 C_W + \Lambda(f)\, C_M + \Lambda(S)\, C_B + \\ + \Lambda(T)\, C_H]\, \mathcal{C}^{coll}_{cW}(K'L, KL), \tag{22}$$

(see (2.5), (2.6) in Ref. 6 and (3) in Ref. 8), where $A_0 = A(A-1)/2$ and $\Lambda(f)$, $\Lambda(S)$, $\Lambda(T)$ correspondingly, denote the eigenvalues of symmetric group class operators, consisting of the orbital, spin and isospin exchange operators (see (10), (11), (12) in Ref. 5), C_W, C_M, C_B, C_H — the exchange constants of the central interaction, we shall use the normalization condition $C_W + C_M + C_B - C_H = -1$[5,6]. The central NN-interaction matrix elements $\mathcal{C}^{coll}_{cW}(K'L, KL)$ will be described later on. Note that (22) has been obtained substituting the space exchange operators by spin-isospin exchange operators, which is valid in the antisymmetric states.

4) For the anticollective kinetic energy term:

$$E_{0k\,coll}(K'L, KL) = \delta(K'K)\, E_{kin}/2; \tag{23}$$

5) for the anticollective Coulomb energy term (see (45) in Ref. 7):

$$E_{0e\,acoll}(K'L, KL) = \delta(K'K)\, e^2 (\mathscr{E}^+_{ae} + 9\mathscr{E}^-_{ae}/5)/6, \tag{24}$$

with $\mathscr{E}^+_{ae}$ and $\mathscr{E}^-_{ae}$ discussed later on;

6) for the anticollective central NN-interacting energy term (see (12.12) in Ref. 9 and (12) in Ref. 8):

$$E_{0c\,acoll}(K'L, KL) = \delta(K'K)\{(C_W + C_M)\mathscr{E}^+_{ac} + [(C_W - C_M) + \\ + 4(C_B + C_H)/5]\mathscr{E}^-_{ac}\}, \tag{25}$$

with $\mathscr{E}^+_{ac}$ and $\mathscr{E}^-_{ac}$ given further on.

Similarly to (7), the U_{A-1} — invariant Wigner interaction matrix elements for Coulomb and effective NN-potential, denoted in (21) and (22) as $\mathscr{E}^{coll}_{eW}(K'L, KL)$ and $\mathscr{E}^{coll}_{cW}(K'L, KL)$, can be written in terms of the collective density matrix components $Q^{E\,coll}_{\varepsilon_0\varepsilon_0}$ and Talmi integrals $I^{e,c}_{\varepsilon_0\varepsilon_0}$

$$\mathscr{E}^{coll}_{eW}(K'L, KL) = \sum_{\varepsilon_0} Q^{E\,coll}_{\varepsilon_0\varepsilon_0}(K'L, KL)\, I^{e}_{\varepsilon_0\varepsilon_0}(\nu_A), \tag{26}$$

$$\mathscr{E}^{coll}_{cW}(K'L, KL) = \sum_{\varepsilon_0} Q^{E\,coll}_{\varepsilon_0\varepsilon_0}(K'L, KL)\, I^{c}_{\varepsilon_0\varepsilon_0}(\nu_A, V_0, r_p, q) \tag{27}$$

(see formulae (4)—(11) in Ref. 8, where more details on the collective density matrix calculation technique are given).

In the case of the NN-potential (16) the Talmi integrals have the expression[9,6]:

$$I^{c}_{\varepsilon_0\varepsilon_0}(\nu_A, V_0, r_p, q) = [V_0\,(2/(\nu_A r_p^2))^{|q|/2}/(1 + 2\sigma/(\nu_A r_p^2))^{\varepsilon_0+((q+3)/2)}] \times$$

$$\times \left\{ \begin{array}{ll} \prod\limits_{\tau=1,2,\ldots}^{\varepsilon_0} (2\tau + q + 1)/(2\tau + 1), & \text{if } \varepsilon_0 > 0 \\ 1 & \text{, if } \varepsilon_0 = 0 \end{array} \right\} (2/(3 + |q|)) \times$$

$$\times \left[\prod_{\tau=0,1,\ldots}^{\mathrm{Int}\,(q/2)+1} (3 + |q| - 2\tau)/2\right] \{2/[1 + (1 - \pi)(2\,\mathrm{Int}\,(|q|/2) - |q|)]^{1/2}\}, \tag{28}$$

where $\mathrm{Int}(x)$ denote the integer part of x. For Coulomb interaction, following from (16) with $V_0 = 1$, $r_p = 1$, $\sigma = 0$ and $q = -1$, (28) gives

$$\left. \begin{array}{l} I^{e}_{00}(\nu_A) = \sqrt{2\nu_A/\pi}, \\ I^{e}_{\varepsilon_0\varepsilon_0}(\nu_A) = \dfrac{2}{3} \cdot \dfrac{4}{5} \cdot \dfrac{6}{7} \cdots \dfrac{2\varepsilon_0}{2\varepsilon_0 + 1} I^{e}_{00}(\nu_A); (\varepsilon_0 = 1, 2, \ldots). \end{array} \right\} \tag{29}$$

Analogically to (26) and (27) matrix elements of the U_3-invariant Coulomb and central NN-interaction in (24) and (25) can be written as

$$\mathscr{E}^{\pm}_{ae} = \sum_{\varepsilon_0} Q^{E(\pm)\,a\,coll}_{\varepsilon_0\varepsilon_0} I^{e}_{\varepsilon_0\varepsilon_0}(\nu_A), \tag{30}$$

$$\mathscr{E}^{\pm}_{ac} = \sum_{\varepsilon_0} Q^{E(\pm)\,a\,coll}_{\varepsilon_0\varepsilon_0} I^{c}_{\varepsilon_0\varepsilon_0}(\nu_A, V_0, r_p, q), \tag{31}$$

where $Q^{E(\pm)\,a\,coll}_{\varepsilon_0\varepsilon_0}$ denotes the anticollective density matrix components. In particular cases they have the simple algebraic expressions (see (62), (63), (65) in Ref. 7 and Ref. 8) in terms of the Casimir operators of the symmetry groups involved, covering all the states considered in this paper.

