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COMMENT ON COLLECTIVE MODELS AND THE p-n INTERACTION

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Collective models that contain no explicit mention of the p-n interaction have been extremely successful. Likewise, it is well known that the residual valence p-n interaction is critical to the development of collectivity, phase transitions and deformation in nuclei. Brief comments are made on the interrelationship of these two statements.

Two threads running through much of nuclear structure physics for the last three decades have been the successful interpretation of collectivity, phase transitions and deformation in nuclei with a series of collective models that, generally, make no *explicit* mention of the p-n interaction, and the simultaneous recognition that it is the residual valence p-n interaction that is responsible for the development of these features in nuclei. It is a discussion of this seeming dichotomy of viewpoints that is the purpose of the present comments, which arise from a convergence of ideas spanning several of our recent publications.

It is clear that the p-n interaction amongst the valence nucleons is intimately related to configuration mixing and hence to collectivity. This was pointed out as early as 1953 by deShalit and Goldhaber[1)], and has been repeatedly emphasized by Talmi[2)]. It is evident in such simple features of nuclear spectra as the near constancy of 2_1^+ energies in singly magic Sn isotopes, in contrast to the immediate descent in these energies when both valence protons *and* neutrons are present. Recently, this relationship has been made explicit through the N_pN_n scheme[3)].

It has been shown, for example, that the systematics of 2_1^+ and 4_1^+ energies tracks smoothly with N_pN_n. Likewise $B(E2{:}0_1^+ \rightarrow 2_1^+)$ values, nuclear charge radii, the energies of intruder states, and nuclear masses, all correlate well with N_pN_n. An example of this is shown in Fig. 1. In contrast, the systematics of *vibrational* ener-

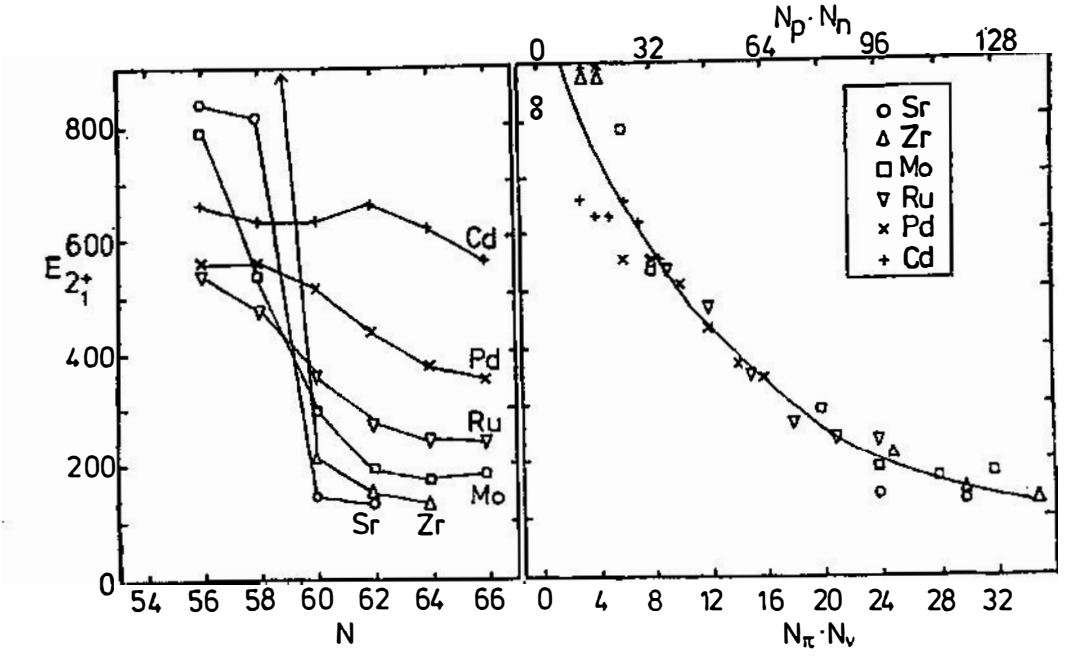

Fig. 1. Normal and N_pN_n plots of $E(2\dagger)$, for the $A = 100$ region. (From. Ref. 3).

