YU ISSN 0015—3206
FZKAAA 22 (2) 439 (1990)

EXTENDED BRST SYMMETRY AND THE PLANAR GAUGE

ANĐELKA ANDRAŠI

Ruđer Bošković Institute, 41001 Zagreb, P. O. B. 1016, Croatia, Yugoslavia

Received 8 April 1990

UDC 539.12

Original scientific paper

BRST identities are not sufficient to control all the counter-terms in non-covariant gauges. We consider an explicit example of the planar gauge to one-loop order in perturbation theory to show how extended BRST symmetry connects renormalization constants.

1. Introduction

Non-covariant gauges are widely used because they are ghost-free and therefore are considered to be physical gauges. In non-covariant gauges, gluons have only transverse degrees of freedom, while in covariant gauges, such as the Feynman gauge, gluons also have longitudinal propagation which is compensated by ghosts. However, the actual evaluation of Feynman diagrams in non-covariant gauges is much more tedious than in covariant gauges. There are problems how to regularize the spurious singularities $(n \cdot k)^{-1}$ appearing in the gluon propagator (n is the constant vector defining the gauge). Also, one can construct more counter-terms using the vector n. The planar gauge[1] is defined by the gauge-fixing Lagrangian

$$L' = L - (2an^2)^{-1}\, n_\lambda A^\lambda \Box n_\mu A^\mu, \tag{1}$$

where L' is the complete Lagrangian containing Yang-Mills fields, ghosts and their respective soucres:

$$L' = L_{YM} - (2an^2)^{-1}\, n_\lambda A^\lambda \Box n_\mu A^\mu + \eta\, n \cdot D\omega + sA + uD\omega + \\ + x\omega + \frac{g}{2} v(\omega \wedge \omega) + \eta y. \tag{2}$$

Although the Feynman diagrams containing ghosts are zero, the diagrams containing sources with open ghost lines are different from zero. They are needed for the complete analysis of BRST (Slavnov-Taylor) and extended BRST (Piguet-Sibold) identities[2)].

The BRST transformations are the usual ones:

$$\delta A = D\omega\delta\xi,$$
$$\delta\eta = -(an^2)^{-1} \Box n \cdot A\delta\xi,$$
$$\delta\omega = -\frac{g}{2}(\omega\wedge\omega)\,\delta\xi, \tag{3}$$

where η is the antighost and $\delta\xi$ is the infinitesimal anticommuting number.

Then the BRST variation of the gauge-fixing term is connected with the variation of the antighost

$$\delta\{-(2an^2)^{-1}\, n \cdot A\Box n \cdot A\} + (\delta\eta)\, nD\omega = 0 \tag{4}$$

because $\delta\xi$ and ω anticommute, ensuring the BRST invariance of the complete Lagrangian. On the other hand, the variation of the Lagrangian with respect to the gauge-fixing vector n is

$$\Delta L' = -(an^2)^{-1}\, \Delta n \cdot A\Box n \cdot A + \eta\Delta nD\omega. \tag{5}$$

When multiplied by infinitesimal anticommuting number $\delta\xi$, it becomes

$$\Delta L'\,\delta\xi = \delta\eta\, \Delta n \cdot A + \eta\Delta n \cdot \delta A = \delta\,(\eta\Delta n \cdot A), \tag{6}$$

where δ is to be taken in the sense of BRST.

Therefore, the variation of the generating functional of connected Green's functions with respect to the gauge-fixing vector n is

$$\Delta e^{iX}\,\delta\xi = \int \mathrm{d}A\, \mathrm{d}\omega\, \mathrm{d}\eta e^{iS} \int \mathrm{d}^4x \cdot i\delta\,(\eta\Delta nA). \tag{7}$$

We shall use the BRST invariance of the Lagrangian to derive the Piguet-Sibold identity for the planar gauge:

$$\delta\,(\eta\Delta n \cdot A) = \eta'\Delta nA' - \eta\Delta nA,$$
$$A + \delta A = A',$$
$$\omega + \delta\omega = \omega',$$
$$\eta + \delta\eta = \eta'. \tag{8}$$

Let us follow what happens with the gluon field A_μ (the transformation of the ghost fields is analogous).

