

## RESTRICTIONS ON THE CLASS OF THE STRONG COUPLING UNIFIED SUSY MODELS

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Group properties of a very general class of SUSY preon models, including already published simple models, are studied. Yukawa mass terms and the resulting proton decays are investigated. Special attention is given to interesting models based on the chain  $E_6 \supset SU(2)_{HC} \otimes SU(3)_C \otimes U(1)^3$ . In those models grand unification requires either extensive symmetry breaking or an assumption that second and third generations are excited states of the basic structure. The proton decay is characterized by the four particle final state  $p \rightarrow \pi^- e^+ e^+ \nu_e$ . It is very difficult to construct SUSY strong coupling model which satisfies a set of constraints (given in the text) which are inspired by physical considerations.

### 1. Introduction

The strong coupling version of the Standard Model (SCSM)<sup>1,2)</sup>, which contains spinor and scalar preons, leads quite naturally to its supersymmetric generalizations<sup>3, 4)</sup> (SSCSM). The unification of the fundamental forces within the supersymmetric gauge group produced a very simple model<sup>4)</sup> which contained a minimal number of superfields and which was in its physical content somehow parallel to the standard  $SU(5)$  grand unified theory<sup>5)</sup> (GUT). Subsequent study of superpotentials, which could generate fermion mass terms<sup>6)</sup> was not quite complete.

As it will be shown, supersymmetry (or more precisely superpotential) provides a very strict limitations on the model building. It turns out that the flavour SU(5) symmetry is the most attractive feature in our preon models<sup>4,6)</sup>, while its SUSY generalization always demands the introduction of some experimentally redundant superfields.

We use adjective »redundant« for those fields which are not yet observed experimentally. Reasonable amount of redundancy at some stage of a model development need not be a bad feature. One has only to think of the phenomenal success of the standard electroweak model. However one can not be comfortable with the superabundance of new fields and new exotics, especially if they appear in experimentally well explored region. The usual way of escape by assigning very large masses to such exotic, is just a transparent disguise for the model deficiencies.

In any SUSY preon-model one can roughly classify the redundancies and the exotics as follows:

- (i) Spin exotics, which naturally appear when one combines all spins contained in a given preon-superfield.
- (ii) Supermassive superfields, which are needed to mass-split preon-superfields in order to carry out an acceptable unification.
- (iii) Flavour exotics appear when, in order to construct superpotentials, one has to introduce preon-fields with new flavours<sup>3)</sup>.

In the following we study possible covering groups and argue that all of the exotic features mentioned above must appear in a SSCSM.

In order to specify this statement let us remember that the underlying group structure of the models studied in Refs. 4 and 6 was based on the sequence

$$SO(10) \supset SU(5) \otimes U(1)_{Q_1} \supset [SU(2)_{HC} \otimes SU(3)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1}. \quad (1.1)$$

Two preon fields ( $5^*, 10$ ) were placed in the single spinorial 16-dimensional representation of SO(10). Their electric charges were given as linear combination of U(1) generators

$$Q_{EM} = a_1 Q_1 + a_2 Q_2. \quad (1.2)$$

One used the »ordinary« ( $a_1 = 0, a_2 = 1/6$ )<sup>4)</sup> and the »flipped« ( $a_1 = -1/5, a_2 = -1/30$ )<sup>6)</sup> embedding of the electric charge. The sequences (1.1), (1.2) which were discussed by us<sup>4,6)</sup>, do not contain any flavour exotics. The supermassive fields (ii) and the spin exotics (i) were unavoidable.

In the »flipped« case one can not construct superpotential which would contain required flavours and which would lead to a renormalizable theory<sup>6)</sup>. With ordinary embedding one can construct superpotential which, unfortunately, does not satisfy all physical requirements. With  $5^*$  and 10 left-handed chiral superfields one can construct only one SUSY and gauge invariant Yukawa coupling

$$W = \lambda_D [10 \otimes 5^* \otimes 5^*]. \quad (1.3)$$

This coupling can account for electron and  $d$ -quark masses.

The combination needed for  $\nu$ -quark mass

$$\lambda_U [10 \otimes 10 \otimes 5] \tag{1.4}$$

is not SUSY invariant, as the representations 5 and 10 have opposite chiralities. One can perhaps argue that such a situation can explain why  $m_d$  is larger than  $m_u$ . Unfortunately, this would not work for the second and third generation of quarks.

The Yukawa coupling of the form (1.3), i. e.

$$\mathcal{W} = \lambda_D \sum_f [10 \otimes 5^* \otimes 5^*] \tag{1.5}$$

(where  $f$  stands for flavours) does not lead to an effective proton decay operator. The introduction of the explicit SUSY breaking term (1.4) can induce a rapid decay of the proton into muon neutrino and kaon

$$p \Rightarrow K^+ + \nu_\mu. \tag{1.6}$$

This decay is a consequence of the SUSY breaking superpotential

$$\mathcal{W} = \lambda_U \sum_f [10 \otimes 10 \otimes 5] + \lambda_D \sum_f [10 \otimes 5^* \otimes 5^*] \tag{1.7}$$

which leads to the effective terms<sup>11)</sup>

$$\int d\theta^2 \Phi_{\alpha_1^i} \Phi_{\alpha_2^i} \Phi_{\alpha_2^i} \Phi_{\beta_1^i} + \int d\theta^2 \Phi_{\alpha_1^i} \Phi_{\alpha_2^i} \Phi_{\alpha_2^i} \Phi_{\beta_2^i}. \tag{1.8}$$

Here the superfields  $\Phi$  are labeled according to their preon content. Upper index refers to a generation while the lower one is a hypercolour index.

Thus

$$(\alpha_1^i \alpha_2^i); \quad i = 1, 2, 3$$

is the hypercolour doublet belonging to  $i$ -th generation.

The amplitude for the process (1.6) is proportional to an unacceptably large factor

$$(\lambda_U \lambda_D)/m_D.$$

The corresponding Feynman diagram is shown in Fig. 1.

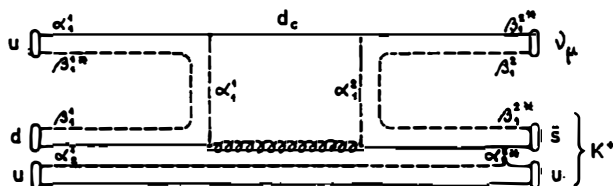


Fig. 1. Proton decay  $p \rightarrow K^+ + \nu_\mu$ . The full lines refer to spinor and the dashed lines to scalar components of superfields. The wavy line is a hypergluino.