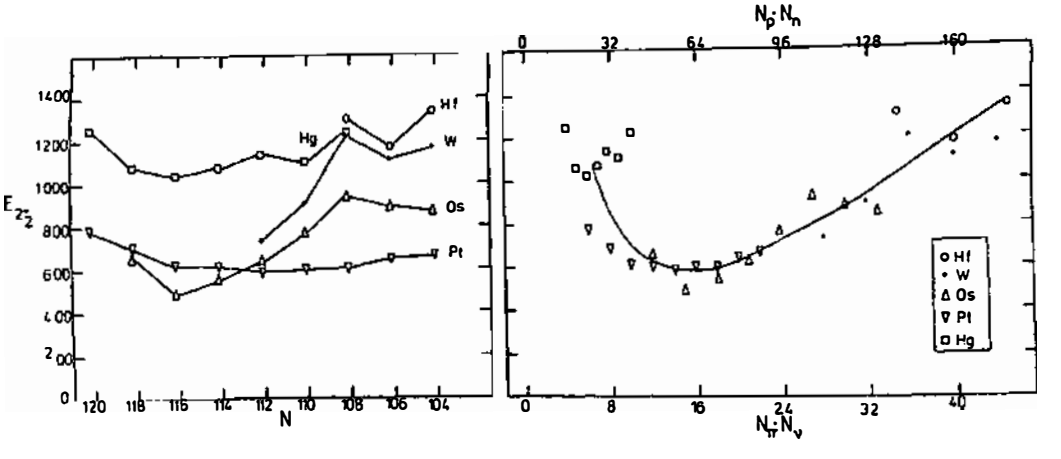

Fig. 2. Normal and N_pN_n plots of $E(2_2^+)$, (generally $E(2_\nu^+)$) for the $A = 190$ region. (From Ref. 3).

gies, such as γ-vibrations or the lowest 3^- states in medium and heavy nuclei, although somewhat smoother against N_pN_n than in normal plots, does *not* display nearly the same *simplicity* of behaviour. This is evident in the undulations in γ vibrational energies in the rare earth region seen in Fig. 2. The relationship between the p-n interaction and collectivity is highlighted even more dramatically by plots of certain observables in *several* mass regions against the so-called P-factor[4], defined as

$$P = \frac{N_pN_n}{N_p + N_n}.$$

Fig. 3 shows plots of the energy ratios $E(4_1^+)/E(2_1^+)$ against P for six phase transitional regions in heavy nuclei. There is a remarkably similar behaviour in all regions. The curves have nearly identical shapes and all pass through the transition region, which can be conveniently defined as the point at which $E(4_1^+)/E(2_1^+) = 3.0$, for P-values in the narrow critical range from 4 to 5.

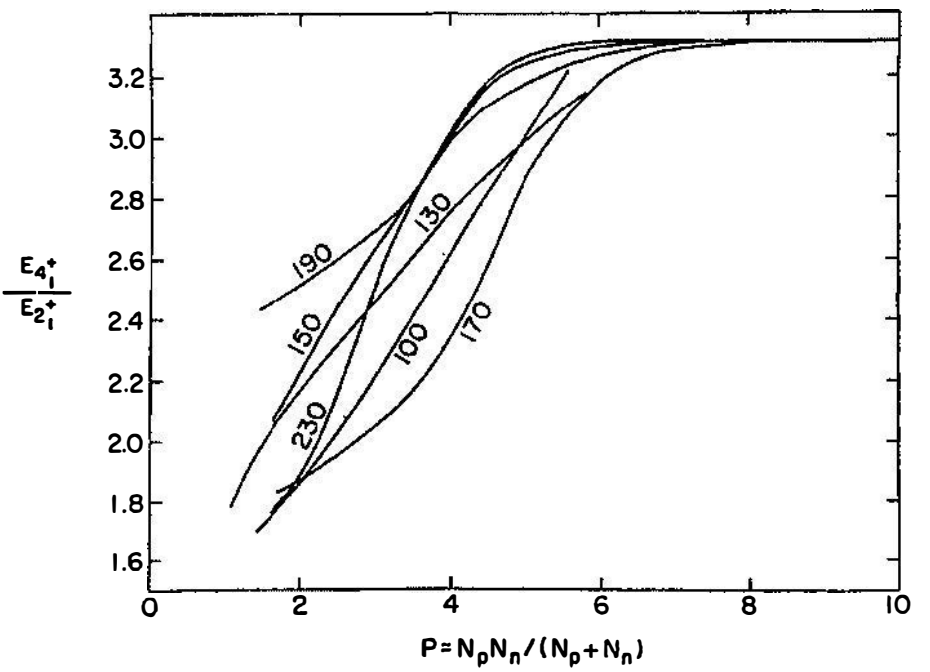

Fig. 3. P-factor plots of $E(4_1^+)/E(2_1^+)$ for six phase transitional regions in medium and heavy nuclei. (From Ref. 4).

