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DEUTERON VERTEX FUNCTIONS INCLUDING MESON EXCHANGE CORRECTIONS

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New fits to the deuteron S and D state vertex functions are presented using a standard relativistic pole expansion. Deuteron static properties and the world collection of the *ed* scattering data were used to constrain the fits. Meson exchange effects for *ed* scattering are explicitly included. The data admit two solutions which should be distinguishable by measuring the tensor polarization in *ed* scattering. This conclusion is not changed by the meson exchange contributions.

1. *Introduction*

In Ref. 1 the relativistic deuteron vertex functions for the pole expansion of Ref. 2 were fitted to the then available *ed* scattering data respecting the constraints from the static deuteron properties which are known with high precision. This procedure has been repeated recently in Ref. 3 to study the influence of new data on the vertex functions. We are expanding this study in the present paper. A reliable parametrization of the deuteron vertex functions is required for many reactions involving the deuteron (few nucleon systems, pion production and absorption, the EMC effect, etc.). The pole expansion has the merit of leading to analytic expressions in many applications. In Refs. 1, 3 the S and D state vertex functions were fitted independently and it was shown that three poles in the expansion are sufficient to describe the available data with high precision. Three poles are in fact the minimum number for describing the D state vertex function²⁾.

Since the time of Ref. 1 new high precision data from electron scattering have become available for the electron deuteron cross section. In Ref. 3 the deuteron wave functions have been refitted in the light of these new data. The new solutions lead to differences in particular for the tensor polarization near the predicted zero of the charge monopole form factor. The tensor polarization in this range of momentum transfers will soon become available from an experiment at Bates.

In Ref. 3 meson exchange effects have been treated only effectively. In this paper we introduce them explicitely into the deuteron form factors. It turns out that all the major conclusions of Ref. 3 remain unchanged. The paper is self-contained which requires repeating the formal part and some of the discussion in Ref. 3. For the benefit of the potential used the present paper gives the closed expressions for the deuteron body form factors for a multipole Yukawa expansion in the Appendix. They are not readily available in the literature.

Section 2 gives the formalism. The details of the fitting procedure are given in Section 3, while the resulting vertex functions and electron deuteron observables are discussed in Section 4. Details are collected in the Appendix.

2. *Formalism*

The basic formalism used in this article is from Ref. 2. Constraints are imposed as in Refs. 1, 3. Contrary to Refs. 1, 3 meson exchange effects are taken into account explicitly by including them into the deuteron form factors with the values calculated in Ref. 4. In what follows we shall use the notation from Ref. 3.

The *dpn* vertex function with one nucleon off-shell[1,2], see Fig. 1, is described in terms of two invariant functions $G_a(t)$ and $G_b(t)$ as follovs:

$$D_\mu(t) = \gamma_\mu G_a(t) - \frac{1}{2}(p_1 - p_2)_\mu G_b(t) \tag{2.1}$$

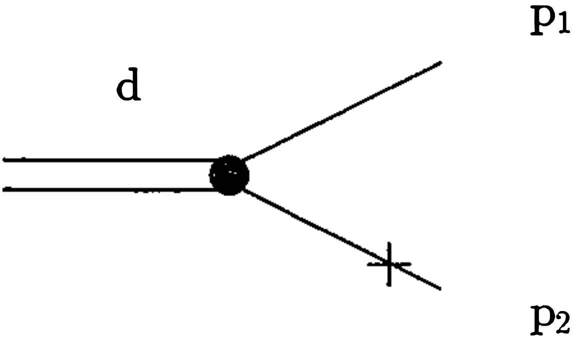

Fig. 1. The *dpn* vertex.
The letters denote four-vectors: the deuteron *d* and nucleon p_2 are on the mass shell: $d^2 = m_d^2$, $p^2 = m^2$, whereas p_1 is off shell: $t = p_1^2$.

with $p_2^2 = m^2$ and $p_1^2 = t$, where m is the nucleon mass* and t is the Mandelstam variable. The G_a and G_b functions essentially describe the off-shell or momentum dependence of the direct and derivative coupling, respectively. It is more convenient to use the linear combinations F_S and F_D for the analysis in the nonrelativistic limit:

$$F_S(t) = G_a(t) + (m^2 - t - 2a^2)(6m)^{-1} F_D(t)$$

$$F_D(t) = G_b(t) - (2m)^{-1} G_a(t)$$

$$a = \text{deuteron wave number} \tag{2.2}$$

which connect directly to the nonrelativistic S and D state wave functions. An invariant pole expansion is introduced for the functions F_S and F_D:

$$F_S(t) = F_S^0 \sum_i c_S^i \frac{t_S^i - m^2}{t_S^i - t}$$

$$F_D(t) = F_D^0 \sum_i c_D^i \frac{t_D^i - m^2}{t_D^i - t}. \tag{2.3}$$

The number of poles, the dimensionless residues $c_{S,D}^i$ and the pole positions $t_{S,D}^i$ will be discussed in the fitting procedure.

