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GAUGED Q BALLS WITH NON-STANDARD COUPLINGS BETWEEN THE GAUGE FIELD AND THE SCALAR FIELD

CARL WOLF

Department of Physics, North Adams State College, North Adams, MA (01247) U. S. A.

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The properties of a gauged Q ball with non-standard coupling of the scalar field to the electromagnetic field is studied. Such configurations may arise from Kaluza-Klein type compactifications or from gravitational theory with a varying gravitational constant.

1. Introduction

An oft forgotten question in physics centers on the problem of global symmetries in particle theory. Global symmetries have a distasteful flavour associated with them because they are not connected with a gauge principle and involve non-local ideas in that the entire system is changed by the same amount independent of the fact that the points are separated by space-like intervals. Lepton number, baryon number, as well as chiral flavour symmetry all have this hidden generic attachment with non-locality[1,2,3]. In fact, it could very well be that a chiral flavour symmetry operating in the preon world could protect the composite quarks and leptons from acquiring a large mass[4]. We might remark that various flavour symmetries exist in the family picture of quarks and leptons and Harari and Seiberg used this in a discussion of quark lepton mass generation from a hierarchial breaking of these symmetries[5]. But, what is the origin of these global symmetries and what do they give rise to in physical systems, it might be that a global symmetry in a physical theory is an accidental symmetry of a G. U. T. theory at a much higher scale wherein at a lower scale, we only perceive the fragmented gauge symmetry, but the global symmetry remains intact. These global symmetries give rise to globally conserved charges that may stabilize global configurations of the corresponding fields. This is the origin of the name Q balls which represents a non-topological soliton whose mass is less than the mass of corresponding pions of the theory[6]. It was Coleman's observation of this idea that led to the above term and has subsequently opened up a new avenue of research. In fact, after Coleman's original work, a subsequent investigation by Coleman, H. Georgi, Mohonar and Cohen on L balls, (which are configurations of scalar and spinor fields inspired by the Gelmini Roncadelli model of neutrino mass generation with a globally conserved lepton number) further cast light on the subject, such configuration were shown to be stable except for slow decay into fermions at the surface[7,8]. In the context of curved space, we have discussed a Q ball in general relativity for a simple scalar quartic potential and demonstrated that Q balls were stable provided they did not get too large[9]. Recently, K. Lee et al. have discussed gauged Q balls and demonstrated that they are stable providing they do not get too large because of the electrostatic repulsion[10]. Inspired by this new progress, we investigate gauged Q balls with a non-standard coupling between a scalar field and a gauge field. Such non-standard couplings might arise as a left over fragment of Kaluza-Klein compactification or from a theory involving a variable gravitational constant[11,12]. In curved space, we develop an approximate solution for the mass of a gauged Q configuration in terms of the Q charge and the electric charge.

2. *Gauged Q balls with non-standard scalar gauge coupling*

We begin our analysis by writing the following lagrangian for the coupling of gravitation, electromagnetism and charged scalar field

$$L = \frac{c^4}{16\pi G} R\sqrt{-g} + \left[\begin{array}{c} D_\mu \varphi (D^\mu \varphi)^* - U(\varphi) \\ - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \overline{\alpha} \frac{\varphi\varphi^*}{4} (F_{\mu\nu} F^{\mu\nu}) \end{array} \right] \sqrt{-g} \tag{2.1}$$

Here $F_{\mu\nu} = \frac{\partial A_\mu}{\partial x^\nu} - \frac{\partial A_\nu}{\partial x^\mu}$; $D\varphi = \left(\partial_\mu + ieA_\mu) \varphi, \; e = \frac{e_e}{\hbar c} \right) \overline{\alpha} =$ constant, where $e_e =$ = unit of electric charge and

$$U(\varphi) = \frac{A_2}{4} \left(\varphi\varphi^* - \frac{A_1}{A_2} \right)^2$$

is the scalar potential for the charged scalar field. We study a stationary configuration with solution of the form

$$\varphi(r, t) = \frac{f(r)}{\sqrt{2}} e^{i\omega t} \tag{2.2}$$

with a true vacuum at $r = 0$

$$|\varphi(r)_{r=0}| = \sqrt{\frac{A_1}{A_2}},$$

and the false vacuum at

$$r = R, \text{ or } |\varphi(r)_{r=R}| = 0, \text{ or } f = \sqrt{\frac{2A_1}{A_2}} \tag{2.3}$$

at $r = 0$ and $f = 0$ at $r = R$.