Obviously one can not use a mass term of the type (1.7). It would be in disagreement with the proton decay experiments. Besides a theory with an explicit SUSY breaking (1.4) might be nonrenormalizable.

One encounters similar difficulties when dealing with other, larger covering groups, as it is shown in detail in the next section. The situation might be resolved by introducing superfield which contains new flavours (Sect. III), which means that flavour exotic (iii) must appear in any SSCSM. In the light of the present experimental knowledge this is a rather unattractive feature of such models.

### 2. Groups and embeddings

Starting from some simple group  $G$  one searches for the maximal regular and non-semisimple embeddings. They can be deduced from Dynkin diagrams by removing a dot in two (2.1) or three (2.2) steps. The subsequent products of (semi-)simple groups and  $U(1)$  factors<sup>7)</sup> are

$$G \supset H \otimes U(1)_{Q_1} \supset [K \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \tag{2.1}$$

and

$$\begin{aligned} G &\supset \tilde{H} \otimes U(1)_{Q_1} \supset [H \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \supset \\ &\supset [K \otimes U(1)_{Q_3}] \otimes U(1)_{Q_2} \otimes U(1)_{Q_1}. \end{aligned} \tag{2.2}$$

The electric charge generator  $Q_{EM}$  is a linear combination of the  $U(1)$  generators  $Q_i$

$$Q_{EM} = \sum_i a_i Q_i. \tag{2.3}$$

The rational coefficients  $a_i$  are determined by a set of simultaneous linear equations of the form

$$Q_{EM}^{(N)} = \sum_i a_i Q_i^{(N)}. \tag{2.4}$$

Here  $N$  refers to every multiplet which appears on the right hand side of the Eqs. (2.1) and (2.2). Nontrivial solutions of (2.4) which are based on the embedding (2.1) correspond to the »flipped« embeddings. The solution based on (2.2) are »double flipped« embeddings. A normal embedding is based on (2.1) with  $a_1 = 0$ . This correspond to the model<sup>4,6)</sup> (1.1), which was discussed in the preceding section.

Only those groups which satisfy certain constraints and assumptions can be employed in the model building. An example of such reasonable assumptions, as proposed for the grand unification program, can be found in Ref. 8. In our case there exist obvious rank constraints for (2.1) and (2.2), respectively:

$$\begin{aligned} \text{rank}(G) &= 1 + \text{rank}(H) = 2 + \text{rank}(K) \\ \text{rank}(G) &= 1 + \text{rank}(\tilde{H}) = 2 + \text{rank}(H) = 3 + \text{rank}(K). \end{aligned} \tag{2.5}$$

Additional assumptions are:

- 1)  $K$  is a gauge group. It must contain the colour group  $SU(3)_C$  and some unitary hypercolour group  $SU(N)_{HC}$ , which confines preons.
- 2)  $G$ ,  $H$  and  $\tilde{H}$  are simple groups.
- 3)  $G$  must have complex representations.
- 4) The theory is free of anomalies.
- 5)  $G$  leads to asymptotic freedom.
- 6) Nontrivial solution of Eq. (2.4) exist.
- 7) SUSY breaks at low energy only.

By assuming that  $SU(N)_{HC}$  is the hypercolour gauge group we are saying that the preon dynamics is analogous to the quark dynamics. This need not be so, but some assumptions has to be made. The demand of asymptotic freedom 5) allows the implementation of grand unification, as it was done in our earlier papers<sup>4,6)</sup>.

The flipped chains (2.1) will be considered first. We use the following notation<sup>8)</sup>

$$\begin{aligned}
 A_N &= SU(N + 1) & C_N &= Sp(2N) \\
 B_N &= SO(2N + 1) & D_N &= SO(2N).
 \end{aligned}
 \tag{2.6}$$

All possible maximal regular non-semisimple embeddings (2.1) are listed in Table 1. They are defined through projection matrices which are listed in Table 2. Only three entries in Table 1 contain the required subgroup  $K$

TABLE 1.

G	H	K
$A_N$	$A_{N-1}$	$A_{N-R-2} \otimes A_R$
$B_N$	$B_{N-1}$	$B_{N-2}$
$C_N$	$A_{N-1}$	$A_{N-R-2} \otimes A_R$
$D_N$	$A_{N-1}$	$A_{N-R-2} \otimes A_R$
$D_N$	$D_{N-1}$	$A_{N-2}$
$D_N$	$D_{N-1}$	$D_{N-2}$
$E_6$	$D_5$	$A_4$
$E_6$	$D_5$	$D_4$
$E_7$	$E_6$	$D_5$

Maximal regular (non-semisimple) embeddings  $G \supset H \otimes U(1)_{Q1} \supset [K \otimes U(1)_{Q2}] \otimes U(1)_{Q1}$ .

$$K \supset (A_{N-4})_{HC} \otimes (A_2)_C \equiv SU(N-3)_{HC} \otimes SU(3)_C; \quad N \geq 5.$$

The symplectic groups  $C_N (\equiv Sp(2N))$  have only real representations. They are unsuitable for the grand unification<sup>9)</sup>. No flipped embeddings exist for unitary groups  $A_N (\equiv SU(N+1))$ <sup>10)</sup>. One also knows that the examples of the anomaly free complex  $SU(N)$  representations are extremely rare. Usually several irreducible representations have to conspire in cancelling the anomalies, which is aesthetically quite unpleasing.

TABLE 2.

$$\begin{array}{c}
 A_N \rightarrow A_R * A_{N-R-1} * U(1) \\
 (0 \leq R \leq N-1) \\
 \begin{array}{c|c|c}
 A_R & A_{N-R-1} & U(1) \\
 \hline
 I_R & 0 & \begin{array}{c} 1 \cdot (N-R) \\ 2 \cdot (N-R) \\ \vdots \\ R \cdot (N-R) \end{array} \\
 \hline
 0 & 0 & (R+1)(N-R) \\
 \hline
 0 & I_{N-R-1} & \begin{array}{c} (R+1)(N-R-1) \\ \vdots \\ (R+1) \cdot 2 \\ (R+1) \cdot 1 \end{array}
 \end{array}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 B_N \rightarrow B_{N-1} * U(1) \\
 (2 \leq N) \\
 \begin{array}{c|c}
 0 & 2 \\
 \hline
 I_{N-1} & \begin{array}{c} 2 \\ 2 \\ \vdots \\ 2 \\ 2 \\ 1 \end{array}
 \end{array}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 C_N \rightarrow A_{N-1} * U(1) \\
 (2 \leq N) \\
 \begin{array}{c|c}
 I_{N-1} & \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ N-2 \\ N-1 \end{array} \\
 \hline
 0 & N
 \end{array}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 D_N \rightarrow A_{N-1} * U(1) \\
 \begin{array}{c|c}
 I_{N-1} & \begin{array}{c} 2 \\ 4 \\ 6 \\ \vdots \\ 2(N-3) \\ 2(N-2) \\ (N-2) \end{array} \\
 \hline
 0 & N
 \end{array}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 D_N \rightarrow D_{N-1} * U(1) \\
 \begin{array}{c|c}
 0 & 2 \\
 \hline
 I_{N-1} & \begin{array}{c} 2 \\ 2 \\ \vdots \\ 2 \\ 1 \\ 1 \end{array}
 \end{array}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 E_6 \rightarrow D_5 * U(1) \\
 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}
 \end{array}
 ;
 \end{array}
 \begin{array}{c}
 E_7 \rightarrow E_6 * U(1) \\
 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}
 \end{array}
 ;
 \end{array}$$

Projection matrices.