The normalization of S and D state functions in Eq. (2.3) is given by:

$$F_S^0 = (8\pi/m)^{1/2} N$$

$$F_D^0 = (8\pi/m)^{1/2}\, 3m\varrho N (\sqrt{2}a^2)^{-1}$$

$$N^2 = 2a(1 + \varrho^2)^{-1}(1 - ar_{eff})^{-1} \tag{2.4}$$

where $a^2 = mB$ (to order B), B is the deuteron binding energy, ϱ is the asymptotic D/S normalization and r_{eff} is the triplet np effective range. In the nonrelativistic limit the Mandelstam variable t can be reduced to:

$$t = m^2 - 2(p^2 + a^2) \tag{2.5}$$

where p is the magnitude of the relative three momentum of the nucleons in the deuteron rest frame. In terms of the momenta β^i the pole positions t^i are given by:

$$t_{S,D}^i = m^2 + 2[(\beta_{S,D}^i)^2 - a^2]. \tag{2.6}$$

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* For calculational purposes we shall use twice the reduced mass in the proton-neutron system.

The nonrelativistic S and D state wave functions in terms of the parameters of Eq. (2.3) are:

$$u(r) = N\left[e^{-\alpha r} - \sum_i c_S^i\, e^{-\beta_S^i r}\right]$$

$$w(r) = \varrho N\left[\alpha r h_2(i\alpha r) - \sum_i (\beta_D^i/\alpha)^2\, c_D^i\, \beta_D^i r h_2(i\beta_D^i r)\right] \tag{2.7}$$

with

$$x h_2(ix) = (1 + 3/x + 3/x^2)\, e^{-x}.$$

The electron deuteron elastic cross section in the laboratory system[5] is given by:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega}\left[A(q^2) + B(q^2)\tan^2(\Theta/2)\right] \tag{2.8}$$

where $d\sigma_0/d\Omega$ is the Mott cross section, Θ is the laboratory scattering angle, $\mathbf{q} = \mathbf{e} - \mathbf{e}'$ is the electron momentum transfer and $q = |\mathbf{q}| = 2p$ in the nonrelativistic approximation. In terms of the charge monopole (F_{Ch}), quadrupole (F_Q) and magnetic (F_{Mag}) form factors of the deuteron Eq. (2.8) reads:

$$A(q^2) = F_{Ch}^2(q^2) + (8/9)\,\eta^2 F_Q^2(q^2) + (2/3)\,\eta\,(1+\eta)\, F_{Mag}^2(q^2)$$

$$B(q^2) = (4/3)\,\eta\,(1+\eta)^2\, F_{Mag}^2(q^2) \tag{2.9}$$

with

$$\eta = q^2 (4m_d^2)^{-1}.$$

The deuteron form factors are given as a sum of the product of the nucleon form factor with the deuteron body form factors[1,3,6] and the meson exchange effect contributions taken from Ref. 4:

$$F_{Ch}(q^2) = 2F_{Ch}^S(q^2)\, C_E(q^2) + F_{Ch}^{mex}(q^2)$$

$$F_Q(q^2) = 2F_{Ch}^S(q^2)\, C_Q(q^2) + F_Q^{mex}(q^2) \tag{2.10}$$

$$F_{Mag}(q^2) = \frac{m_d}{m}\left[2F_{Mag}^S(q^2)\, C_S(q^2) + F_{Ch}^S(q^2)\, C_L(q^2)\right] + F_{Mag}^{mex}(q^2).$$

The static limit of Eq. (2.10) is:

$$F_{Ch}(0) = 1, \qquad F_Q(0) = m_d^2 Q, \qquad F_{Mag}(0) = 2m_d \mu_d \tag{2.11}$$

where m_d, μ_d and Q are the deuteron mass, the dipole and the quadrupole moment, respectively.