For α-cluster type nuclei with $f_0 = [4\ldots4]$ from (25) and (22) there follow the expressions

$$E_{0c\,acoll}(K'L, KL) = \delta(K'K)(C_1\mathscr{E}^{+}_{c} + C_2\mathscr{E}^{-}_{c}), \tag{32}$$

$$E_{0c\,coll}(K'L, KL) = [(3C_1 - 5C_2)A/4 + (3C_1 + 5C_2)A^2/16]\,\mathscr{E}^{coll}_{eW}(K'L, KL), \tag{33}$$

with

$$\left.\begin{aligned} C_1 &= V_0(C_W + C_M), \\ C_2 &= V_0[(C_W - C_M) + 4(C_B + C_H)/5]. \end{aligned}\right\} \tag{34}$$

Therefore, in the case of $f_0 = [4\ldots4]$ and effective NN-potential (16) with definite integer $q/2$, the matrix elements of H_0 depend on 3 adjustable parameters, namely on C_1, C_2 and r_p.

Let us denote by $\mathscr{E}_{0\,coll}$ the matrix elements (11) of collective Hamiltonian with $\Gamma' = fN_0EK'L$ and $\Gamma = fN_0EKL$, listed in Table 1, and present them in the form

$$\mathscr{E}_{0\,coll}(K'L, KL) = E_{0k\,coll}(K'L, KL) + E_{0e\,coll}(K'L, KL) + E_{0c\,coll}(K'L, KL), \tag{35}$$

with the corresponding terms following from (19), (21) and (33). In practical calculations we employ the orthogonal basis constructed from the Elliott's states, using the explicit expression of the overlap integrals[10,11]. The diagonalization procedure gives the eigenvalues and states of H_0.

In most cases of α-cluster type nuclei considered, when (λ, μ) have a form

$$\left.\begin{aligned} (\lambda, \mu) &= (\lambda, 0) = (\varkappa, 0), \\ (\lambda, \mu) &= (0, \mu) = (0, \varkappa) \end{aligned}\right\} \tag{36}$$

only one Elliott's band exists with $K = 0$, $L = J = 0, 2, 4, \ldots, \varkappa$. For states (36) the energy expression for SRDM Hamiltonian (8) is very simple and the eigenvalue E_0 of SRDM Hamiltonian is equal to the mean value of H_0. In order to find the linear equations for C_1 and C_2, using both the [formulae (32), (33), (19), (21), (23), (24) and the experimental data, we present E_0 in the form

$$C_1 \mathscr{E}_1 (L, \nu_A, q, r_p) + C_2 \mathscr{E}_2 (L, \nu_A, q, r_p) + \mathscr{E}^0 (L, \nu_A) = 0, \tag{37}$$

where

$$\left.\begin{aligned} \mathscr{E}_1 (L, \nu_A, q, r_p) &= \mathscr{E}^+_{ac} (L, \nu_A, q, r_p) + \frac{3A\,(A/4 + 1)}{4} \mathscr{E}^{coll}_{cW} (L, \nu_A, q, r_p), \\ \mathscr{E}_2 (L, \nu_A, q, r_p) &= \mathscr{E}^-_{ac} (L, \nu_A, q, r_p) + \frac{5A\,(A/4 - 1)}{4} \mathscr{E}^{coll}_{cW} (L, \nu_A, q, r_p), \end{aligned}\right\} \tag{38}$$

and all other terms of E_0, not depending on the effective NN-potential parameters q and r_p, as well as the experimental binding energy $E_b^{exp}(L = 0)$ and excited level energy $E_{ex}^{exp}(L)$ values, are put together in the expression

$$\mathscr{E}^0 (L, \nu_A) = E_{0k\,coll} (\nu_A) + E_{0k\,acoll} (\nu_A) + E_{0e\,coll} (L, \nu_A) +$$

$$+ E_{0e\,acoll} (\nu_A) - (E_b^{exp} (L = 0) + E_{ex}^{exp} (L)). \tag{39}$$

The system of equations (37) is useful for both the energy and fitted parameters calculations. The number of equations in (37) depend on the number of the experimental data used. With respect to C_1 and C_2 the system (37) has been solved for α-cluster type nuclei with $A = 4$, 8, 12, 16, 20, 40 using the standard LSQF procedure for the linear equation system. For a given $q/2$ such a value of r_p in the interval $0.6 < r_p < 2.0$ fm has been found which minimize the mean square deviation $\Delta E = [\sum_{i=1}^{n} (E_i^t - E_i^e)^2/n]^{1/2}$ of the calculated energy values E^t form the experimental ones E^e. The numerical values of C_1, C_2 and r_p as well as the calculated energies, given in Table 2, show a reasonable agreement with the experimental data.

The results of calculations listed in Table 2 have been obtained with the complete set of the collective density matrix components. The Tables 3 and 4 contain some preliminary results of calculations, performed by taking into account only the SU_3-irreducible component [42] of the collective density matrix. The values of the fitted parameters, given in Table 4, are calculated minimizing ΔE by the standard method of LSQF, covering the values of C_1, C_2 and r_p in the intervals $-300 < C_1 < -150$ MeV, $-150 < C_2 < -50$ MeV and $0.6 < r_p < 1.0$ fm.