Here R is the outer radius of the configuration of charged scalar field and the electromagnetic field. The lagrangian Eq. (2.1) is invariant under the global symmetry $\delta\varphi = \mathrm{i}\alpha\varphi$, $\delta\varphi^* = -\mathrm{i}\alpha\varphi^*$ which generates the conserved Q charge by writing

$$\frac{\partial L}{\partial\varphi}\delta\varphi + \frac{\partial L}{\partial\varphi_{,\mu}}\delta\varphi_{,\mu} + \frac{\partial L}{\delta\varphi^*}\partial\varphi^* + \frac{\partial L}{\partial\varphi_{,\mu}^*}\delta\varphi_{,\mu}^* = 0 \tag{2.4}$$

Using the field equations in Eq. (2.4) we find upon integration over the spherical coordinates $\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}\bar{\varphi}$

$$\iiint \frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial\varphi_{,\mu}}\delta\varphi + \frac{\partial L}{\partial\varphi_{,\mu}^*}\delta\varphi^*\right]\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}\bar{\varphi} = 0 \tag{2.5}$$

or

$$Q = \iiint\left[\frac{\partial L}{\partial\varphi_{,4}}\delta\varphi + \frac{\partial L}{\partial\varphi_{,4}}\delta\varphi^*\right]\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}\bar{\varphi} \tag{2.6}$$

is the conserved Q charge since the values of the integral vanish for $U = 1$, r; from the form of the lagrangian and the invariance condition $\delta\varphi = \mathrm{i}\alpha\varphi$, $\delta\varphi^* = -\mathrm{i}\alpha\varphi^*$.

Also, the value of the integral for $u = 2$, Θ and $u = 3$, $\bar{\varphi}$ are zero, since the fields φ, φ^* are independent of Θ, $\bar{\varphi}$. Since the integral over r, Θ, $\bar{\varphi}$ of the fourth component vanishes in Eq. (2.5) this implies, after taking the derivative outside the integral, that the quantity inside is a constant of motion which we identify as the Q charge in Eq. (2.6). We next employ the metric $(\mathrm{d}s)^2 = e^\nu(\mathrm{d}x^4)^2 - e^\lambda(\mathrm{d}r)^2 - r^2\,\mathrm{d}\Theta^2 - r^2\sin^2\Theta\,(\mathrm{d}\bar{\varphi})^2$ where $x^4 = ct$; e^λ, e^ν depend on r only. Eq. (2.6) becomes, with the only surviving component of the electromagnetic field being the scalar potential $A_4 = \psi$, also using the stationary solution for

$$\varphi = \frac{f}{\sqrt{2}}\mathrm{e}^{i\omega t}$$

from Eq. (2.2) and the invariance condition $\delta\varphi = \mathrm{i}\alpha\varphi$, $\delta\varphi^* = -\mathrm{i}\alpha\varphi^*$.

$$Q = \iiint\left(-\frac{\mathrm{i}\omega}{c}\varphi^* - \mathrm{i}e\psi\varphi^*\right)e^{-\nu}r^2e^{(\nu+\lambda)/2}\sin\Theta\,(\mathrm{i}\alpha\varphi)\,\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}\bar{\varphi} \tag{2.7}$$

$$+ \iiint\left(\frac{\mathrm{i}\omega\varphi}{c} + \mathrm{i}e\psi\varphi\right)e^{-\nu}r^2e^{(\nu+\lambda)/2}\sin\Theta\,(-\mathrm{i}\alpha\varphi^*)\,\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}\bar{\varphi}$$

giving

$$Q = 4\pi \int_0^R \left(\frac{\omega}{c} f^2 + e\psi f^2\right) r^2 e^{(\nu-\lambda)/2}\, \mathrm{d}r \tag{2.8}$$