The definition of the integrals needed for the electric monopole and quadrupole, and for the magnetic scalar and longitudinal body form factor of the deuteron, $C_{E,Q,S,L}$ as well as their explicit form for the invariant pole expansion Eq. (2.3) are given in the Appendix.

In the former analysis[1)] the parametrization of nucleon form factors has been taken from Ref. 7. In Ref. 3 a new representation of nucleon form factor from Ref. 8 has been used, which differs significantly from the old neutron electric and magnetic form factors. The more recent fit has been triggered by new data for neutron form factors mainly for momentum transfers $q^2 > 1\ \mathrm{GeV}^2$ but it differs at all q^2. While the parametrization of the nucleon form factors is not crucial it matters for questions of detail. The present minimization is based on the nucleon form factors of Ref. 8.

In our normalization we have:

$$F_{Ch}^{S}(q^2) = F_1^S(q^2) - \frac{q^2}{4m^2} F_2^S(q^2)$$

$$F_{Mag}^{S}(q^2) = F_1^S(q^2) + F_2^S(q^2) \tag{2.12}$$

where following Ref. 8

$$F_1^S(q^2) = \left[\frac{m_\omega^2}{m_\omega^2 + q^2} \frac{g_\omega}{f_\omega} + \left(1 - \frac{g_\omega}{f_\omega}\right)\right] \frac{F_1(q^2)}{2}$$

$$F_2^S(q^2) = \left[\frac{m_\omega^2}{m_\omega^2 + q^2} \frac{\varkappa_\omega\, g_\omega}{f_\omega} + \left(\varkappa_S - \frac{\varkappa_\omega\, g_\omega}{f_\omega}\right)\right] \frac{F_2(q^2)}{2}. \tag{2.13}$$

The form factors $F_1(q^2)$ and $F_2(q^2)$ are represented by:

$$F_1(q^2) = \frac{\Lambda_1^2}{\Lambda_1^2 + \hat{q}^2} \frac{\Lambda_2^2}{\Lambda_2^2 + \hat{q}^2}$$

$$F_2(q^2) = \frac{\Lambda_1^2}{\Lambda_1^2 + \hat{q}^2} \left[\frac{\Lambda_2^2}{\Lambda_2^2 + \hat{q}^2}\right]^2 \tag{2.14}$$

with

$$\hat{q} = q^2 \log\left(\frac{\Lambda_2^2 + q^2}{\Lambda_{QCD}^2}\right) / \log\left(\frac{\Lambda_2^2}{\Lambda_{QCD}^2}\right).$$

The parameters from Ref. 8 are:

$$m_\omega = 0.784\ \mathrm{GeV}, \quad \frac{g_\omega}{f_\omega} = 0.411, \quad \varkappa_\omega = 0.163,$$

$$\varkappa_S = -0.12 \qquad \Lambda_{QCD} = 0.29\ \mathrm{GeV}.$$

Finally the *ed* tensor polarization t_{20} is given by:

$$t_{20} = -\sqrt{2}\,\frac{\frac{4}{9}\eta^2 F_Q^2 + \frac{4}{3}\eta F_{Ch} F_Q + \frac{1}{3}\eta(1+\eta) f(\Theta) F_{Mag}^2}{F_{Ch}^2 + \frac{8}{9}\eta^2 F_Q^2 + \frac{4}{3}\eta(1+\eta) f(\Theta) F_{Mag}^2}$$

$$f(\Theta) = \frac{1}{2} + (1+\eta)\tan^2\left(\frac{\Theta}{2}\right) \tag{2.15}$$

where Θ is the *ed* scattering angle in the center of mass system.

3. *Data selection and fitting procedure*

The data sets used for the determination of the pole parameters in Eq. (2.3) are as follows. The old *ed* scattering data $A(q^2)$ and $B(q^2)$, from Refs. 9, 10, the new data for momentum transfers $q^2 > 0.7$ GeV^2, from Refs. 11, 12, and the new measurement of $A(q^2)$ at lower momentum transfers[13)]. One should also keep in mind that the new nucleon form factor parametrization[8)] increases the size of *ed* form factors[14)], producing an additional source for differences in the minimization procedure. Note also that meson exchange corrections are explicitly included in the present paper.

The parameters of the multipole expansion Eq. (2.3) are constrained by:

- the measured static deuteron properties
- the conditions of regularity at the spatial origin for the S and D state deuteron wave functions
- the measured deuteron electromagnetic structure functions, Eq. (2.9).