TABLE 2.

	$q/2 =$	−2	−1	0	1	2	3	4
	r_p	1.94345	1.26043	0.998665	0.851845	0.754987	0.685018	0.631447
	C_1	− 10.7600	− 69.4785	−157.064	−223.836	−241.549	−214.755	−164.898
	C_2	− 2.05411	− 13.3174	− 30.8029	− 44.5563	− 48.5585	− 43.4715	− 33.5499
	ΔE [MeV]	1.280	1.266	1.278	1.288	1.294	1.299	1.303
Nuclei	$J\pi$	E^e [MeV]	Calculated energies E^t [MeV]					
^{4}He	0^+	− 28.30	− 30.19	− 29.95	− 29.86	− 29.82	− 29.79	− 29.77	− 29.76
^{8}Be	0^+	− 56.50	− 58.79	− 58.03	− 57.80	− 57.69	− 57.62	− 57.58	− 57.55
	2^+	3.04	5.60	5.31	5.22	5.18	5.15	5.13	5.12
	4^+	11.40	15.94	15.40	15.25	15.19	15.15	15.12	15.11
^{12}C	0^+	− 92.16	− 90.41	− 90.32	− 90.33	− 90.33	− 90.34	− 90.35	− 90.35
	2^+	4.44	3.98	3.87	3.85	3.84	3.84	3.83	3.83
	4^+	14.08	12.23	12.00	11.97	11.96	11.96	11.96	11.96
^{16}O	0^+	−127.62	−127.10	−127.68	−127.87	−127.96	−128.01	−128.05	−128.07
^{20}Ne	0^+	−160.65	−159.95	−160.22	−160.32	−160.38	−160.42	−160.44	−160.46
	2^+	1.63	0.93	0.94	0.95	0.95	0.95	0.95	0.96
	4^+	4.25	3.09	3.11	3.13	3.14	3.15	3.15	3.16
	6^+	8.78	6.39	6.44	6.48	6.50	6.52	6.53	6.54
	8^+	11.95	10.75	10.83	10.91	10.95	10.98	11.00	11.01
^{40}Ca	0^+	−342.05	−341.79	−341.27	−341.08	−340.99	−340.93	−340.89	−340.86

Least square fit solutions for $A = 4, 8, 12, 16, 20, 40$ nuclei.

Note: All energy values are calculated with respect to the experimental binding energies $E_B^e(L = 0) < 0$, but for the convenience the excited level energies are presented assuming $E_B^e(L = 0) = 0$. For the References of the experimental data — See Ref. 8.

TABLE 3.

Nuclei	K	$E(L=0)$		$E(L=2)$		$E(L=3)$		$E(L=4)$		$E(L=5)$		$E(L=6)$		$E(L=7)$		$E(L=8)$	
		Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.
^{4}He	0	− 25.55	− 28.30	—	—	—	—	—	—	—	—	—	—	—	—	—	—
^{8}Be	0	− 56.94	− 56.50	5.08	3.04	—	—	16.93	11.40	—	—	—	—	—	—	—	—
^{12}C	0	− 90.33	− 92.16	4.26	4.44	—	—	14.19	14.08	—	—	—	—	—	—	—	—
^{16}O	0	−125.93	−127.62	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	0	−161.86	−160.65	1.11	1.63	—	—	3.70	4.25	—	—	7.76	8.78	—	—	13.31	11.95
^{20}Ne	0	−197.21	−198.26	0.83	1.37	—	—	2.66	4.12	—	—	5.51	8.11	—	—	9.31	13.25
^{24}Mg	2	—	—	1.25	4.24	2.08	5.24	3.33	6.01	6.79	7.81	6.60	9.53	8.90	12.35	10.48	14.15
	4	—	—	—	—	—	—	4.40	8.44	9.02	10.58	7.52	12.00	9.30	?	11.87	?
	0	−232.00	−236.54	0.66	1.78	—	—	2.20	4.62	—	—	4.61	8.54	—	—	7.91	?
^{28}Si	0	−267.87	−271.78	0.74	2.23	—	—	2.51	4.46	—	—	5.31	?	—	—	9.07	?
^{32}S	2	—	—	0.57	4.28	1.30	5.41	2.22	6.41	5.11	?	4.78	?	6.82	?	8.14	?
	4	—	—	—	—	—	—	1.79	6.85	2.94	?	4.43	?	6.34	?	8.45	?
^{36}Ar	0	−304.45	−306.72	0.59	1.97	—	—	1.96	4.41	—	—	4.12	?	—	—	7.06	?
^{40}Ca	0	−343.77	−342.05	—	—	—	—	—	—	—	—	—	—	—	—	—	—

Calculated and experimental energies (in MeV) for $A = 4, 8, \ldots, 40$ nuclei at $q/2 = 0$.

Notes: 1) See Note of Table 2. 2) See Table 4 at $q/2 = 0$ for adjustable parameter values.

TABLE 4.