where we have factored out the arbitrary parameter α. To obtain a solution to the scalar field $f(r)$ and the scalar potential $\psi(r)$ we substitute

$$\varphi = \frac{f}{\sqrt{2}}\, e^{i\omega t}, \qquad A_4 = \psi$$

into Eq. (2.1) to obtain for the scalar electromagnetic part of the lagrangian

$$L = \begin{bmatrix} -\frac{1}{2}(f_{,r})^2\, e^{-\lambda} + \left[\frac{\omega^2}{2c^2} f^2 + \frac{e\psi f^2}{c}\,\omega + \frac{e^2\psi^2 f^2}{2}\right] e^{-\nu} \\ +\frac{1}{8\pi}(\psi_{,r})^2\, e^{-(\nu+\lambda)} - \frac{\overline{\alpha} f^2}{4}(\psi_{,r})^2\, e^{-(\nu+\lambda)} \\ -\frac{A_2}{4}\left(\frac{f^2}{2} - \frac{A_1}{A_2}\right)^2 \end{bmatrix} r^2 e^{(\nu+\lambda)/2} \sin\Theta. \tag{2.9}$$

If we vary Eq. (2.9) with respect to f and ψ after integration over Θ, $\overline{\varphi}$ we obtain by varying with respect to f, ψ and neglecting the dependence on the metric

$$\left(\frac{\omega^2}{c^2} f + \frac{2e\psi f\omega}{c} + e^2\psi^2 f\right) r^2 - \frac{\overline{\alpha} f}{2}(\psi_{,r})^2\, r^2 \tag{2.10}$$

$$- \frac{A_2}{2} f\left(\frac{f^2}{2} - \frac{A_1}{A_2}\right) r^2 + \frac{\mathrm{d}}{\mathrm{d}r}(f_{,r}\, r^2) = 0$$

$$\left(\frac{ef^2 w}{c} + e^2\psi f^2\right) r^2 - \frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{1}{4\pi}(\psi_{,r})\, r^2 - \left(\frac{\overline{\alpha} f^2 \psi_{,r}}{2}\right) r^2\right] = 0. \tag{2.11}$$

To solve Eqs. (2.10) and (2.11) in an approximate manner, we use a power series cut off at the third term for f and ψ

$$\begin{aligned} f &= f_0 + f_1 r + f_2 r^2 + f_3 r^3 \\ \psi &= \psi_0 + \psi_1 r + \psi_2 r^2 + \psi_3 r^3. \end{aligned} \tag{2.12}$$

We substitute Eq. (2.12) into Eq. (2.10), Eq. (2.11) and insist that the coefficients only obey the equations up to the third power in r, upon neglecting the term f^3 in the third term in Eq. (2.10), this gives (f_0 to be determined),

$$f_1 = 0, \quad f_2 = -\frac{1}{6}\left[\frac{w^2}{c^2} f_0 + \frac{2ew\,\psi_0 f_0}{c} + e^2 f_0 \psi_0^2 + \frac{A_1 f_0}{2}\right], \quad f_3 = 0 \tag{2.13}$$

$$\psi_0 \text{ (arbitrary)}, \quad \psi_1 = 0, \quad \psi_2 = \frac{ef_0^2 w/c + e^2 f_0^2 \psi_0}{\frac{3}{2\pi} - 3\overline{a} f_0^2}, \quad \psi_3 = 0. \tag{2.14}$$

We next add another term to the power series and insist that the solution obey the boundary conditions at $r = 0$, $r = R$

$$f = f_0 + f_1 r + f_2 r^2 + f_3 r^3 + f_4 r^4$$
$$\psi = \psi_0 + \psi_1 r + \psi_2 r^2 + \psi_3 r^3 + \psi_4 r^4 \tag{2.15}$$

we have

$$|f|_{r=0} = \sqrt{\frac{2A_1}{A_2}},$$

at $r = 0$, $|f|_{r=R} = 0$ at $r = R$. (2.16)