Thus we are left with the orthogonal groups  $D_N (\cong SO(2N))$  only. All those groups, with one exception, are anomaly free and posses complex representations. The exception is  $D_3$  which is isomorphic to  $A_3$ .

The orthogonal group based emebdding relies upon the flipped chain

$$D_N \supset A_{N-1} \otimes U(1)_{Q_1} \supset [(A_{N-4})_{HC} \otimes (A_2)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1}. \quad (2.7)$$

Only odd  $N = 2k + 1$  is allowed by our assumptions.  $D_{2k+1}$  has two complex spinor representations  $\Delta_{\pm}$  which are conjugate to each other and whose dimen-

sions are  $2^{2k}$ . For even  $N$  ( $N = 2k$ ) there exist two self-conjugate and nonequivalent spinors. The hypercolour gauge group

$$(A_{2k-3})_{HC} \equiv SU(2k-2)_{HC}; \quad k \geq 2 \quad (2.8)$$

limits the value of  $k$  from below. For an orthogonal group  $D_N$  the  $b_N$  coefficients<sup>1)</sup>

$$b_N = 11(2N-2)/3 - (2/3)2^{N-4} \quad \text{non-SUSY} \quad (2.9)$$

$$b_N = 3(2N-2) - 2^{N-4} \quad \text{SUSY} \quad (2.10)$$

have to be positive, i. e.  $b_N > 0$  in order to have the asymptotic freedom. One finds

$$b_N > 0; \quad N = 5, 7, 9 \quad (k = 2, 3, 4). \quad (2.11)$$

The corresponding covering groups and their hypercolour groups are:

$$\begin{aligned} G = SO(10) & \quad SU(2)_{HC} \\ G = SO(14) & \quad SU(4)_{HC} \\ G = SO(18) & \quad SU(6)_{HC}. \end{aligned} \quad (2.12)$$

The first row corresponds to already published case<sup>4,6)</sup>.

All conditions 1)–7) are satisfied by the groups (2.12). However, in order to construct a preon model one has to specify the preon content of the group representations. The most economical and elegant procedure, which is also anomaly free, is to put all model's particles in the complex spinor  $\Delta_+$ . This can be formulated as an additional, 8<sup>th</sup>, assumption:

8) Model's particles are in the representation  $\Delta_+$ .

It would be aesthetically pleasing if one could build superpotentials with this single preon field. This means:

9) There exist a SUSY invariant superpotential, containing  $\Delta_+$  only, which can generate quark and lepton masses.

This condition was not used in our earlier papers<sup>4,6)</sup> which were based on the following scheme

$$SO(10) \supset SU(2)_{HC} \otimes SU(3)_C \otimes U(1) \otimes U(1) \quad (2.13)$$

$$\Delta_+ = 10_{SO(10)} = (10 + 5^* + 1)_{SU(5)}.$$

<sup>1</sup> The last terms in (2.9, 10) come from  $n_R T(R)$  where  $T(R)$  is the trace of the group generator matrices and  $n_R$  denotes the number of representations. We have only one spinor, i. e.  $n_R = 1$ .

There the superpotential (see also Section 1) was not constructed at the SO(10) but at the SU(5) level, meaning at the K-group (2.1) level. Analogous procedure can be carried out for larger G or K groups. Some details about the decomposition of the respective complex spinor representation  $\Delta_+$  can be found in Appendix.

It is easy to see that the condition 9) can not be fulfilled. SUSY allows only bilinear and trilinear couplings. However, at least four spinors  $\Delta_+$  are needed for a gauge invariant coupling

$$[\Delta_+ \otimes \Delta_+ \otimes \Delta_+ \otimes \Delta_+] \in 1. \tag{2.14}$$

Such coupling leads to nonrenormalizable theory. In the model building one has to employ additional real fundamental vector  $2N$  representation. For example, in SO(10) one can construct

$$[16 \otimes 16 \otimes 10] + [10 \otimes 10]. \tag{2.15}$$

As physically needed preons sit in the representation 16 (2.13), the representation 10 must contain new exotic flavours. The combination (2.15) is not a typical mass term which at SU(5) level must look like (1.3). Therefore the choice (2.15) is neither attractive nor acceptable.

Contrary to the SO(10) case (1.2) it is impossible to construct invariant trilinear coupling in SO(14) and SO(18) if one uses the minimal sets of anomaly free SU(N) representation only. The corresponding SU(N) decomposition (see Appendix) of the SO(2N) spinor representations are

$$\begin{aligned} 64_{\text{SO}(14)} &= (1 + 7^* + 21 + 35^*)_{\text{SU}(7)} \\ 256_{\text{SO}(18)} &= (1 + 9^* + 36 + 84^* + 126)_{\text{SU}(7)}. \end{aligned} \tag{2.16}$$

In the Young tableau picture (see Appendix) one can write (2.16) as

$$\begin{aligned} 1 + 7^* + 21 + 35^* &= [0] + [6] + [2] + [4] \\ 1 + 9^* + 36 + 84^* + 126 &= [0] + [8] + [2] + [6] + [4]. \end{aligned} \tag{2.17}$$

In order to construct an invariant trilinear coupling in SU(N) one has to arrange three representations  $[n]$  in such a way as to produce a scalar, which has a sum of entries zero (mod N), which is obviously impossible.

In the SO(14) embedding one could construct<sup>13)</sup> some invariant couplings at SU(7) level by using anomaly free combinations

$$5 \cdot 7^* + 35 + 21 \tag{2.18}$$

or

$$2 \cdot 35 + 21^* + 7^* \tag{2.19}$$

which do not constitute  $\Delta_+$  (64) of SO(14). Obviously more fields and thus new flavours are involved. Other possibilities are analogous to (2.15) in SO(10). One

can introduce  $14 = 7 + 7^*$  in  $SO(14)$  and  $18 = 9 + 9^*$  in  $SO(18)$ . with the same consequences as before.