	$q/2 =$	0	1	2	3	4
	r_p	0.905243	0.869043	0.697512	0.618438	0.609351
	C_1	−194.37062	−210.13020	−284.72494	−268.04043	−174.76728
	C_2	− 98.90730	− 55.23642	−138.12101	−141.94256	− 71.43705
	ΔE [MeV]	2.09	2.29	2.12	2.12	2.16
Nuclei	$E_b^e\,(L=0)$	Calculated binding energies $E_b\,(L=0)$				
^{4}He	− 28.30	− 27.55	− 28.79	− 27.52	− 27.45	− 27.69
^{8}Be	− 56.50	− 56.94	− 56.12	− 56.63	− 57.02	− 55.98
^{12}C	− 92.16	− 90.33	− 89.83	− 90.20	− 90.54	− 89.75
^{16}O	−127.62	−125.93	−128.18	−126.17	−125.97	−126.73
^{20}Ne	−160.65	−161.86	−161.69	−161.86	−162.07	−161.67
^{24}Mg	−198.26	−197.21	−195.91	−197.11	−197.45	−196.61
^{28}Si	−236.54	−232.00	−230.98	−231.92	−232.13	−231.59
^{32}S	−271.78	−267.87	−267.41	−267.82	−267.84	−267.74
^{36}Ar	−306.72	−304.45	−305.52	−304.52	−304.18	−305.00
^{40}Ca	−342.05	−343.77	−346.80	−343.98	−343.21	−345.16

Calculated and experimental binding energies (in MeV) for $A = 4, 8, \ldots, 40$ nuclei at $q/2 = 0, 1, \ldots 4$.

Notes: 1) The adjustable parameter and mean square deviation ΔE values refer to complete set of experimental energies ($n = 43$) included in calculation. 2) for the References of the experimental data on $E_b^e\,(L=0)$ values — see Ref. 8.

3. *Density distribution and formfactors*

The nucleon density $\varrho(\vec{r})$ or proton density $\varrho_Z(\vec{r})$ at the point $\vec{r}$ is given by the mean value of the operators

$$\left.\begin{aligned} \hat{\varrho}(\vec{r}) &= \frac{1}{A}\sum_{i=1}^{A}\delta(\vec{r}_i - \vec{r}), \\ \hat{\varrho}_Z(\vec{r}) &= \frac{1}{Z}\sum_{i=1}^{A}\left(\frac{1}{2} - \hat{t}_0^1(i)\right)\delta(\vec{r}_i - \vec{r}), \end{aligned}\right\} \tag{40}$$

where $\vec{r}_i$ and $\vec{r}$ are the nucleon and density variable vectors in the centrum-of-mass frame and $\hat{t}_0^1(i)$ — z-projections of the nucleon isospin momenta. Using the algebraic technique of the symmetric groups $S_A^{(r)}$ and $S_A^{(\tau)}$, acting correspondingly on the indices of space and isospin variables of nucleons, we obtain the expression (for details — see Ref. 12)

$$\hat{\varrho}_Z(\vec{r}) = \hat{\varrho}(\vec{r}) + \Delta\hat{\varrho}(\vec{r}) \tag{41}$$

with $S_A^{(r)} \times S_A^{(\tau)}$ — scalar term $\hat{\varrho}(\vec{r})$ and $\Delta\hat{\varrho}(\vec{r})$, characterized by the $S_A^{(r)} \times S_A^{(\tau)}$ — irreducible representation $s_1 \equiv [A-1, 1]$. In calculations we shall use the density operator presented in the form (41).

In the RDM approach we calculate the mean value of (41) in the H_{RD} states. In the case of α-cluster type nuclei, when $f = f_0 = [4...4]$, from the selection rules for the decomposition of the direct product $s_1 \times f_0$ it follows (see (9.27) in Ref. 9) that $s_1 \times f_0$ do not contain f_0. Thus the matrix element of $\Delta\hat{\varrho}(\vec{r})$ vanishes identically, and the mean value of $\hat{\varrho}_Z(\vec{r})$ is equal to that for $\hat{\varrho}(\vec{r})$.

In order to calculate the matrix elements of $\hat{\varrho}(\vec{r})$, we shall use the following expression of the δ-function in terms of the spherical harmonics $Y_{M_0}^{L_0}$:

$$\delta(\vec{r_i} - \vec{r}) = \frac{\delta(r_i - r)}{r_i^2} \sum_{L_0 M_0} Y_{M_0}^{L_0}(\vec{r_i}) Y^{*L_0}_{M_0}(\vec{r^0}), \tag{42}$$

where r, r_i denote the absolute values of $\vec{r}$, $\vec{r_i}$ and $\vec{r^0}$, $\vec{r_i^0}$ give the spherical angles of $\vec{r}$, $\vec{r_i}$. Using (42) we present the mean value of $\varrho(\vec{r})$,

$$\varrho(\vec{r}) = \varrho_Z(\vec{r}) = \langle \Gamma f_0 | \hat{\varrho}(\vec{r}) | \Gamma f_0 \rangle \tag{43}$$

in the states characterized by Γf_0 in the form

$$\varrho(\vec{r}) = \sum_{L_0 M_0} \varrho_{L_0 M_0}(r) Y^{*L_0}_{M_0}(\vec{r^0}), \tag{44}$$

with $L_0 M_0$ taking the values, allowed by the set of quantum numbers Γ used.

We shall find the radial part $\varrho_{L_0 M_0}(r)$ of $\varrho(\vec{r})$, using the density matrix technique permitting to present $\varrho_{L_0 M_0}(r)$ in the form

$$\varrho_{L_0 M_0}(r) = \sum_{\substack{slm \\ s'l'm'}} Q^{L_0 M_0}_{slm, s'l'm'} \langle slm | \hat{\varrho}^{L_0}_{M_0} | s'l'm' \rangle, \tag{45}$$

where $Q^{L_0 M_0}$ denote the one-particle type density matrix SO_3-irreducible components and

$$\hat{\varrho}^{L_0}_{M_0} = \frac{\delta(r_A - r)}{r_A^2} \hat{Y}^{L_0}_{M_0}(\vec{r^0_A}). \tag{46}$$

The matrix elements in the r. h. s. of (45) are taken in the three-dimensional harmonic oscillator basis $\varphi_{slm}(\vec{\xi})$, depending on the components of the Jacobi vector

$$\vec{\xi} = -\sqrt{\frac{A}{A+1}}\vec{r}_A. \tag{47}$$

The expression (45) has been obtained in Ref. 12. The method of derivation used is explained in detail on the similar example in § 23 of Ref. 9.