$$\int \mathrm{d}A\mathrm{d}\eta\mathrm{d}\omega \exp\{\mathrm{i}\int \mathrm{d}^4y\,(\ldots sA \ldots)\} \int \mathrm{d}^4x\,(\eta'\Delta nA' - \eta\Delta nA) =$$

$$= \int \mathrm{d}\eta\mathrm{d}\omega \exp\left\{\mathrm{i}\int \mathrm{d}^4y\left[-\frac{1}{4}F^2 + \ldots s\,(A' - \delta A)\ldots\right]\right\} \int \mathrm{d}^4x\eta'\Delta nA'\mathrm{d}A -$$

$$- \int \mathrm{d}\eta\mathrm{d}\omega \exp\left\{\mathrm{i}\int \mathrm{d}^4y\left(-\frac{1}{4}F^2 + \ldots sA\ldots\right)\right\} \int \mathrm{d}^4x\eta\Delta nA\mathrm{d}A =$$

$$= \int \mathrm{d}\eta\mathrm{d}\omega \exp\left\{\mathrm{i}\int \mathrm{d}^4y\left(-\frac{1}{4}F'^2 + \ldots sA'\ldots\right)\right\} \int \mathrm{d}^4x\eta'\Delta nA'\,(1 - \mathrm{i}s\delta A\ldots)\,\mathrm{d}A' -$$

$$- \int \mathrm{d}\eta\mathrm{d}\omega \exp\left\{\mathrm{i}\int \mathrm{d}^4y\left(-\frac{1}{4}F^2 + \ldots sA\ldots\right)\right\} \int \mathrm{d}^4x\eta\Delta nA\mathrm{d}A =$$

$$= - \int \mathrm{d}\eta\mathrm{d}\omega \exp\left\{\mathrm{i}\int \mathrm{d}^4y\left(-\frac{1}{4}F'^2 + \ldots sA'\ldots\right)\right\} \int \mathrm{d}^4x\eta'\Delta nA' \cdot \mathrm{i}\,(s\delta A + \ldots)\,\mathrm{d}A'. \tag{9}$$

We are dealing with infinitesimal quantities Δn and δA, so the change in the generating functional to lowest order becomes

$$\mathrm{i}\Delta X \mathrm{e}^{\mathrm{i}X}\delta\xi = - \int \mathrm{d}A\mathrm{d}\omega\mathrm{d}\eta\mathrm{e}^{\mathrm{i}S} \int \mathrm{d}^4x\mathrm{i}\eta\Delta nA \cdot \mathrm{i}\,(s\delta A + x\delta\omega + \delta\eta y) =$$

$$= \left\{\frac{\mathrm{d}}{\mathrm{d}\lambda}\int \mathrm{d}A\mathrm{d}\omega\mathrm{d}\eta \exp\{\mathrm{i}\int \mathrm{d}^4x\,(L' + \eta\Delta nA\lambda)\} \cdot \mathrm{i}\,(s\delta A + x\delta\omega + \delta\eta y)\right\}_{\lambda=0}$$

$$= \left[\frac{\mathrm{d}}{\mathrm{d}\lambda}U_\lambda\right]_{\lambda=0}. \tag{10}$$

The auxiliary functional U_λ is related to the original generating functional by

$$U_\lambda = \left\{s\frac{\delta}{\delta u} - x\frac{\delta}{\delta v} + y\,(an^2)^{-1}\,\Box\, n_\mu\frac{\delta}{\delta s_\mu}\right\}[\mathrm{e}^{\mathrm{i}X_\lambda}]\,\mathrm{d}\xi, \tag{11}$$

where

$$\mathrm{e}^{\mathrm{i}X_\lambda} = \int \mathrm{d}A\ \mathrm{d}\omega\mathrm{d}\eta \exp\{\mathrm{i}\int \mathrm{d}^4x\,(L' + \eta\Delta n \cdot A\lambda)\}. \tag{12}$$

The new action contains the ghost-gluon transition with the variation of the gauge-fixing parameter. To get identities satisfied by the generating functional of the one-particle irreducible Green's functions, we take the Legendre transform defined by

$$X_\lambda = \int \mathrm{d}^4x\,(sA + x\omega + \eta y) + \Gamma(A, \omega, \eta; u, v),$$

$$s = -\frac{\delta\Gamma}{\delta A}, \qquad x = \frac{\delta\Gamma}{\delta\omega}, \qquad y = -\frac{\delta\Gamma}{\delta\eta}. \tag{13}$$