For the potential we have, at $r = R$

$$\psi(R) = \frac{Q_{EM}}{R}$$
$$E(R) = -\left(\frac{d\psi}{dr}\right)_R = \frac{Q_{EM}}{R^2} \tag{2.17}$$

where Q_{EM} = total electric charge of Q configuration. Thus, Eq. (2.16), and Eq. (2.17) give upon substituting Eq. (2.15) into the boundary conditions; Eq. (2.16), Eq. (2.17)

$$f_0 = \sqrt{\frac{2A_1}{A_2}}, \quad \sqrt{\frac{2A_1}{A_2}} + f_2 R^2 + f_4 R^4 = 0$$

$$\psi_0 + \psi_2 R^2 + \psi_4 R^4 = \frac{Q_{EM}}{R} \tag{2.18}$$

$$2\psi_2 R + 4\psi_4 R^3 = -\frac{Q_{EM}}{R^2}$$

giving

$$f_4 = -\frac{\sqrt{2A_1/A_2} - f_2 R^2}{R^4}, \qquad \psi_4 = -\frac{Q_{EM}/R^2 - 2\psi_2 R}{4R^3}$$

$$\psi_0 = \frac{Q_{EM}}{R} - \psi_2 R^2 - \psi_4 R^4. \tag{2.19}$$

We summarize

$$f_0 = \sqrt{\frac{2A_1}{A_2}}, \quad f_1 = 0, \quad f_2 = -\frac{1}{6}\left[\frac{\omega^2}{c^2} f_0 + \frac{2e\omega f_0}{c}\psi_0 + e^2 f_0 \psi^2 + \frac{A_1 f_0}{2}\right]$$

$$f_3 = 0, \quad f_4 = -\frac{\sqrt{2A_1/A_2} - f_2 R^2}{R^4} \tag{2.20}$$

and

$$\psi_1 = 0, \quad \psi_2 = \frac{e f_0^2 \omega/c + e^2 f_0^2 \psi_0}{\frac{3}{2\pi} \quad 3\bar{a} f_0^2}, \quad \psi_3 = 0$$

$$\psi_0 = \frac{\frac{5 \cdot Q_{EM}}{2R^3} - \frac{e f_0^2 \omega/c}{\left(\frac{3}{2\pi} 3\bar{a}^2 f_0^2\right)}}{\frac{2}{R^2} + \frac{e^2 f_0^2}{\frac{3}{2\pi} - 3a f_0^2}} \tag{2.21}$$

$$\psi_4 = -\frac{Q_{EM}/R^2 - 2\psi_2 R}{4R^3}$$

where we have used Eq. (2.14) and Eq. (2.18) to find ψ_0. Eq. (2.20) and Eq. (2.21) represent the solutions for f and ψ for the region $0 \leq r \leq R$ satisfying the boundary conditions of the false vacuum at $r = R$ and the true vacuum at $r = 0$. The scalar potential and the electric field at $r = R$ match that of an electric charge Q_{EM} enclosed within the sphere of radius R. For the exsterior metric, we have for the 4 component of the Einstein equation

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 \, T_4^4$$

which gives

$$e^{-\lambda} = 1 - \frac{2GM}{rc^2} + \frac{GQ_{EM}}{r^2 c^4} - \frac{8\pi G r^2}{3c^4}\left(\frac{A_1^2}{4A_2}\right), \text{ for } r < R, \tag{2.22}$$

which is the metric for the field of an electric charge Q_{EM} in the presence of false vacuum energy density $A^2/4A_2$ from Eq. (2.1) where

$$U(0) = \frac{A_1^2}{4A_2}.$$

To calculate the interior metric, we have for the energy momentum tensor from Eq. (2.1)

$$T_{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\partial L}{\partial g^{\mu\nu}}=D_\mu\varphi D_\nu\varphi^*+D_\mu\varphi^* D_\nu\,\varphi-g_{\mu\nu}\left(D_\alpha\varphi\,(D^\alpha\varphi)^*\right)+g_{\mu\nu}U(\varphi)+$$

$$+\frac{g_{\mu\nu}}{16\pi}F_{\alpha\beta}F^{\alpha\beta}-\frac{1}{4\pi}F_{\mu\alpha}F^{\alpha}_{\nu}-\frac{a\varphi\varphi^*}{4}(F_{\alpha\beta}F^{\alpha\beta})\,g_{\mu\nu}+a\,(\varphi\varphi^*)\,F_{\mu\alpha}F^{\alpha}_{\nu}. \quad (2.23)$$