Let us present $\hat{\varrho}(\vec{r})$ in the form[12)]

$$\hat{\varrho}(\vec{r}) = \hat{\varrho}_0(\vec{r}) + \hat{\varrho}'(\vec{r}), \tag{48}$$

where

$$\hat{\varrho}_0(\vec{r}) = \hat{\varrho}_{coll}(\vec{r}) + \hat{\varrho}_{acoll}(\vec{r}). \tag{49}$$

In SRDM approach we neglect the last term in (48) and take $\hat{\varrho}_0(\vec{r})$ possessing the symmetry of the Hamiltonian H_0, i. e. assuming that $\hat{\varrho}_{coll}(\vec{r})$ is the U_{A-1}-scalar term and $\hat{\varrho}_{acoll}(\vec{r})$ is the U_3-scalar term of $\hat{\varrho}(\vec{r})$. In this paper we shall discuss only the radial density distribution given by the mean values of the operator $\hat{\varrho}_0(\vec{r})$, obtained averaging $\hat{\varrho}_0(\vec{r})$ with respect to the spherical angles of $\vec{r}$.

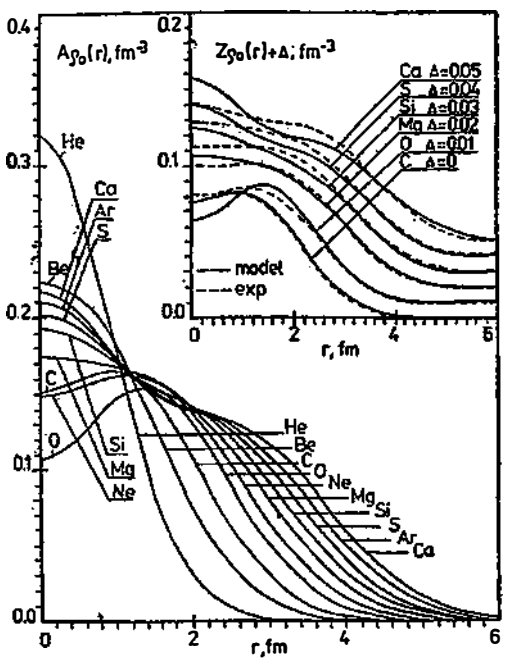

Fig. 1. Nucleon density distributions for α-cluster type nuclei.

The calculated density of the nucleon distribution $A\varrho_0(r)$ in the ground states of H_0 of the α-cluster type nuclei with the frequency parameter (17) used is shown in Fig. 1 with the symbols of nuclei indicated for each curve. It is remarkable that calculated curves essentially differ (compare, for instance, $A\varrho_0(r)$ for ^{4}He, ^{16}O and ^{32}S), thus the theory gives non-trivial dependence on A. In the top right-

hand corner of Fig. 1 the proton density distribution $Z\varrho_0(r)$ is compared with the experimental data. For the sake of clearness in this fragment of Fig. 1 curves are shifted by the constant Δ.

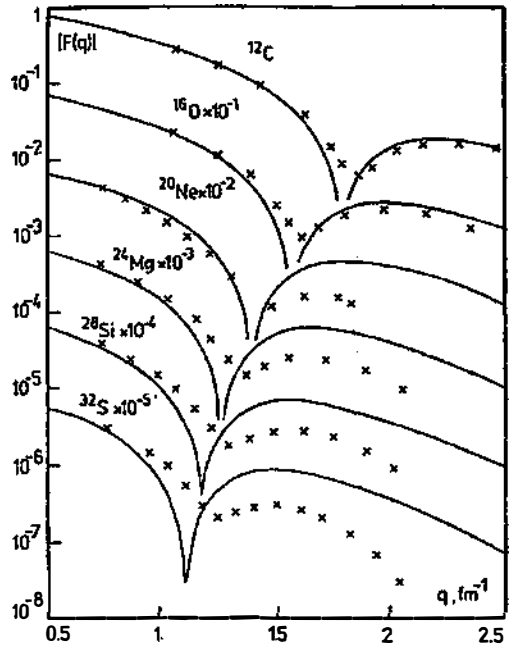

Fig. 2. Charge formfactors for α-cluster type nuclei.

In Fig. 2 the charge formfactors $F(q)$ are shown, related with $\varrho_0(r)$ by means of the Fourier transformation

$$F(q) = \frac{1}{4\pi}\int \varrho_0(r)\frac{\sin(qr)}{qr}\,\mathrm{d}r. \tag{50}$$

Calculated curves (solid) and experimental data (indicated by asterisks) for α-cluster type nuclei with $A = 12, 16, \ldots, 32$ are shifted by the numerical factor, adopted to the logarithmic scale for each nucleus separately. From Fig. 2 it can be seen that the difference between the calculated and measured formfactors increases with both A and the values of the transfered momenta q. The observed diffractional minimum is also presented in the theoretical curves.

The results shown in Fig. 1 and Fig. 2 indicate that both the radial density distribution and the formfactors, calculated in H_0 states, give a reasonable description of the density in the real nuclei, including the specific dependence on A. The analysis of the density distribution for the collective and the anticollective terms of $\hat{\varrho}_0$ has been also carried out, showing the essential contribution of both terms of $\hat{\varrho}_0$ to the mean value of the density operator $\hat{\varrho}_0$.