As discussed in Appendix, one can not identify all needed preons and left handed charge conjugate fields in (2.16). The mass term for one quark only, say  $u$ , can be constructed.

Exceptional groups  $E_N$  provide some additional possibilities. The embedding based on the  $E_6$  group

$$\begin{aligned} E_6 &\supset SU(6) \otimes SU(2) \supset SU(6) \otimes U(1)_{Q_1} \supset \\ &\supset [SU(3)_{HC} \otimes SU(3)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \end{aligned} \quad (2.20)$$

for example, satisfies the conditions 1)–7). However, its vector representation 27 do not have the required  $SU(3)_C \otimes SU(3)_{HC}$  content. It decomposes as

$$\begin{aligned} 27_{E_6} = (6^* + 6^* + 15)_{SU(6)} = &[[ (3^*, 1) + (1, 3^*) ] + [ (3^*, 1) + \\ &+ (1, 3^*) ] + [ (3^*, 1) + (1, 3^*) + (3, 3) ] ]_{SU(3) \otimes SU(3)}. \end{aligned} \quad (2.21)$$

In (2.21) there are no singlets  $(1, 1)$  which could be interpreted as  $e^c$  or  $\nu^c$  fields.

Available option may be based on the chain

$$E_6 \supset D_5 \supset A_4 \quad (2.22)$$

which will be considered at length in the next section.

### 3. New flavours

The chain (Table 1)

$$E_6 \rightarrow D_5 (SO(10)) \rightarrow A_4 (SU(5)) \quad (3.1)$$

naturally leads to the double flipped embedding

$$E_6 \supset SU(2)_{HC} \otimes SU(3)_C \otimes U(1)_{Q_3} \otimes U(1)_{Q_2} \otimes U(1)_{Q_1}. \quad (3.2)$$

This is closely related to the regular<sup>4)</sup> and to the flipped<sup>6)</sup> embeddings which were described in Section 1 of this paper. However model's particles are now in  $E_6$  representation 27, whose decomposition at  $SU(5)$  level

$$27_{E_6} \rightarrow (10 + 5 + 5_a^* + 5_b^* + 1_a + 1_b)_{SU(5)} \quad (3.3)$$

has two unequivalent  $5^*$  and two unequivalent singlet representations. They obviously must contain new preon flavours and thus new exotic quarks and leptons.

There exist three possibilities for the charge embedding, which are analogous to the ways described by Ref. 4 in the context of the normal quark-lepton models. One can construct three preon models, with electromagnetic charges  $Q_{EM}^{14,15}$

$$\begin{aligned}
 \text{MODEL I} & \quad Q_{EM} = Q_3/6 \\
 \text{MODEL II} & \quad Q_{EM} = -Q_3/30 - Q_2/5 \\
 \text{MODEL III} & \quad Q_{EM} = -Q_3/30 + Q_2/20 + Q_1/4. \tag{3.4}
 \end{aligned}$$

The particle content of the SU(5) representation in (3.3), labeled as  $N(SU(5), Q_2, Q_1)$  is shown in Table 3. This table contains exotics  $B^c, B, H^c, H$ , two of which we identify with additional preon fields. Composite quarks and leptons with left-handed helicity are constructed as usual<sup>1-4)</sup>. For the first generation, in the Model I for example (see Table 4).

$$\begin{aligned}
 u_L & \cong (\alpha H^c) & d_L & \cong (\alpha H) \\
 e_L & \cong (\beta H) & \nu_L & \cong (\beta H^c). \tag{3.5}
 \end{aligned}$$

Here the field  $H (H^c)$  has taken the role of the  $\beta$  field in the simple old model<sup>4)</sup>. The IVB's must be built from the scalar components of  $H$  and  $H^c$  superfields. There are some differences in the treatment of the second and the third generation. In the Model I for example one could introduce generation dependent representations

$$\begin{aligned}
 (10, -1, -1)_x & = (\alpha_x, u_x^c, e_x^c) \\
 (5^*, 3, 1)_x & = (d_x^c, \beta_x) \\
 (1, -5, 1)_x & = \nu_x^c. \tag{3.6}
 \end{aligned}$$

TABLE 3.

Model	(10, -1, 1)	(5*, 3, 1)	(5*, -2, -2)	(5, 2, -2)	(1, -5, 1)	(1, 0, 4)
I	$\alpha$	$d^c$	$B^c$	$B$	$\nu^c$	$S^c$
	$u^c e^c$	$\beta$	$H$	$H^c$		
II	$\alpha$	$u^c$	$B^c$	$B$	$e^c$	$S^c$
	$d^c \nu^c$	$\beta$	$H^c$	$H$		
III	$\alpha$	$d^c$	$u^c$	$B$	$\nu^c$	$e^c$
	$B^c S^c$	$H^c$	$H$	$\beta$		

\*  $\alpha, \beta, u^c, d^c, e^c, \nu^c$  are notations from Ref. 4 and 6.

Particle contents  $(N(SU(5), Q_2, Q_1)^*$ .

TABLE 4.

	Model I	Model II	Model III
$\lambda_1$ $(10, -1, 1) \times (5^*, 3, 1)$ $\times (5^*, -2, -2)$	$\alpha Hd^c + \alpha \beta b^c +$ $\beta e^c + u^c d^c B^c$	$\alpha u^c H^c + \alpha BB^c +$ $\beta \nu^c H^c + u^c d^c B^c$	$\alpha d^c H + \alpha u^c H^c +$ $HH^c S^c + u^c d^c B^c$
$\lambda_2$ $(5^*, 3, 1) \times (5, 2, -2)$ $\times (1, -5, 1)$	$\beta \nu^c H^c + d^c \nu^c B$	$\beta H e^c + u^c e^c B$	$\beta H^c \nu^c + d^c \nu^c B$
$\lambda_3$ $(5^*, -2, -2) \times (5, 2, -2)$ $\times (1, 0, 4)$	$HH^c S^c + BB^c S^c$	$HH^c S^c + BB^c S^c$	$\beta e^c H + u^c e^c B$
$\lambda_4$ $(10, -1, 1) \times (10, -1, 1)$ $\times (5, 2, -1)$	$\alpha \alpha B + \alpha u^c H^c +$ $u^c e^c B$	$\alpha \alpha B + \alpha Hd^c +$ $d^c \nu^c B$	$\alpha \alpha B + \alpha \beta B^c +$ $BB^c S^c$

Couplings of matter fields.