In the face of this obvious correlation between the p-n interaction and collectivity, how is it that an abundance of collective models, both macroscopic and microscopic, have succeeded so thoroughly with no explicit introduction of the p-n interaction? To approach a possible resolution of this dilema, we note that those collective observables that behave smoothly *and* simply against N_pN_n or

the P-factor all relate to the shape and collectivity in the *ground state* configuration. This certainly applies to 2_1^+ and 4_1^+ energies and, of course, to $B(E2: 0_1^+ \rightarrow 2_1^+)$ values, nuclear radii and masses. It also applies to the energies of intruder states if the concept of »ground state« is taken generally to mean the »base« state of a family of states. This leads us to the statement that the valence residual p-n interaction essentially determines the *mean field*, that is, the shape or equilibrium deformation and, thereby, the degree of ground state collectivity. More specifically, it is primarily the $T = 0$ component of the p-n interaction that correlates the proton and neutron mass distributions so as to create a single »fluid«, rather than separate ones with distinct shapes. Thus, the p-n interaction provides the »starting point« on top of which collective excitations are built. The structure and energies of the latter depend on the properties of the operators, such as $Y_{2,K}$ or $Y_{3,K}$ that create them.

This view of the role of the p-n interaction in determining the mean field is pointed to rather directly by the P-factor plot shown in Fig. 3, in particular, the value $P_{crit} \approx 4$—5. The valence p-n interaction is known[5] to have a typical strength in heavy nuclei of 200 to 300 keV. The pairing interaction, on the other hand, has a strength on the order of ≈ 1 MeV. The P-factor may be viewed as the ratio of the number of valence p-n interactions to the number of valence pai-

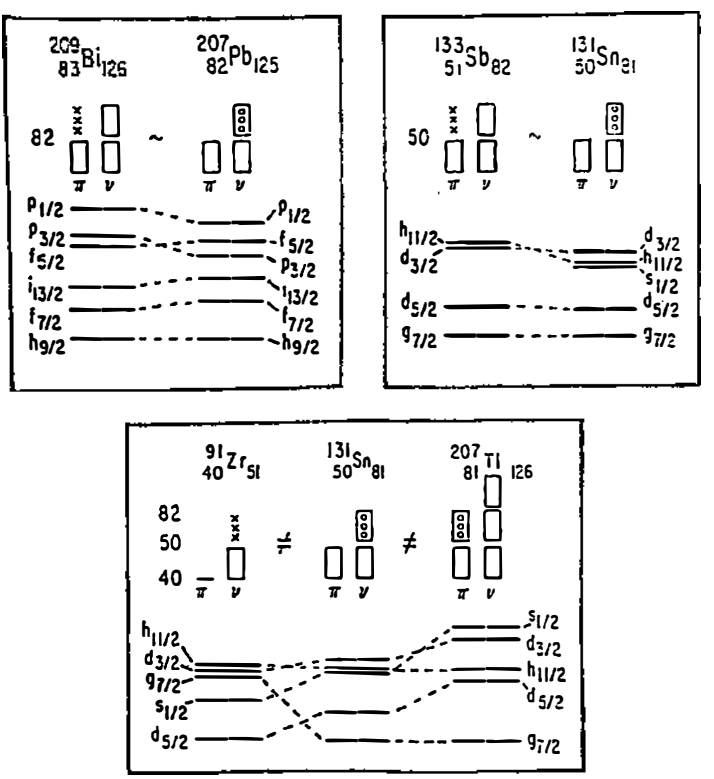

Fig. 4. Comparison of single particle (or hole) energies in different regions.

ring interactions. Thus, the point at which deformation and high collectivity develop corresponds to just the region where the integrated p-n interaction strength is beginning to dominate the integrated valence pairing strength.

A second, equally important, role played by the p-n interaction focuses on its monopole component. This component acts to shift the underlying single particle shell model energies[6]. Fig. 4 illustrates this. The top left panel shows that neutron single particle energies in ^{207}Pb are nearly identical to proton energies in ^{209}Bi. The top right shows the same idea for the 50—82 shell. The bottom panel, in contrast, shows the dramatic effect on neutron single particle energies when the $Z = 40$—50 proton shell is filled between Zr and Sn. The high spatial overlap of the $1g_{9/2\pi}$ and $1g_{7/2\nu}$ orbits leads to a strong attractive contribution to the energy of the latter, lowering it from well above, to well below, the $2d_{5/2\nu}$ energy.