4. *E2 transition probabilities*

We take $E2$ transition operator $\hat{Q}^{2m}$ in the irreducible form with respect to the transformations of the O_{A-1} and $U_3 \times U_{A-1}$-groups[13]. In the SRDM approach we use the expression

$$\hat{Q}^{2m} = \hat{Q}_0^{2m} + \hat{Q}'^{2m}, \tag{51}$$

where $\hat{Q}_0^{2m}$ is the U_{A-1}-scalar term of $\hat{Q}^{2m}$ with the symmetry of the strictly restricted collective Hamiltonian $H_{0\,coll}$. The inspection shows that up to the simple factors, $\hat{Q}_0^{2m}$ is equal to the $E2$ transition operator expressed in Ref. 14 in terms of the SU_3-infinitesimal operators. In calculation we have used a closed formula of the reduced transition probabilities $B(E2)$ for the operator $\hat{Q}_0^{2m}$, acting within the (λ, μ)-bands (for details — see Ref. 13). Note, that in case of the $E2$ transition probabilities within the (λ, μ)-bands, for the states with $f_0 = [4...4]$ only the first term in (51) contributes. The final expression for the calculation of $B(E2)$ values is[13,15]

$$B(E2; J'K'L' \to JKL) = \delta(SS')\,\delta(M_T M_T')\,\delta(TT')\,\delta(\omega\omega') \times$$

$$\times\, \delta(\alpha\alpha')\,\delta(ff')\,\delta(\tilde{\alpha}\tilde{\alpha}')\,\delta((\lambda, \mu), (\lambda', \mu')) \times \tag{52}$$

$$\times\, (2L+1)(2J+1) \begin{Bmatrix} J & 2 & J' \\ L' & S & L \end{Bmatrix}^2 \tilde{B}(E2; K'L' \to KL),$$

where α and $\tilde{\alpha}$ are the multiplicity indices in group chains $O_{A-1} \supset S_A$ and $U_4 \supset$ $\supset SU_2 \times SU_2$ and

$$\tilde{B}(E2; K'L' \to KL) = \frac{15e^2 (A - 2M_T)^2}{32\pi A^2 \nu_A^2} \left[\langle {}^{(\lambda,\mu)}_{KL} \| \hat{Q}^S \| {}^{(\lambda,\mu)}_{K'L'} \rangle \right]^2. \tag{53}$$

Making use of the overlap integrals, we obtain

$$\langle {}^{(\lambda,\mu)}_{KL} \| \hat{Q}^S \| {}^{(\lambda,\mu)}_{K'L'} \rangle = (2L'+1)(2L+1)^{-1} \times$$

$$\times \sum_{K''\nu} C^{2L'L}_{\nu K' K''}\, b^{(\lambda,\mu)}_{L'K';LK''}\, P^{(\lambda,\mu)K''}_{KL(00)} / [P^{(\lambda,\mu)K}_{KL(00)}\, P^{(\lambda,\mu)K'}_{K'L'(00)}]^{1/2} \tag{54}$$

with P explicitly given in Refs. 10 and 11, and[14,16]

$$b^{(\lambda,\mu)}_{L'K';LK''} = \delta(K''K) \frac{1}{\sqrt{6}} \begin{bmatrix} 2\lambda + \mu + 3 + (L - L')(L + L' + 1)/2 \\ -\lambda - 2\mu - 3 - (L - L')(L + L' + 1)/2 \end{bmatrix} -$$

$$- \delta(K'', K'+2)\left[\frac{\sqrt{(\mu - K')(\mu + K' + 2)/4}}{\sqrt{(\lambda - K')(\lambda + K' + 2)/4}}\right] - \delta(K'', K-2) \times \tag{55}$$

$$\times \left[\frac{\sqrt{(\mu + K')(\mu - K' + 2)/4}}{\sqrt{(\lambda + K')(\lambda - K' + 2)/4}}\right],$$

where the upper row corresponds to the case $\lambda \geqslant \mu$ and lower row — to the case $\lambda < \mu$.

In the case, when Elliott's states are mixed with the amplitudes C_{KL}, instead of (52), one must use

$$B(E2; J' \to J) = \Big| \sum_{K', K} C_{K'L'} C_{KL} \left[B(E2; J'K'L' \to JKL)\right]^{1/2} \Big|^2. \tag{56}$$

Let us discuss the simplification of (52)—(55) for the states (36). For $E2$ transitions between the levels with $J' = L' = L + 2$ and $J = L$ of the Elliott's band with $K = 0$ taking into account the explicit formulae for the Clebsch-Gordan coefficient

$$C^{2\ L+2\ L}_{0\ 0\ 0} = \sqrt{\frac{3(L+1)(L+2)}{2(2L+3)(2L+5)}} \tag{57}$$

and the overlap integral[17]

$$P^{(\varkappa,0)0}_{0L(00)} = P^{(0,\varkappa)0}_{0L(00)} = \frac{(2L+1)\, 2^L\, \varkappa!\, ((\varkappa + L)/2)!}{(L + \varkappa + 1)!\, ((\varkappa - L)/2)!}, \tag{58}$$

from (52)—(55) follows that

$$B(E2; L+2 \to L) = \frac{15e^2 (A - 2M_T)^2 (\varkappa - L)(L+1)(L+2)(\varkappa + L + 3)}{32\pi A^2 \nu_A^2 \qquad (2L+3)(2L+5)}. \tag{59}$$

This formula differ only by simple factors from the similar expression obtained in Ref. 14. Comparing (59) with the analogous formula for $B(E2)$ given by the phenomenological rotational model in the case of ground state band of deformed even-even nucleus

$$B(E2; J_a + 2 \to J) = \frac{15e^2}{32\pi} Q_0^2 \frac{(J+1)(J+2)}{(2J+3)(2J+5)}, \tag{60}$$

we see, that the dependence of $B(E2; J+2 \to J)$ on the total angular momentum of nucleus $J = L$ in SRDM differs from the phenomenological expression by factor $(\varkappa - L)(\varkappa + L + 3)$. From (60) one can see that $(A - 2M_T)^2/(A^2 \nu_A^2) \sim$ $\sim Q_0^2$, which shows that ν_A^{-1} is proportional to the phenomenological intrinsic quadrupole momentum Q_0.