The number of poles has been restricted to three for the S and D state vertex functions. For the D state the regularity condition near the origin, $w(r) \approx r^3$, requires in fact three poles[2)].

Data selection

The deuteron static properties have not changed recently, so the values from Ref. 1 are reproduced in Table 1 for the sake of completeness.

The new $B(q^2)$ data at $q^2 > 0.7$ GeV^2 from Ref. 12 agree well with the old set[10)] showing a smooth transition to the higher momentum transfers. We have, therefore, formed a combined data set for $B(q^2)$ by adding these data. Creating a self-consistent data set for $A(q^2)$ to serve as input to the minimization procedure is, however, not straightforward. As it has already been reported[13,20)] the new Saclay data[13)] show typically a 5% inconsistency when compared to the world colection of data[9,11)] at momentum transfers $q^2 < 0.7$ GeV^2. The origin of the

TABLE 1.

$r_{eff} = 1.764 \pm 0.006$ fm.	Ref.15
$Q = 0.2860 \pm 0.0015$ fm^2.	Ref.16
$B = 2.224575 \pm 0.000009$ MeV.	Ref.17
[$\alpha = 45.7024$ MeV,	see Eq. (2.4)]
$\varrho = 0.0270 \pm 0.0015$.	Averaged Refs.18
$r_d = 1.9660 \pm 0.0068$ fm.	Ref.19

Static properties of the deuteron. The symbols are explained in the text.

discrepancy has not yet been identified. As the mentioned 5% reduction significantly influences the extraction of the neutron electric form factor[13] from the A (q^2) data (the slope at $q^2 \approx 0$ is noticeably different) we are performing the analysis for the world collection[9,11] and the Saclay data[13] separately.

As in Ref. 3 the following two input data sets will be used:

Set 1

- deuteron static properties
- world collection of $A(q^2)$ data *without* Saclay data
- world collection of $B(q^2)$ data.

Set 2

- deuteron static properties
- the Saclay data for $A(q^2)$ at $q^2 < 0.7$ GeV^2 and world collection for $A(q^2)$ at $q^2 > 0.7$ GeV^2
- world collection of $B(q^2)$ data.

Including the additional data for $q^2 > 0.7$ GeV^2 in Set 2 is essential to ensure realistic higher momentum transfer behaviour for both $A(q^2)$ and $B(q^2)$. We do not see any first order discontinuity between the two sets at the matching point.

Fitting procedure

The fitting procedure is from Ref. 3. However, before searching for the deuteron parameters the meson exchange corrections from Ref. 4 have been applied to the form factors Eq. (2.10).

i) The deuteron static properties are treated as parameters in the fitting procedure itself by including them as a contribution to the total χ^2 with the very high statistical weight which corresponds to their low experimental error as in Ref. 3.

ii) Following Ref. 3 we do not minimize the total χ^2 but the χ^2 per data point for three subsets of data:

$$\chi^2 = \sum_i^3 \frac{\chi_i^2}{n_i}$$

$$n_i = \text{number of points in set } i$$

where $i = 1$ labels the data for $A(q^2)$, $i = 2$ the data for $B(q^2)$ and $i = 3$ the static properties of the deuteron, respectively. In this way we avoid the dominance of the data set with the largest number of points, similar to the procedure in Ref. 7.

iii) As the variation range of $A(q^2)$ and $B(q^2)$ in the fitting domain is as big as seven orders of magnitude, we minimize the logarithm of the function instead of the function itself. In this way small and large values of the function enter in a symmetrical way:

$$\chi_i^2 = \sum_{i=1}^{N_{points}} \left(\frac{\log(A^{exp}) - \log(A^{th})}{w_{log}} \right)^2$$

$$w_{log} = \frac{A^{err}}{A^{exp}} \log e. \tag{3.1}$$

This speeds up the search for the solution with minimum χ^2.

Finally we minimize an eight parameter space using the CERN minimization routine MINUIT, Release 89.05b on a VAX 8650.

4. *Results and conclusions*

Choosing three poles for the S and D state vertex functions Eq. (2.3) we apparently have 12 parameters. The S state regularity fixes one of the S state residues, while the D state regularity condition can be used to express all D state residues by means of the pole positions, see Refs. 1, 2. Therefore, there are only eight free parameters.