Here the index  $x$  refers to three generations. The meaning of the notation is

$$\begin{aligned}
 u_2^c &= c^c & e_2^c &= \mu^c \\
 u_3^c &= t^c & e_3^c &= \tau^c \\
 d_2^c &= s^c & \nu_2^c &= \nu_\mu^c \\
 d_3^c &= b^c & \nu_3^c &= \nu_\tau^c.
 \end{aligned}
 \tag{3.7}$$

The rest of representations can be common to all generations, as this automatically explains the universality of the effective (i. e. due to the composite IVB exchange) weak interactions. The proposed combination is also anomaly free.

One can introduce the generations in Model II in an analogous way. The Model III however would require generation tripling of the fields ( $5^*$ ,  $-2$ ,  $-2$ ) and ( $5$ ,  $2$ ,  $-2$ ) also. As those field contain H and  $H^c$  particles, IVB bosons must be made of  $H_1$ ,  $H_2$ ,  $H_3$ ,  $H_1^c$ ,  $H_2^c$  and  $H_3^c$  fields. One has to make careful adjustments of the model, in order to have the weak interaction universality. An analogous possibility has been already discussed in Ref. 4. However, it would be more attractive and natural to assume that generations 2 and 3 are just exciting states of the first generation. Such a possibility will be mentioned in the next section in the context of the unification of forces. Needless to say, the models I and II can be, if desired, interpreted in the same way also.

As it will be described in detail, all conditions 1)–8) from Section 2 can be met, but at the price. The redundancies are increased, as mentioned in iii), Section 1, of this paper.

The most general  $SU(2)_{HC} \otimes SU(3)_C$  invariant superpotential is

$$\begin{aligned}
 W &= \delta_1 (\alpha H^c u^c) + \delta_2 (\alpha H d^c) + \delta_3 (\beta H e^c) + \delta_4 (H H^c S^c) + \\
 &+ \delta_5 (B B^c S^c) + \delta_6 (B u^c e^c) + \delta_7 (\alpha \beta B^c) + \delta_8 (B d^c \nu^c) + \delta_9 (\alpha \alpha B) + \\
 &+ \delta_{10} (B^c u^c d^c) + \delta_{11} (\beta H \nu^c).
 \end{aligned}
 \tag{3.8}$$

Only the couplings,  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$ ,  $\delta_4$ ,  $\delta_5$ ,  $\delta_9$  and  $\delta_{11}$ , are acceptable. The others would lead to a rapid proton decay<sup>16)</sup>.

A more general expression can be written at the  $SO(10)$  or the  $SU(5)$  level, respectively:

$$\begin{aligned}
 g_1 [(16, 1) \otimes (16, 1) \otimes (10, -2)]_{SO(10)} &\rightarrow \lambda_1 [(10, -1, 1) \otimes (5^*, 3, 1) \otimes \\
 &\otimes (5^*, -2, -2)]_{SU(5)} + \lambda_2 [(5^*, 3, 1) \otimes (5, 2, -2) \otimes \\
 &\otimes (1, -5, 1)]_{SU(5)} + \lambda_4 [(10, -1, 1) \otimes (10, -1, 1) \otimes (5, 2, -2)]_{SU(5)} \\
 g_2 [(10, -2) \otimes (10, -2) \otimes (1, 4)]_{SO(10)} &\rightarrow \lambda_3 [(5^*, -2, -2) \otimes \\
 &\otimes (5, 2, -2) \otimes (1, 0, 4)]_{SU(5)}.
 \end{aligned}
 \tag{3.9}$$

Here we have used the notations  $(N(\text{SO}(10), Q_1))$  and  $(N(\text{SU}(5), Q_2, Q_1))$  respectively. Obviously, for an unbroken  $\text{SO}(10)$  symmetry one has

$$g_1 = \lambda_1 = \lambda_2 = \lambda_4 \tag{3.10}$$

$$g_2 = \lambda_3.$$

The particle content of the superpotential (3.9) varies from model to model as it is shown in Table 4. All terms from that table can be identified in the general expression (3.8).

The unwanted mass terms in (3.8) can be formally gotten rid off at  $\text{SU}(5)$  level by imposing the  $Z_2$  discrete symmetry<sup>11)</sup>

$$(10, -1, 1) \xrightarrow{Z_2} (-) (10, -1, 1)$$

$$(5^*, -2, -2) \xrightarrow{Z_2} (-) (5^*, -2, -2)$$

$$(5, 2, -2) \xrightarrow{Z_2} (-) (5, 2, -2) \tag{3.11}$$

The couplings  $\lambda_2$  and  $\lambda_4$  must disappear if the symmetry (3.11) is to be satisfied. In those couplings quark (or quark-like) and lepton (or lepton-like) fields are mixed. The condition (3.11) makes the unification at  $\text{SO}(10)$  level impossible. It can be tried at  $\text{SU}(5)$  level.

#### 4. The unification of forces

The unification procedure is closely connected with a generation. With generation dependent preon fields, some very tricky and quite unconvincing «fine tuning» is needed for the unification.

Relative magnitudes of the coupling constants  $\alpha_G$  ( $G = \text{HC}, \text{C}$ ) are determined by  $b_N$  factors which appear in the one loop equation

$$\frac{1}{\alpha_G(q)} = \frac{1}{\alpha_G(q_G)} + (b_N/2\pi) \ln(q/q_G)$$

$$b_N = 3N - \sum_R n_R T(R). \tag{4.1}$$

Here  $N = 2$  for the  $\text{SU}(2)_{\text{HC}}$  group or  $N = 3$  for the  $\text{SU}(3)_{\text{C}}$  group. Factor  $n_R$  denotes the number of  $\text{SU}(2)_{\text{HC}}$  doublets or  $\text{SU}(3)_{\text{C}}$  triplets. The Casimir invariant  $T(R)$  is in all cases  $1/2$ . It is easy to see that the combination of preons (3.3) contains 6 doublets and 6 triplets. Thus

$$b_{\text{HC}} = 6 - \frac{1}{2} 6 = 3$$

$$b_{\text{C}} = 9 - \frac{1}{2} 6 = 6. \tag{4.2}$$

These results are valid for all models (3.4). The necessary condition for the asymptotic behaviour which leads to an unification,  $b_N > 0$ , is fulfilled. However, model requires  $b_{HC} > b_C$ , which can be achieved only by introducing further symmetry breaking, mass-splitting and some additional mass-split multiplets<sup>3,4</sup>.