TABLE 5.

Nuclei	$[E_1, E_2, E_3]$	(λ, μ)	$K'L' \to KL$	E_t [MeV]	E_f [MeV]	$B(E2)^t$ [$e^2 \cdot b^2$]	$B(E2)^e$ ($\Delta B(E2)$) [$e^2 \cdot b^2$]
^{12}C	[4, 4, 0]	(0, 4)	02 → 00	4.44	0	$4.54 \cdot 10^{-4}$	$7.74\,(43) \cdot 10^{-4}$
^{20}Ne	[12, 4, 4]	(8, 0)	02 → 00	1.63	0	$1.92 \cdot 10^{-3}$	$6.65\,(38) \cdot 10^{-3}$
			04 → 02	4.25	1.63	$2.43 \cdot 10^{-3}$	$7.16\,(69) \cdot 10^{-3}$
^{24}Mg*	[16, 8, 4]	(8, 4)	02* → 00	1.37	0	$2.84 \cdot 10^{-3}$	$8.55\,(17) \cdot 10^{-3}$
			04 → 02	4.12	1.37	$2.78 \cdot 10^{-3}$	$9.32\,(170) \cdot 10^{-3}$
			22 → 00	4.24	0	$3.38 \cdot 10^{-4}$	$4.34\,(14) \cdot 10^{-4}$
			24 → 22	6.01	4.24	$1.93 \cdot 10^{-3}$	$5.88\,(59) \cdot 10^{-3}$
			24 → 02	6.01	1.37	$9.07 \cdot 10^{-4}$	$4.09\,(9) \cdot 10^{-4}$
			44 → 22	8.44	4.24	$1.57 \cdot 10^{-4}$	$2.96\,(15) \cdot 10^{-4}$
			44 → 02	8.44	1.37	$5.52 \cdot 10^{-7}$	$2.33\,(7) \cdot 10^{-4}$
			45 → 23	10.58	5.24	$4.11 \cdot 10^{-6}$	$2.46\,(35) \cdot 10^{-4}$
^{28}Si	[20, 8, 8]	(12, 0)	02 → 00	1.78	0	$4.72 \cdot 10^{-3}$	$6.52\,(2) \cdot 10^{-3}$
			04 → 02	4.62	1.78	$6.36 \cdot 10^{-3}$	$7.00\,(67) \cdot 10^{-3}$
			06 → 04	8.54	4.62	$6.26 \cdot 10^{-3}$	$4.84\,(135) \cdot 10^{-3}$
^{32}S*	[20, 16, 8]	(4, 8)	02* → 00	2.23	0	$3.35 \cdot 10^{-3}$	$5.5\,(10) \cdot 10^{-3}$
			04 → 02	4.46	2.23	$3.18 \cdot 10^{-3}$	$8.8\,(22) \cdot 10^{-3}$
			22 → 00	4.28	0	$5.17 \cdot 10^{-4}$	$1.23\,(43) \cdot 10^{-3}$
			24 → 02	6.41	2.23	$1.25 \cdot 10^{-3}$	$1.41\,(24) \cdot 10^{-3}$
^{36}Ar	[20, 20, 12]	(0, 8)	02 → 00	1.97	0	$2.81 \cdot 10^{-3}$	$5.95\,(52) \cdot 10^{-3}$

Calculated and experimental $B(E2)$ values for α-cluster type nuclei.

Notes: 1) All $B(E2)^t$ values were evaluated via the formula (59) with the exception of ^{24}Mg and ^{32}S nuclei were the expressions (52)—(56) have been used with the mixing amplitude values obtained in the energy calculations at $q/2 = 0$ (see Tables 3, 4). For oscillator frequency parameter ν_A values used, see Table 1. 2) * — for these nuclei $K'L'$, KL denotes the main components of SU_3-basis functions. 3) for the References of the experimental data on $B(E2)$ values — see Ref. 15.

The theoretical values of $B(E2)^t$ $[e^2b^2]$ calculated for the transitions between the ground band SRDM states of the α-cluster type nuclei agree with the experimental $B(E2)^e$ data by the order of magnitude. Similar agreement holds for most transitions between the states of Elliott's bands with mixed $K = 0, 2, 4$ in ^{24}Mg and ^{32}S (see Table 5).

5. *Concluding remarks*

The main conclusion following from the results discussed in this paper is that in the framework of the SRDM the self-consistent description of the α-cluster type nuclei radii, binding energies, level schemes, density distributions and reduced transition probabilities $B(E2)$ can be obtained. This conclusion confirms the applicability of the Restricted Dynamics Models to the studies of the nuclear structure.

The Strictly Restricted Dynamics (SRD) Model represents the most simple version of the Restricted Dynamics Models. In order to discuss the possibility to improve the results described in this paper let us recall some specific features of the SRDM and its relationship with more general Restricted Dynamics Models. Because of the extremely high symmetry, the SRDM Hamiltonian H_0 acts within the finite subspace of the Hilbert space. As the consequence the Schrödinger equation for H_0 in the case of any set of NN-potentials used can be exactly solved (for details — see Ref. 1). Therefore, all the results discussed in this paper have been obtained in the exactly solvable model, having little left of the dynamics presented in the Schrödinger equation for the original Hamiltonian H we have started with. The important feature of the SRDM is very high postulated symmetry stressing two effects of the very opposite nature, namely, the collective and anti-collective (the two-particle relative state correlation) effects. The first ones play the main role in the creation of the collective band level scheme, while the latter essentially contributes to the binding energy. The interplay between both effects in the proportion, given by the restricted dynamics decomposition of H, allows to choose the effective potentials, suitable to describe both binding energy and the lowest band level schemes. The wave function localization parameter ν_A, correlated with the nuclear radii data, helps to obtain both the proper picture of the nucleon density distribution and, at least in case of the α-cluster type nuclei, the agreeable absolute values for $B(E2)$.