The parameters resulting from the minimization are given in Table 2. The agreement of the fit with the *ed* scattering observables $A(q^2)$ and $B(q^2)$ is illustrated in Figs. 2 and 3, respectively. The existing t_{20} data[21] have not been fitted, but they agree with the predictions, see Fig. 4. The full lines in Figs. 2—4 are the result of the fit for Set 1 as input (Solution 1), while the dashed lines correspond to the results of Set 2 (Solution 2). The contributions of charge monopole, quadrupole and magnetic form factors to $A(q^2)$ are shown separately by the thiner lines in Fig. 2. The vertex functions F_S and F_D are shown in Fig. 5.

TABLE 2.

	Parameters	S-state $i = 1$	S-state $i = 2$	S-state $i = 3$	D-state $i = 1$	D-state $i = 2$	D-state $i = 3$
Sol.	$\beta^i_{S,D}$	0.43578	0.68201	1.07930	0.24286	0.94827	0.94736
1	$c^i_{S,D}$	4.39042	−4.71728	1.32687	1.14015	35.44026	−35.58041
Sol.	$\beta^i_{S,D}$	0.47875	0.75750	2.37620	0.23969	0.99111	1.10520
2	$c^i_{S,D}$	5.40419	−5.65548	1.25129	1.12015	−1.27497	1.15482

Solution	N_A	N_B	χ^2_{st}	χ^2_A/N_A	χ^2_B/N_B	χ^2_{tot}	r^{fit}_{eff}	Q^{fit}	r^{fit}_d
Solution 1	57	62	0.94	3.62	3.08	7.64	1.7625	0.2860	1.9724
Solution 2	72	62	1.74	4.52	7.76	14.02	1.7632	0.2862	1.9749

Deuteron vertex function parameters. Values of the pole position $\beta^i_{S,D}$ in GeV and the corresponding dimensionless residues $c^i_{S,D}$ of Eqs. (2.3), (2.5) and (2.6). Upper lines in the column correspond to the pole positions, while the lower lines give the corresponding residues. The lower box shows the minimization parameters: number of points, χ^2 per data point the total χ^2 and the corresponding static properties.

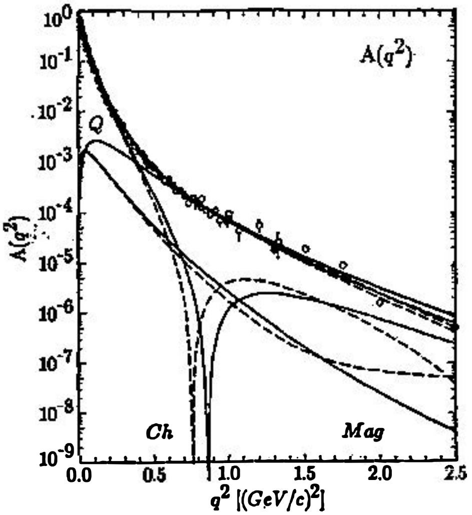

Fig. 2. The function $A(q^2)$ Eq. (2.9) for elastic *ed* scattering.

The open circles are old experimetal points [9], full circles are new experimental points [11] and full triangles are Saclay experimental points [13]. The errors are smaller than symbol size when no error bars are shown. The solid and dashed lines correspond to the solutions of Set 1 and Set 2, respectively. The thin curves marked Ch (charge monopole), Q (quadrupole) and Mag (magnetic) show the contributions due to the correspoding form factors $F_{Ch,Q,Mag}$ from Eq. (2.10).

At this point let us comment on the role of the meson exchange corrections. Their size is typically of order 10%. A comparison of Table 2 with the corresponding values in Ref. 3 shows that there are minor changes in the pole positions and residues. However, the resulting cross sections, the tensor polarization and the vertex functions in Figs. 2—5 are practically identical to the corresponding

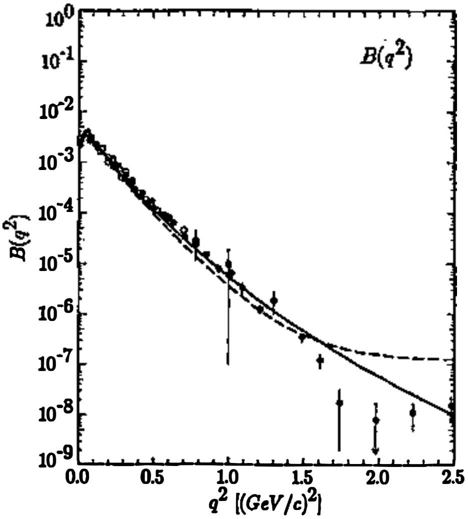

Fig. 3. The function $B(q^2)$ Eq. 2.9 for elastic *ed* scattering.