This effect on single particle energies has two consequences. First, it affects the quadrupole p-n interaction (which is primarily responsible for the growth of collectivity) by altering the effective valence space in which it acts. Thus, for example, shell gaps envolve with N and Z. Indeed, it is this evolution that is crucial to the $A = 100$ and 150 transition regions[7]. The second consequence of shifts in single particle energies is their effect on the available basis states near the Fermi surface that are the *input* for the microscopic structure of collective excitations. This structure is determined by the favoured matrix elements of their associated operators acting on the available orbits. These matrix elements have particular selection rules [e. g. $\Delta n_z = 0$, $\Delta\Lambda = \pm 2$ for Y_{22} (γ-vibration) in the Nilsson scheme] and thus vibrational energies will have a complex variation dependent on the availability of appropriate orbit combinations. For 3^- states, this point has recently been discussed by Zamfir et al.[8] and Cottle and Bromley[9]. Thus, the p-n interaction does in fact affect the structure of these vibrations, but only indirectly.

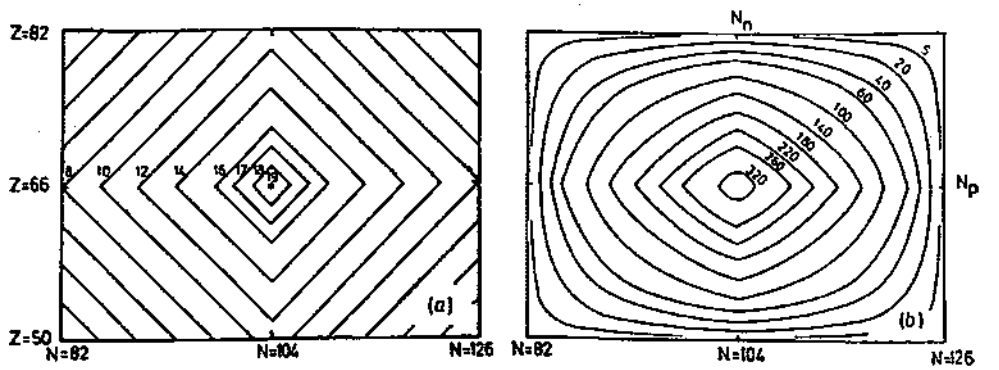

Fig. 5. Contours of a) constant $N_p + N_n$, giving, approximately, contours of constant Fermi surface and b) constant N_pN_n (From Ref. 10).

This last point probably explains why N_pN_n or P-factor plots of observables relating to those vibrations are somewhat smoothed relative to normal plots against N, Z or A. Fig. 5 shows contours of constant $N_p + N_n$ (part a) and N_pN_n (part b)

for a typical pair of shells. In general the patterns are quite different. This is especially so for small numbers of valence particles, that is, just in the phase transitional regions where N_pN_n works so well. However, well within the shells, that is, in the deformed region, N_pN_n and $N_p + N_n$ behave quite similarly (similar »diamond« shapes). Thus, the reason quantities such as $E(2^+_\gamma)$ or $E(3^-_1)$ are relatively smooth against N_pN_n is largely a fortuitous consequence of the scaling of N_pN_n with $N_p + N_n$. In midshell regions, N_pN_n approximately measures the Fermi surface and, as we have said, that is the critical factor determining the systematics of vibrational properties. The fact that their systematics against N_pN_n, though relatively smooth, is not simple, reflects the detailed operation of their respective creation operators acting, near the Fermi surface, on the local orbits provided, in part, by the p-n interaction.

Thus, to return to the initial dilemma, collective models that do not explicitly involve the p-n interaction nonetheless contain it in two important *implicit* ways, namely, first, in choosing the equilibrium deformation (β_2, β_4, γ, etc) and in choosing the same shapes for both the proton and neutron distributions and, secondly, in choosing the set of single particle energies used for microscopic calculations of collective excitations. [Of course, modern collective models do both of these tasks simultaneously and self-consistently through a selection of a set of single particle energies which are then used to obtain both the equilibrium deformation, via an energy minimization procedure, and the collective vibrations, as well as excited deformed or superdeformed configurations.] Perhaps, though, the present remarks suggest that insufficient attention is sometimes paid to the dynamic behaviour of single particle energies due to the p-n interaction. An interesting initiative in just this area is that of Nazarewicz, Dobaczewski and coworkers who carried out Hartree-Fock calculations for the $A = 100$ region and then analyzed the resulting correlations in the context of the p-n interaction.

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KOMENTAR KOLEKTIVNIH MODELA I p-n INTERAKCIJA

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Izuzetno uspješni su bili kolektivni modeli koji ne sadrže eksplicite p-n interakciju. Dobro je poznato da je rezidualna p-n interakcija kritična za razvoj kolektivnosti, fazne prijelaze i deformacije u jezgri. Komentirana je veza između te dvije tvrdnje.