When discussing the numerical results we often refer to the (λ, μ)-band structure. In order to explain its origin in the SRDM let us analyze the diagram in Fig. 3, illustrating the relationship between various collective submodels, contained in the most General Microscopic Collective Model resting on the Hamiltonian H_{coll}. In Fig. 3 we see three branches, where two of them, the extreme left and extreme right, end with the well known Bohr-Mottelson, Interacting Boson and Elliott's Models. The SRD Model, being in-between the Elliott's Model and the General Microscopic Collective Model, can be treated as the most general model employing the arbitrary NN-potentials but still conserving (λ, μ). The SU_3 bands (λ, μ) are also not mixed by $H_{0\,acoll}$, therefore (λ, μ)-band structure is present in the spectrum of H_0.

The scheme shown in Fig. 3 also helps to explain the place of the collective term of the SRDM Hamiltonian in the collective branch of the RDM family.

Violating the U_{A-1}-symmetry, but still keeping the O_{A-1}-symmetry, we remain within the frame of the O_{A-1}-dynamics. The typical feature of the H_{coll} spectrum is the existence of the infinite superbands, containing (in favourable cases) the »classical« collective bands, following from the well-known phenomenological approaches. Generalization of $H_{0\,coll}$ up to H_{coll}, according to the scheme shown on Fig. 3, extremely enriches the collective dynamics and one can expect the B ($E2$) values to be sensitive to it.

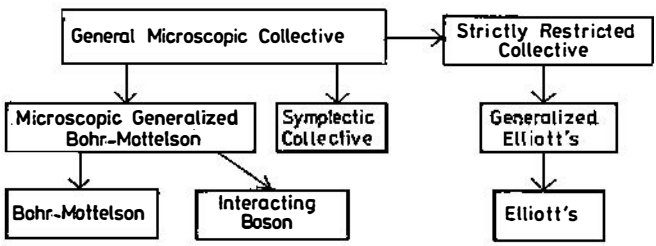

Fig. 3. The collective Hamiltonians.

Our main intention in applying the simplified version of the SRDM to the α-cluster type nuclei region was just to give for the first time the new description of their features in the framework of the restricted dynamics approach.

References

1) V. Vanagas, *The Microscopic Nuclear Theory Within the Framework of the Restricted Dynamics*, Lecture Notes, University of Toronto (Dep. of Physics), 1977;
2) V. Vanagas, *The Restricted Dynamics Nuclear Models*, Poiana Brasov Intern. School Lecture Notes, Springer-Verlag, 1986, N 279, p. 135;
3) V. Vanagas, *Algebraic Foundation of the Microscopic Nuclear Theory*, Nauka, Moscow, 1988;
4) L. Sabaliauskas, V. Vanagas, T. Guseva, J. Ruža and J. Tambergs, *Binding Energies and Level Schemes of Heavy Nuclei in the Simplified Strictly Restricted Dynamics Model*, Preprint LAFI-117, Salaspils 1987;
5) L. Sabaliauskas, V. Vanagas, T. Guseva, J. Ruža, J. Tambergs, Izv. AN USSR, Fiz. Ser. **52** (1988) 838;
6) V. Vanagas, L. Sabaliauskas and K. Eriksonas, Liet. fiz. rinkinys **22** (1982) 12.
7) J. Ruža, T. Guseva, J. Tambergs and V. Vanagas, *Coulomb Interaction in the Supermultiplet Basis*, Preprint LAFI-145, Salaspils 1989;
8) J. Ruža, T. Guseva, J. Tambergs, J. A. Castilho Alacaras, L. Sabaliauskas and V. Vanagas, Ukrainian Phys. Journ. **34** (1989) 1458;
9) V. Vanagas, *Algebraic Methods in Nuclear Theory*, Mintis, Vilnius, 1971;
10) S. Alisauskas, Liet. fiz. rinkinys **22** (1982) 3;
11) J. A. Castilho Alcaras and V. Vanagas, Journ. Math. Phys. **28** (1987) 1995;
12) V. Vanagas and O. Katkevičius, Liet. fiz. rinkinys;
13) O. Katkevičius and V. Vanagas, Liet. fiz. rinkinys **28** (1988) 279;
14) J. P. Elliott, Proc. Roy. Soc. **A245** (1958) 128; **A245** (1958) 562;
15) V. Vanagas, J. Ruža, T. Guseva and J. Tambergs, Izv. AN USSR, Fiz. Ser. **54** (1990) 8;
16) S. Ališauskas, Phys. Elem. Particl. Atom. Nucl. **14** (1983) 1336;
17) S. Ališauskas, Liet. fiz. rinkinys **18** (1978) 567.

ZNAČAJKE JEZGRI TIPA α-GROZDA U OKVIRU OGRANIČENOG DINAMIČKOG MODELA

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UDK 539.142

Originalni znanstveni rad

U okviru ograničenog dinamičkog modela proučavane su energije vezanja, sheme pobuđenih nivoa, raspodjele naboja, formfaktori i $E2$ prijelazne vjerojatnosti za jezgre tipa α-grozda. Postignuto je zadovoljavajuće slaganje teorije i eksperimenta.