The open circles are old experimental points[10] and full circles are new experimental points[12]. The errors are smaller than symbol size when no error bars are shown. The solid dashed lines correspond to the solutions of Set 1 and Set 2, respectively.

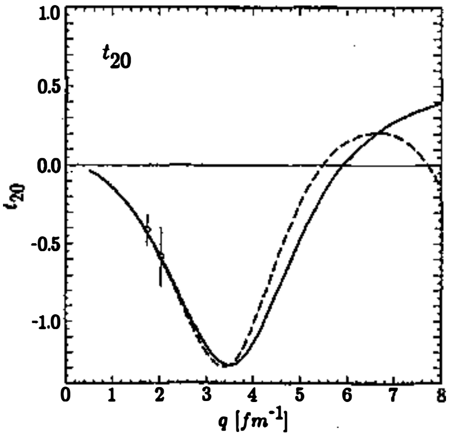

Fig. 4. The tensor polarization t_{20} Eq. (2.15) for elastic *ed* scattering at the electron scattering angle $\Theta = 70°$.

The open circles are experimental points[21]. The solid and dashed lines correspond to the solutions of Set 1 and Set 2, respectively.

figures in Ref. 3. The discussion of the two solutions in Table 2 is therefore very closely the same as in Ref. 3. In particular the new high precision data for $A(q^2)$ from Ref. 13 for $q^2 < 0.7$ GeV^2 seem not to match easily (in slope) with the remaining data. This feature has not changed by the explicit inclusion of meson exchange contributions in the present paper.

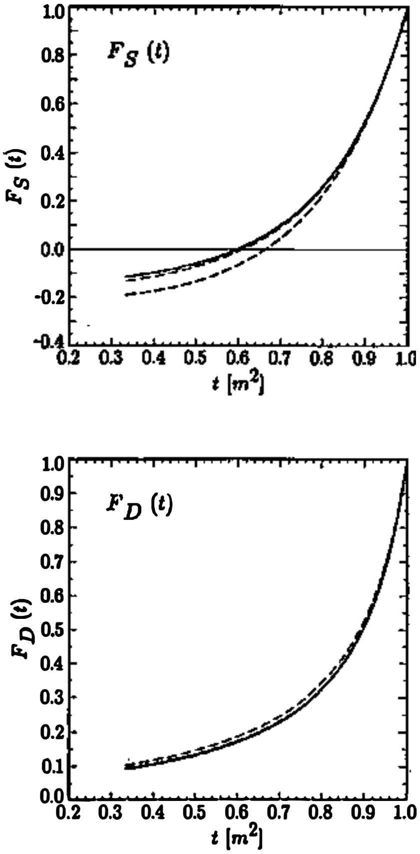

Fig. 5. The (normalized) vertex functions F_S and F_D as given in Eq. (2.3).
Solid lines: Solution 1; dashed lines: Solution 2. Thin thin dashed lines close to solution 1 are showing old results from Ref. 1. The abscissa is given in the units of nucleon mass squared.

The deuteron vertex functions F_S and F_D are shown in Fig. 5 where solid lines correspond to Solution 1 and dashed lines to Solution2. The thin dashed lines show old 1984 results[1)] which are very close to Solution 1. Again the comments of Ref. 3 remain valid after the inclusion of meson exchange corrections in the present fitting procedure. The only sizable difference between Solution 1 and 2 occurs for $F_S(t)$. It is detectable as a shift in the minimum of the charge monopole form factor of the deuteron and in the tensor polarization at higher momentum transfers. The available measurements[21)] of t_{20} cannot distinguish but the forthcoming data[22)] will be able to do so.

Acknowledgement

This note has been stimulated by discussions with Barbara Badelek. We thank S. Platchkov for providing the Saclay data in numerical form and Ingo Sick for a discussion.