This leads to

$$U_\lambda = -\mathrm{e}^{\mathrm{i}X_\lambda} \cdot \mathrm{i}\left[\frac{\delta\Gamma_\lambda}{\delta A}\frac{\delta\Gamma_\lambda}{\delta u} + \frac{\delta\Gamma_\lambda}{\delta\omega}\frac{\delta\Gamma_\lambda}{\delta v} + (an^2)^{-1}\frac{\delta\Gamma_\lambda}{\delta\eta}\Box\, n \cdot A\right]\delta\xi. \tag{14}$$

The equation of motion for the η-field becomes

$$y + n^{\mu} \frac{\partial}{\partial u_{\mu}} X + \lambda \Delta n^{\mu} \frac{\partial}{\partial s_{\mu}} X = 0 \tag{15}$$

or, in terms of Legendre transforms,

$$\frac{\delta \Gamma_{\lambda}}{\delta \eta} = n^{\mu} \frac{\delta \Gamma_{\lambda}}{\delta u_{\mu}} + \lambda \Delta n^{\mu} A_{\mu}. \tag{16}$$

Using (16) and introducing

$$\widetilde{\Gamma}_{\lambda} = \Gamma_{\lambda} + (2an^2)^{-1}\, n \cdot A \,\Box\, n \cdot A, \tag{17}$$

we get

$$U_{\lambda} = -\mathrm{e}^{\mathrm{i}X_{\lambda}} \cdot \mathrm{i} \left\{ \frac{\delta \widetilde{\Gamma}_{\lambda}}{\delta A} \frac{\delta \widetilde{\Gamma}_{\lambda}}{\delta u} + \frac{\delta \widetilde{\Gamma}_{\lambda}}{\delta \omega} \frac{\delta \widetilde{\Gamma}_{\lambda}}{\delta v} + \lambda\, (an^2)^{-1} \,\Box\, n \cdot A \Delta n^{\mu} A_{\mu} \right\} \delta \xi. \tag{18}$$

When taking $\left.\frac{\mathrm{d}U_{\lambda}}{\mathrm{d}\lambda}\right|_{\lambda=0}$, we recall the BRST identities

$$\frac{\delta \widetilde{\Gamma}_0}{\delta A} \frac{\delta \widetilde{\Gamma}_0}{\delta u} + \frac{\delta \widetilde{\Gamma}_0}{\delta \omega} \frac{\delta \widetilde{\Gamma}_0}{\delta v} = 0. \tag{19}$$

Defining

$$\Gamma' = \frac{\delta \widetilde{\Gamma}}{\delta \lambda}, \tag{20}$$

we arrive at the Piguet-Sibold identity

$$\Delta \widetilde{\Gamma} = \frac{\delta \widetilde{\Gamma}_0}{\delta u} \frac{\delta \widetilde{\Gamma}'}{\delta A} + \frac{\delta \widetilde{\Gamma}_0}{\delta A} \frac{\delta \widetilde{\Gamma}'}{\delta u} + \frac{\delta \widetilde{\Gamma}_0}{\delta v} \frac{\delta \widetilde{\Gamma}'}{\delta \omega} + \frac{\delta \widetilde{\Gamma}_0}{\delta \omega} \frac{\delta \widetilde{\Gamma}'}{\delta v}, \tag{21}$$

or, with the notation of Zinn-Justin,

$$\Delta \widetilde{\Gamma} = \widetilde{\Gamma}_0 * \widetilde{\Gamma}'. \tag{22}$$

Here $\sim$ means that we have taken the action without the gauge-fixing term.

2. *Solution of the Piguet-Sibold identity*

Let us now consider Eq. (22) to one-loop order. In terms of loop expansion, we write

$$\Gamma = \sum_{n=0}^{\infty} \hbar\, \Gamma_n, \qquad \Gamma' = \sum_{n=0}^{\infty} \hbar\, \Gamma_n, \tag{23}$$

with

$$\Gamma_0 = S, \qquad \Gamma_0' = S'. \tag{24}$$

Then (22) implies that

$$\Delta\Gamma_1 = S * \Gamma_1' + \Gamma_1 * S'. \tag{25}$$

This is certainly satisfied if Γ_1 can be written as

$$\Gamma_1 = S * G_1, \tag{26}$$

because then

$$\begin{aligned}\Delta\Gamma_1 &= \Delta S * G_1 + S * \Delta G_1 = (S*S') * G_1 + S * \Delta G_1 = \\ &= (S * G_1) * S' + S * (\Delta G_1 + G_1 * S),\end{aligned} \tag{27}$$