With three generations of preons (i. e. three different combinations of SU(5) representations (3.3))  $b_{HC}$  factor becomes negative. One has 18 doublets and 18 triplets which results in

$$b_{HC} = 6 - \frac{1}{2} 18 = -3 < 0$$

$$b_C = 9 - \frac{1}{2} 18 = 0. \quad (4.3)$$

Extensive fine tuning is needed for an unification. First one assumes that all SU(5) representations in (3.3) are mass-split in such a way that SU(2)<sub>HC</sub> doublet in 10 becomes superheavy and decouples. The same goes for the SU(3)<sub>C</sub> triplets in 5 and 5\* representations. This can be achieved through some complicated couplings to representations 75, 50, 50\*, 24, 15, 15\*, 10 and 10\*<sup>3,4,17</sup>. One ends with 3 doublets and 1 triplet per generation. For three generations one finds

$$b_{HC} = 3/2; \quad b_C = 15/2. \quad (4.4)$$

In order to have  $b_C < b_{HC}$  one can add 7 combinations of the mass-split representations 10 and 10\* which introduce 14 additional triplets, thus

$$b_C = 15/2 - 14/2 = 1/2.$$

Thus one needs 3 additional sets of representations 75, 50, 50\*, 24, 15, 15\*, 10 and 10\* and 7 additional sets of 10 and 10\*, altogether more than 30 unobservable superfields. It looks too messy and too unconvincing.

An alternative is to use just one (3.3) set of preons and to assume that the generations correspond to a set of bound states of the same preons. This analogy with the energy levels of, for example, hydrogen atom, has its drawbacks. One observes many atomic states, but there are only three generations. One has to assume some huge energy gap after first three bound states, which must be appreciatively larger than the gap which is responsible for the nuclear magic numbers.

Nevertheless we will analyze the unification and the proton decay. This is intended to serve as an illustration for the complexity of the model and does not pretend to be a true picture (not even in the model sense) of quark and lepton structure.

The result (4.2) can be improved by introducing 4 mass-split combinations 10 and 10\* (using coupling  $24 \otimes 15^* \otimes 10$  and  $24 \otimes 15 \otimes 10^*$ , mentioned above), which add 8 SU(3)<sub>C</sub>-triplets so that  $b_C$  decreases. With

$$b_C = 6 - \frac{1}{2} 8 = 2$$

$$b_{HC} = 3 \quad b_C < b_{HC} \quad (4.5)$$

one can find the unification scale  $M$  from the formula<sup>4)</sup>

$$M = q_{HC}^{b_{HC}/(b_{HC}-b_C)} q_C^{b_C/(b_C-b_{HC})}. \quad (4.6)$$

A list for the acceptable values for the unification scale  $M$  with the corresponding strengths of the universal coupling constant  $\alpha(M)$  is given in Table 5.

TABLE 5.

$q_{HC}$ (GeV)	$q_C$ (GeV)	$\alpha(M)$	$M$ (GeV)
$5 \times 10^4$	0.1	$7.39 \times 10^{-2}$	$1.25 \times 10^{16}$
$1 \times 10^4$	0.1	$8.34 \times 10^{-2}$	$1.00 \times 10^{14}$
$5 \times 10^3$	0.1	$8.92 \times 10^{-2}$	$1.25 \times 10^{13}$
$1 \times 10^3$	0.1	$1.02 \times 10^{-1}$	$1.00 \times 10^{11}$
$5 \times 10^4$	0.2	$7.77 \times 10^{-2}$	$3.12 \times 10^{15}$
$1 \times 10^4$	0.2	$8.82 \times 10^{-2}$	$2.50 \times 10^{13}$
$5 \times 10^3$	0.2	$9.37 \times 10^{-2}$	$3.12 \times 10^{12}$
$1 \times 10^3$	0.2	$1.09 \times 10^{-1}$	$2.50 \times 10^{10}$
$5 \times 10^4$	0.5	$8.34 \times 10^{-2}$	$5 \times 10^{14}$
$1 \times 10^4$	0.5	$9.56 \times 10^{-2}$	$4 \times 10^{12}$
$5 \times 10^3$	0.5	$1.19 \times 10^{-1}$	$5 \times 10^{11}$
$1 \times 10^3$	0.5	$1.09 \times 10^{-1}$	$4 \times 10^9$

Unification scale  $M$ .

The electromagnetic coupling can be also unified. At the one loop level one finds

$$\begin{aligned} \text{MODELS I, III} & \quad b_E = -27 \\ \text{MODEL II} & \quad b_E = -47/3. \end{aligned} \quad (4.7)$$

By using electric charge operators (3.4) one can calculate for the Model I, for example

$$\alpha_E^{-1}(100 \text{ GeV}) = 162.2 \quad (M = 1.25 \cdot 10^{16} \text{ GeV}, \alpha(M) = 7.39 \cdot 10^{-2}). \quad (4.8)$$

This obviously can be bettered by selecting some suitable  $q_{HC}$  and  $q_C$  values. It is easier to get closer to the experimental value

$$\alpha_E^{-1}(100 \text{ GeV}) \simeq 137 \quad (4.9)$$

in models II and III, where one finds

$$\frac{1}{\alpha_E(q)} = \frac{1}{15 \alpha_3(q)} + \frac{8}{5 \alpha_2(q)} \quad (\text{Model II})$$

and

$$\frac{1}{\alpha_B(q)} = \frac{1}{675\alpha_3(q)} + \frac{73}{400\alpha_2(q)} + \frac{33}{16\alpha_1(q)} \quad (\text{Model III}). \quad (4.10)$$

Here the indices  $k$  in  $\alpha_k$  correspond to the indices of  $Q$  in (3.4). Also

$$\alpha_3(M) = \alpha(M)$$

and

$$\alpha_2(M) \neq \alpha_1 \neq \alpha(M). \quad (4.11)$$

As already discussed in Ref. 6 one can easily find  $\alpha_1(M)$  and  $\alpha_2(M)$  which would give the desired value (4.9).

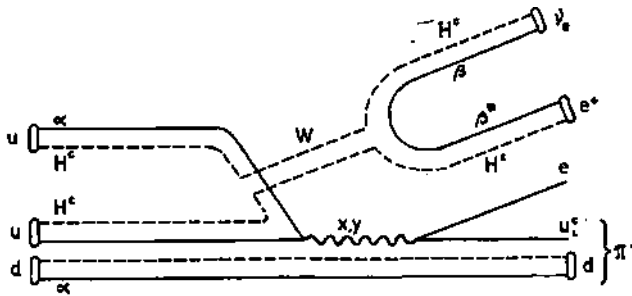


Fig. 2. Proton decay  $p \rightarrow \pi^- + e^+ + e^+ + \nu_e$ . The full lines symbolize a spinor and the dashed lines a scalar component of a superfield. The wavy line (X, Y) is a gauge boson.