Appendix

The integrals needed for the electric monopole and quadrupole, and for the magnetic scalar and longitudinal body form factors of the deuteron ($C_{E,Q,S,L}$) are:

$$C_E(q^2) = \int_0^\infty [u^2(r) + w^2(r)]\, j_0\left(\frac{qr}{2}\right) \mathrm{d}r$$

$$C_Q(q^2) = \frac{3\sqrt{2}}{2\eta} \int_0^\infty \left[u(r)\, w(r) - \frac{w^2(r)}{2\sqrt{2}}\right] j_2\left(\frac{qr}{2}\right) \mathrm{d}r$$

$$C_S(q^2) = \int_0^\infty [u^2(r) - \frac{1}{2} w^2(r)]\, j_0\left(\frac{qr}{2}\right) \mathrm{d}r$$

$$+ \frac{1}{\sqrt{2}} \int_0^\infty \left[u(r)\, w(r) + \frac{w^2(r)}{\sqrt{2}}\right] j_2\left(\frac{qr}{2}\right) \mathrm{d}r$$

$$C_L(q^2) = \frac{3}{2} \int_0^\infty w^2(r) \left[j_0\left(\frac{qr}{2}\right) + j_2\left(\frac{qr}{2}\right)\right] \mathrm{d}r \qquad \text{(A.1)}$$

with

$$\eta = \frac{q^2}{4m_D^2}, \quad q = |\mathbf{q}|. \qquad \text{(A.2)}$$

The functions j_0 and j_2 are Bessel functions, $\mathbf{q}$ is the electron momentum transfer and m_D is the deuteron mass. Using the explicit parametrization (2.7) for the wave functions the form factor integrals (A.1) are all elementary. Formal singularities of individual terms at $r = 0$ cancel if the regularity conditions for the wave functions are imposed.

For the S-state the minimal requirement for $r \to 0$ is

$$u(r) \sim r \quad \text{or} \quad \sum_i c_S^i = 1.$$

For the D-state regularity at $r = 0$ requires:

$$w(r) \sim r^3 \text{ or } \begin{cases} \sum_i c_D^i = 1 \\ \sum_i (\beta_D^i)^2 c_D^i = \alpha^2 \\ \sum_i (\beta_D^i)^4 c_D^i = \alpha^4. \end{cases}$$

Explicit expressions for the body form factor integrals are

$$C_E(q^2) = C_E^{(1)}(q^2) + C_E^{(2)}(q^2)$$

with

$$C_E^{(1)}(q^2) = \int_0^\infty u^2(r) j_0\left(\frac{qr}{2}\right) \mathrm{d}r = N^2 \Bigg[I_1\left(2\alpha, \frac{q}{2}\right) - 2 \sum_{\beta_S} c_{\beta_S} I_1\left(\alpha + \beta_S, \frac{q}{2}\right) +$$

$$+ \sum_{\beta_S, \beta_S'} c_{\beta_S} c_{\beta_S'} I_1\left(\beta_S + \beta_S', \frac{q}{2}\right)\Bigg]$$

$$C_E^{(2)}(q^2) = \int_0^\infty w^2(r) j_0\left(\frac{qr}{2}\right) \mathrm{d}r = \frac{\varrho^2 N^2}{\alpha^4} \sum_{\beta_D, \beta_D'} c_{\beta_D} c_{\beta_D'} \cdot \Bigg\{ \beta_D^2 \beta_D'^2 \Bigg[I_7\left(0, 2\alpha, \frac{q}{2}\right) -$$

$$- I_7\left(0, \alpha + \beta_D, \frac{q}{2}\right) - I_7\left(0, \alpha + \beta_D', \frac{q}{2}\right) + I_7\left(0, \beta_D + \beta_D', \frac{q}{2}\right)\Bigg] +$$

$$+ 6\beta_D^2 \Bigg[\alpha I_7\left(-1, 2\alpha, \frac{q}{2}\right) - \alpha I_7\left(-1, \alpha + \beta_D, \frac{q}{2}\right) - \beta_D' I_7\left(-1, \alpha + \beta_D', \frac{q}{2}\right) +$$

$$+ \beta_D' I_7\left(-1, \beta_D + \beta_D', \frac{q}{2}\right)\Bigg] + (9\alpha^2 + 6\beta_D^2) I_7\left(-2, 2\alpha, \frac{q}{2}\right) -$$

$$- (9\alpha\beta_D + 6\beta_D^2) I_7\left(-2, \alpha + \beta_D, \frac{q}{2}\right) - (9\alpha\beta_D' + 6\beta_D^2) I_7\left(-2, \alpha + \beta_D', \frac{q}{2}\right) +$$

$$+ (9\beta_D\beta_D' + 6\beta_D^2) I_7\left(-2, \beta_D + \beta_D', \frac{q}{2}\right) + 18 \Bigg[\alpha I_7\left(-3, 2\alpha, \frac{q}{2}\right) -$$