where we have used a sort of Jacobi identity which follows from the definitions of the $*$ operation (G_1 and S' are anticommuting quantities). Thus (25) is recovered from (26) provided that[3)]

$$\Gamma_1' = \Delta G_1 + G_1 * S'. \tag{28}$$

3. Consequences for the planar gauge

We discussed the planar gauge to one-loop order in Ref. 4. The divergent parts of Γ_1 are given by (26) with

$$G_1 = \int \mathrm{d}^4x\, [a_3 A_\mu^a (u_\mu^a + n_\mu \bar{c}^a) + a_4 n_\mu A_\mu^a (n_\nu u_\nu^a + n^2 \bar{c}^a) - a_5 c^a v^a], \tag{29}$$

where a_3, a_4 and a_5 are divergent constants. The values were given in Ref. 4. Then Γ_1' defined by Eq. (28) becomes

$$\Gamma_1' = a_4 \int \mathrm{d}^4x\, [\Delta n_\mu A_\mu^a (n_\nu u_\nu^a + n^2 \bar{c}^a) + n_\mu A_\mu^a \Delta n_\nu u_\nu^a], \tag{30}$$

because

$$S' = \Gamma_0' = - \int \mathrm{d}^4x\, \eta \Delta n_\mu \cdot A_\mu. \tag{31}$$

It is easy to check that Eq. (28) is satisfied.

4. Diagrammatic approach

Diagrams contributing to Γ_1' (Eq. (30)) to one-loop order are shown in Figs. 1 and 2. Figs. 1a and 1b show diagrams for the gluon-source transition. Fig. 2 shows diagrams for the antighost-gluon transition. The cross signifies the action of the term $\eta \Delta n \cdot A_\lambda$ in Eq. (12) and the circle denotes the source. The extended BRST symmetry gives the constraint that the divergent constant for these graphs

Fig. 1.a. The gluon — source transition.

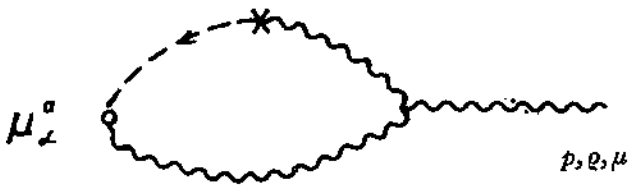

Fig. 1.b. The gluon — source transition.

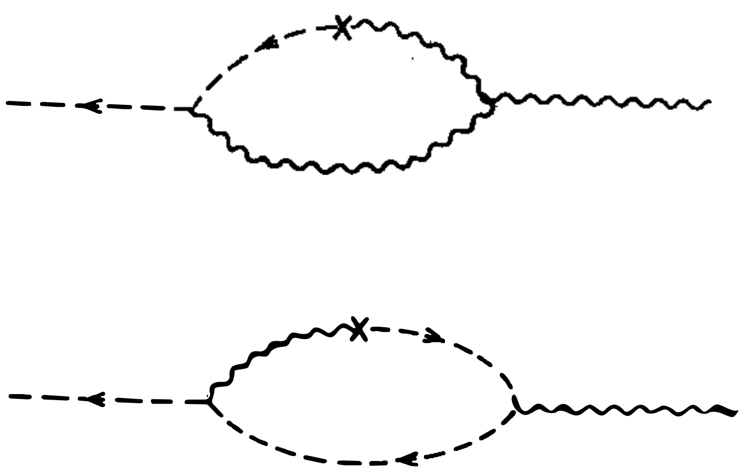

Fig. 2. The antighost — gluon transition.

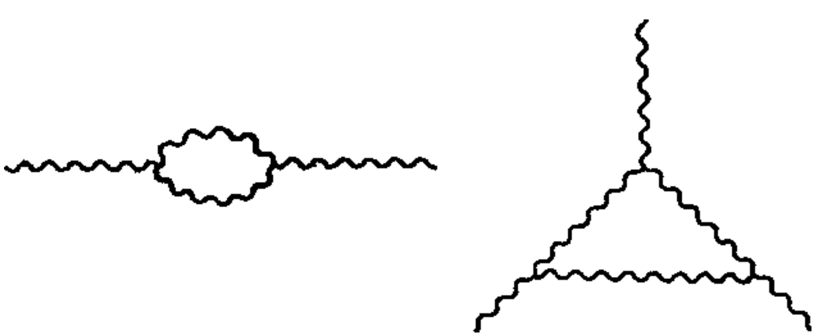

Fig. 3. The self — energy and vertex graphs.