In this, one generation approach, neither of the discussed models can have dangerous effective proton-decay Lagrangian terms of dimension 4 or 5, which were discussed in Introduction, formula (1.8). The only terms which can lead to the proton decay have standard dimension 6 with mediating boson being either a gauge boson or one of the Higgses. The effective Lagrangian for the Model I would have the form similar to one which was discussed in Ref. 18, in somewhat different physical content. As shown in Fig. 2 this leads to the decay

$$p \rightarrow \pi^- + e^+ + e^+ + \nu_e \quad (4.12)$$

which has three leptons in the final state. One encounters the same 3 lepton signature in Model II where the effective Lagrangian leads to the decay

$$p \rightarrow \pi^0 + e^+ + \nu_e + \bar{\nu}_e. \quad (4.13)$$

Both (4.12) and (4.13) are the dominant decay modes. The amplitude has an order of magnitude

$$f^2 g m_p^3 \Lambda_{HC}^{-1} M_X^{-2} M_W^{-2}. \quad (4.14)$$

Here  $f$  is the coupling constant of a gauge boson  $X$  with mass  $M_x$  (or of a suitable Higgs), while  $(g M_W^{-2})$  is of the order of the Fermi coupling constant. Obviously the proton decay must be slow. As it results in four particle final state, the phase space is also small.

## 5. Conclusion

As it has been already discussed<sup>3,4,6)</sup> SUSY appears as a natural symmetry for the strong coupling models which contain both spinor and scalar preons. The unification of all interactions in a such models requires some large covering group. An aesthetical requirement is that all model's particles are contained in the complex spinor representation. This is the most economical and elegant way to construct an anomaly free model. Additional requirement is the possibility to construct group invariant and renormalizable Yukawa couplings, which would give masses to quarks and leptons. They must not lead to rapid proton decay. Finally, one would like to be able to carry through the unification with all three known generations included.

Unfortunately it turns out that one can not fulfill all these requirements in a reasonable and convincing way. In the simplest model<sup>4,6)</sup> mass giving Yukawa terms can be constructed only by breaking SUSY. They lead to rapid proton decay. If SUSY is kept, only electron and d-quark can get masses and proton does not decay. However, one must think of some other mechanism which would give appropriate masses to all physical particles.

Models based on larger symmetry group, such as SO(10), SO(14) and SO(18) do not lead to acceptable mass terms. Even by going to subgroups one fails to produce all necessary mass terms and one is forced to include exotics. One can also construct some invariant couplings by using anomaly free combinations which do not constitute a complex spinor representation of a larger covering group. But one has to pay a heavy price by introducing numerous new redundant, flavours. Model loses elegance and simplicity and becomes an involved and unconvincing construction, a patchwork of ad hoc added pieces.

The  $E_6$  based models seem to be particularly attractive. The  $E_6$  SUSY models is vaguely suggested by the superstring theories. As this exceptional group contains SU(5) as subgroup, one can hope that such models will contain a minimum of the redundant exotics. Unfortunately, it turns out that such models can not be unified if they contain more than one generation. One is left with a difficult task to interpret generations as excited states of one basic preon structure. Alternatively one can resort to heavy, and thus quite unconvincing, SU(5) symmetry breaking.

This paper does not contain any clear-cut «no go» theorem. Certain things are mathematically forbidden under certain assumptions. Those assumptions can always be relaxed and all symmetries can be always arbitrarily broken. Lost somewhere along such way, however, are the credibility and the simplicity. Something, some new physical ingredient or new physical idea, seems to be missing. This is strongly indicated by the fact that there is no natural way to build models. All aesthetically pleasant symmetries usually have to be either abandoned or broken and the result looks as a wretched invalid who shuffles along on crutches.

APPENDIX

In the decomposition

$$D_{2k+1} \supset A_{2k} \otimes U(1)_{Q_1}; \quad N = 2k + 1, \quad 4 \geq k \geq 2 \quad (A1)$$

the  $2^{2k}$  dimensional spinors decompose as

$$\Delta_+ = 2^{2k} = \sum_{n=0}^{2k} [2n]_{4n-(2k+1)} = [0]_{-(2k+1)} + [2]_{3-2k} + \dots \quad (A2)$$

$$\Delta_- = (2^{2k})^* = \sum_{n=0}^k [2n+1]_{2k+1-4n} = [1]_{2k+1} + [3]_{2k-3} + \dots \quad (A3)$$

Here  $[2n]$  is an irreducible representation of  $A_{2k}$ , which is an antisymmetric tensor with  $2n$  upper (lower) indices. The general rule about dimensions is

$$2^{N-1} (\text{spinor}) = \sum_{n=0}^k \binom{N}{2n} (\text{tensors}). \quad (A3')$$

The subscripts in (A2) give the  $Q_1$  values.

The actual decomposition can be obtained by using projection matrices, Table 2. The procedure is straightforward, although sometimes rather tedious. We refer the reader to Ref. 7 for more details.

In the further decomposition

$$A_{2k} \supset (A_{2k-3})_{HC} \otimes (A_2)_C \quad (A4)$$

one uses the following rule which refers to the Young tableaux

$$[i] = \bigoplus_{j+l=k} [j, l] \quad (A5)$$

Here  $[i]$  etc. gives the number of boxes in a column. The decomposition of  $\Delta_+ = 16$  in  $SO(10)$  has already been given in Ref. 6. Here we list the decomposition of  $\Delta_+ = 64$  in  $SO(14)$  and of  $\Delta_+ = 256$  in  $SO(18)$ .