For the $^4_4$ component of Eq. (2.23), we have using

$$\varphi=\frac{fe^{i\omega t}}{\sqrt{2}}$$

and $A_4=\psi$

$$T^4_4=e^{-\nu}\left[\frac{\omega^2}{2c^2}f^2+\frac{f^2\omega e\psi}{c}+\frac{e^2\psi^2f^2}{2}\right]+e^{-\lambda}\left(\frac{f_r^2}{2}\right)+\frac{A_2}{4}\left(\frac{f^2}{2}-\frac{A_1}{A_2}\right)^2+$$

$$+\frac{1}{8\pi}(\psi_{,r})^2\,e^{-(\nu+\lambda)}-\frac{1}{4}af^2\,(\psi_{,r})^2\,e^{-(\nu+\lambda)}. \quad (2.24)$$

If we neglect the metric in Eq. (2.24), we have

$$T^4_4\cong\frac{\omega^2}{2c^2}f^2+\frac{f\omega e\psi}{c}+\frac{e^2\psi^2f^2}{2}+\left(\frac{f_{,r}}{2}\right)^2+\frac{A_2}{4}\left(\frac{f^2}{2}-\frac{A_1}{A_2}\right)^2+$$

$$+\frac{1}{8\pi}(\psi_{,r})^2-\frac{\bar{a}}{4}f^2\,(\psi_{,r})^2. \quad (2.25)$$

From Eq. (2.25) we calculate the metric for $r\leq R$ from the $^4_4$ component of the Einstein equation

$$e^{-\lambda}=1-\frac{8\pi G}{c^4r}\int_0^R r^2T^4\mathrm{d}r \quad (2.26)$$

after evaluating $(re^{-\lambda})=0$ at $r=0$.

If we match Eq. (2.26) and Eq. (2.22), we may calculate the mass of the Q matter coupled to electromagnetism using the coefficients in Eq. (2.20), Eq. (2.21) for the expansion of $f\,(r)$, $\psi\,(r)$ in Eq. (2.15). To eliminate ω, we use the expression for the Q charge in Eq. (2.7) where we approximate $e^\nu\cong e^\lambda\cong 1$ to compute the Q charge

$$Q=\int_0^R 4\pi r^2\left(\frac{\omega f^2}{c}+e\psi f^2\right)\mathrm{d}r. \quad (2.27)$$

Eq. (2.27) can be evaluated using f, ψ from Eq. (2.15) and solved for ω in terms of Q. Thus, the mass of the configuration can be computed from the matching of the metric at $r=R$ from Eq. (2.26) and Eq. (2.22) with ω substituted from Eq. (2.27) with the resulting mass finally expressed in terms of Q, Q_{EM} and the coupling constants.

3. Conclusions

In the above calculation, we have at least developed an approximate solution for a gauged Q ball with a non-standard scalar electromagnetic coupling. A study of the mass of the Q ball as a function of the Q charge and the electric charge Q_{EM} and the coupling constants $e_c/\hbar c$, $\overline{\alpha}$ would reveal regions of stability. From the form of the energy momentum tensor, we would find that the non-standard coupling would work against the coulomb repulsion because it contributes a negative term to T_4^4. In an astrophysical setting, any theory that leads to this non-standard coupling would have this stabilizing feature associated with it. In the future we will report on a more precise study of the stability of the above non-standard gauged Q ball.

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BAŽDARSKE Q LOPTE S NESTANDARDNIM VEZANJIMA IZMEĐU BAŽDARSKOG I SKALARNOG POLJA

CARL WOLF

Department of Physics, North Adams State College, North Adams, MA (01247), U. S. A.

UDK 539.12

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Proučavaju se svojstva baždarske Q lopte s nestandardnim vezanjem skalarnog polja na elektromagnetsko polje. Takove konfiguracije mogu se pojaviti u kompaktifikacijama tipa Kaluza-Klein ili u gravitacionoj teoriji s promjenljivom gravitacionom konstantom.