should be a_4 from Ref. 4 where it was determined by the graphs shown here in Fig. 3:

$$a_4 = -\frac{g^2}{4\pi^2 n^2}\frac{1}{\varepsilon} C_{YM}\, a. \tag{32}$$

Indeed, the diagrams in Fig. 1 for the planar gauge with $\alpha = 1$ give

$$\Gamma_1'(a) = g^2 C_{YM} \int \frac{d^4k}{(2\pi)^4}\frac{\Delta n_\lambda}{n\cdot k}\frac{1}{k^2}\left[\delta_{\lambda\mu} - \frac{k_\lambda n_\mu + k_\mu n_\lambda}{n\cdot k}\right][\delta_{\mu\beta}(p-2k)_\varrho +$$

$$+ \delta_{\varrho\beta}(k-2p)_\mu + \delta_{\mu\varrho}(k+p)_\beta]\frac{1}{(p-k)^2}\left\{\delta_{\alpha\beta} - \frac{(p-k)_\alpha n_\beta + (p-k)_\beta n_\alpha}{n(p-k)}\right\}. \tag{33}$$

The logarithmic divergence in (30) and therefore in (33) is independent of the external momentum p and in order to isolate it, we can set $p = 0$. Using the same method as in Ref. 4, we obtain the term $a_4 \Delta n_\mu A_\mu^a n_\nu u_\nu^a$:

$$\Gamma_1'(a) = -\frac{g^2}{4\pi^2 n^2}\frac{1}{\varepsilon} C_{YM}\Delta n_\varrho n_\alpha = a_4 \Delta n_\varrho n_\alpha, \tag{34}$$

where $\varepsilon = d - 4$ in dimensional regularization. In the same way, $\Gamma_1'(b)$ correctly gives $a_4 n^\mu A_\mu^a \Delta n_\nu u_\nu^a$:

$$\Gamma_1'(b) = -\frac{g^2}{4\pi^2 n^2}\frac{1}{\varepsilon} C_{YM} n_\varrho \Delta n_\alpha = a_4 n_\varrho \Delta n_\alpha. \tag{35}$$

The graphs in Fig. 2 do not exist for the light-cone gauge. They give $a_4 \Delta n \cdot A \cdot n^2 \eta$ in the planar gauge.

5. Conclusion

In Ref. 5 we discussed the diagrammatic approach to new identities in the planar gauge. In this paper we have formulated these identities within the BRST formalism. On an explicit example to one-loop order we have shown how the Piguet-Sibold identities connect the renormalization constants of the ordinary Feynman diagrams with the diagrams arising from the action which contains the insertion of the infinitesimal change in the gauge-fixing term with respect to the gauge-fixing vector.

Acknowledgement

The author wishes to thank Prof. J. C. Taylor for invaluable help and discussions.

Referencse

1) Yu. L. Dokshitsev, D. I. D'yakanov and S. I. Troyan, Phys. Reports **58** (1980) 269;
2) C. Becchi, A. Rouet and R. Stora, in Renormalization Theory, eds. G. Velo and A. S. Wightman (D. Reidel, Holland and Boston, Mass., 1976);
A. A. Slavnov, Theor. Math. Phys. **10** (1972) 99;
J. C. Taylor, Nucl. Phys. **B33** (1971) 436;
O. Piguet and K. Sibold, Nucl. Phys. **B253** (1985) 517;
3) A. Andraši and J. C. Taylor, DAMTP 86-11 preprint, unpublished;
4) A. Andraši and J. C. Taylor, Nucl. Phys. **B192** (1981) 283;
5) A. Andraši and J. C. Taylor, Fizika **19** (1987) 345.

PROŠIRENA BRST SIMETRIJA I PLANARNI BAŽDARNI UVJET

ANĐELKA ANDRAŠI

Institut »Ruđer Bošković«, 41001 Zagreb

UDK 539.12

Originalni znanstveni rad

BRST identiteti nisu dovoljni za kontrolu svih kontračlanova u nekovarijantnim baždarnim uvjetima. Razmatran je eksplicitni primjer planarnog baždarnog uvjeta do reda jedne petlje u perturbacionoj teoriji da bi se pokazalo kako proširena BRST simetrija povezuje konstante renormalizacije.