One has

$$\begin{aligned} D_7 &\supset A_6 \otimes U(1)_{Q_1} \supset [(A_3)_{HC} \otimes (A_2)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \\ (SO(14) &\supset SU(7) \otimes U(1)_{Q_1} \supset SU(4)_{HC} \otimes SU(3)_C \otimes U(1)_{Q_2} \otimes U(1)_{Q_1}) \\ \Delta_+ = 64 &= (1 + 7^* + 2\bar{1} + 35^*)_{SU(7)} = [(1, 1)_{0,-7}] + [(1, 3^*)_{-4,5} + \\ &+ (4^*, 1)_{3,5}] + [(6, 1)_{-6,-3} + (4, 3)_{1,-3} + (1, 3^*)_{8,-3}] + \\ &+ [(1, 1)_{-12,1} + (4^*, 3)_{-5,1} + (6, 3^*)_{2,1} + (4, 1)_{9,1}]. \end{aligned} \quad (A6)$$

The labeling is based on the following scheme

$$[N(SU(4)), L(SU(3))]_{Q_2, Q_1}$$

Using the same method (A.5) one finds

$$\begin{aligned} D_9 &\supset A_8 \otimes U(1)_{Q_1} \supset [(A_5)_{HC} \otimes (A_2)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \\ (SO(18) &\supset SU(9) \otimes U(1)_{Q_1} \supset [SU(6)_{HC} \otimes SU(3)_C \otimes U(1)_{Q_2}] \otimes U(1)_{Q_1} \\ \Delta_+ &= 256 = (1 + 9^* + 36 + 84^* + 126)_{SU(9)} = [(1, 1)_{0,9}] + \\ &+ [(6^*, 1)_{3,7} + (1, 3^*)_{-6,7}] + [(6, 3)_{3,-5} + (15, 1)_{-6,-5} + (1, 3^*)_{12,-5}] \\ &[(1, 1)_{-18,3} + (6^*, 3)_{-9,3} + (15^*, 3^*)_{0,3} + (20, 1)_{9,9}] + [(15^*, 1)_{-12,-1} + \\ &+ (20, 3)_{-3,-1} + (15, 3^*)_{6,-1} + (6, 1)_{15,1}]. \end{aligned} \tag{A.7}$$

The notation corresponds to

$$[N(SU(6)), L(SU(3))]_{Q_2, Q_1}$$

The electric charge is determined by

$$Q_{EM} = a_1(n) Q_1 + a_2(n) Q_2. \tag{A.8}$$

The index  $n$ , in  $a_1(n)$  for example, is to remind us that for each embedding i. e. for each K-group one needs different  $a(n)$ 's.

For the convenience of the reader we list also some terms appearing in the 16 of SO(10) which were used in Refs. 4 and 6:

$$\begin{aligned} \Delta_+ &= (10 + 5^* + 1)_{SU(5)} = [(1, 1)_{6,-1} + (2, 3)_{1,-1} + (1, 3^*)_{-4,-1}] + \\ &+ [(2, 1)_{-3,3} + (1, 3^*)_{2,3}]. \end{aligned} \tag{A.9a}$$

Labeling here is

$$[N(SU(2))_{HC}, L(SU(3)_C)]_{Q_2, Q_1}$$

The charges are given by

$$Q_{EM} = a_1 Q_1 + a_2 Q_2. \tag{A.9b}$$

Normal embedding means

$$a_1 = 0 \qquad a_2 = 1/6 \tag{A.9c}$$

while the flipped one is

$$a_1 = -1/3 \qquad a_2 = -1/30. \tag{A.9d}$$

One can easily identify the physically needed preons in (A.9). It is not so with the decomposition (A.6) and (A.7). In the SU(7) case (A.6), for example, the normal embedding contains states whose charges are determined by

$$Q_{EM} = (1/6) Q_2.$$

Thus the possible candidates for the  $\alpha$  or  $\beta$  type of preons<sup>4,6)</sup> are

$$(4^*, 1)_{3,5}; (4^*, 3)_{-5,1}; (4, 1)_{3,1}; (4, 3)_{1,-3}. \quad (\text{A.10})$$

The left handed charge conjugate fields such as  $u^c, d^c, e^c, \nu^c$ <sup>4,6)</sup> might be among SU(4)<sub>HC</sub> singlets

$$(1, 3^*)_{8,-3}; (1, 3^*)_{-4,5}; (1, 1)_{0,-7}; (1, 1)_{-12,1}; \quad (\text{A.11})$$

Among all candidates listed in (A.10) and (A.11) one can identify only

$$\begin{aligned} (1, 3^*)_{-4,5} &\cong u^c(1, 3, -2/3) & (1, 1)_{0,-7} &\cong \nu(1, 1, 0) \\ (4, 3)_{1,-3} &\cong \alpha(4, 3, 1/6) & (4^*, 1)_{3,5} &\cong \beta^*(4, 1, 1/2). \end{aligned} \quad (\text{A.12})$$

One can easily build the composite  $u_L$  quark

$$(\alpha\beta^*) \cong (1, 3, 2/3) = u_L \quad (\text{A.13})$$

but one would need an another state

$$\beta(4^*, 1, -1/2) \quad (\text{A.14})$$

to build the  $d_L$  composite. (Note: only within SU(2)<sub>HC</sub> the representations 2 and 2\* are equivalent). Thus one can build the mass term for u-quark only:

$$[(4, 3)(4^*, 1)](1, 3^*). \quad (\text{A.15})$$

There exist an another possibility. One can introduce somewhat different preon states, say  $\alpha'$  and  $\beta'$  which are

$$\begin{aligned} (4^*, 3)_{-5,1} &\cong \alpha'(4^*, 3, -5/6) \\ (4, 1)_{9,1} &\cong \beta'(4, 1, 9/6) \\ (\alpha'\beta') &\cong (1, 3, 2/3) = u_L. \end{aligned} \quad (\text{A.16})$$

Even with this new preons only the u-quark mass term

$$[(4^*, 3)(4, 1)](1, 3^*) \quad (\text{A.17})$$

can be constructed. Moreover, one has to decide which are the physical preons, (A.12) or (A.16) ones. Flipping cannot change this unsatisfactory situation.

In the double-flipped embedding, which is discussed in Section 3, one has

$$Q_{EM} = a_1 Q_1 + a_2 Q_2 + a_3 Q_3 \quad (\text{A.18})$$

Here  $Q_2$  and  $Q_3$  are analogous to the  $Q_1$  and  $Q_2$  in (A.9b). In the case of normal embedding the charges are given by (3.4).

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# OGRANIČENJA NA KLASU UNIFICIRAJUĆIH PREONSKIH SUSY MODELA

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Studirana su grupno-teorijska svojstva šire klase SUSY preonskih modela (uključujući i prije objavljeni jednostavni model). Analizirani su Yukawa maseni članovi i protonski raspad. Posebna je pažnja posvećena modelima temeljenim na grupnom lancu  $E_6 \supset SU(2)_{HC} \otimes SU(3)_C \otimes U(1)^3$ . Zbog unifikacije, ovi modeli zahtijevaju jako narušenje simetrije ili pretpostavku da su drugo i treće pokoljenje pobuđenja osnovne strukture. Raspad protona karakteriziran je sa četiri čestice u konačnom stanju,  $p \rightarrow \pi^- e^+ e^+ \nu_e$ . Općenito, vrlo je teško konstruirati supersimetrični preonski model koji zadovoljava ograničenja dana u tekstu, a koja su inspirirana fizikalnim razmatranjima.