$$- \alpha I_7\left(-3, \alpha + \beta_D', \frac{q}{2}\right) - \beta_D I_7\left(-3, \alpha + \beta_D, \frac{q}{2}\right) +$$

$$+ \beta_D I_7\left(-3, \beta_D + \beta_D', \frac{q}{2}\right)\Bigg] + 9 \Bigg[I_7\left(-4, 2\alpha, \frac{q}{2}\right) - I_7\left(-4, \alpha + \beta_D', \frac{q}{2}\right) -$$

$$- I_7\left(-4, \alpha + \beta_D, \frac{q}{2}\right) + I_7\left(-4, \beta_D + \beta'_D, \frac{q}{2}\right)\Big]\Big\}. \tag{A.3}$$

$$C_Q(q^2) = \frac{3\sqrt{2}}{2\eta}\left[C_Q^{(1)}(q^2) - C_Q^{(2)}(q^2)\right]$$

with

$$C_Q^{(1)}(q^2) = \int_0^\infty u(r)\, w(r)\, j_2\left(\frac{qr}{2}\right) \mathrm{d}r = \frac{\varrho N^2}{\alpha^2} \sum_{\beta_S, \beta'_D} c_{\beta_S}\, c_{\beta'_D} \Big\{\beta'^2_D \Big[I_8\left(0, 2\alpha, \frac{q}{2}\right) -$$

$$- I_8\left(0, \alpha + \beta_S \frac{q}{2}\right) - I_8\left(0, \alpha + \beta'_D, \frac{q}{2}\right) + I_8\left(0, \beta_S + \beta'_D, \frac{q}{2}\right)\Big] +$$

$$+ 3\Big[\alpha I_8\left(-1, 2\alpha, \frac{q}{2}\right) - \alpha I_8\left(-1, \alpha + \beta_S, \frac{q}{2}\right) - \beta'_D I_8\left(-1, \alpha + \beta'_D, \frac{q}{2}\right) +$$

$$+ \beta'_D I_8\left(-1, \beta_S + \beta'_D, \frac{q}{2}\right)\Big] + 3\Big[I_8\left(-2, 2\alpha, \frac{q}{2}\right) - I_8\left(-2, \alpha + \beta_S, \frac{q}{2}\right) -$$

$$- I_8\left(-2, \alpha + \beta'_D, \frac{q}{2}\right) + I_8\left(-2, \beta_S + \beta'_D, \frac{q}{2}\right)\Big]\Big\}$$

$$C_Q^{(2)}(q^2) = \frac{1}{2\sqrt{2}} \int_0^\infty w^2(r)\, j_2\left(\frac{qr}{2}\right) \mathrm{d}r = \frac{\varrho^2 N^2}{2\sqrt{2}\,\alpha^4} \sum_{\beta_D, \beta'_D} c_{\beta_D}\, c_{\beta'_D} \Big\{\beta^2_D \beta'^2_D \Big[I_8\left(0, 2\alpha, \frac{q}{2}\right) -$$

$$- I_8\left(0, \alpha + \beta_D, \frac{q}{2}\right) - I_8\left(0, \alpha + \beta'_D, \frac{q}{2}\right) + I_8\left(0, \beta_D + \beta'_D, \frac{q}{2}\right)\Big] +$$

$$+ 6\beta^2_D \Big[\alpha I_8\left(-1, 2\alpha, \frac{q}{2}\right) - \alpha I_8\left(-1, \alpha + \beta_D, \frac{q}{2}\right) - \beta'_D I_8\left(-1, \alpha + \beta'_D, \frac{q}{2}\right) +$$

$$+ \beta'_D I_8\left(-1, \beta_D + \beta'_D, \frac{q}{2}\right)\Big] + (9\alpha^2 + 6\beta^2_D)\, I_8\left(-2, 2\alpha, \frac{q}{2}\right) -$$

$$- (9\alpha\beta_D + 6\beta^2_D)\, I_8\left(-2, \alpha + \beta_D, \frac{q}{2}\right) - (9\alpha\beta'_D + 6\beta^2_D)\, I_8\left(-2, \alpha + \beta'_D, \frac{q}{2}\right) +$$

$$+ (9\beta_D\beta'_D + 6\beta^2_D)\, I_8\left(-2, \beta_D + \beta'_D, \frac{q}{2}\right) + 18\Big[\alpha I_8\left(-3, 2\alpha, \frac{q}{2}\right) -$$

$$- \alpha I_8 \left(-3, \alpha + \beta'_D, \frac{q}{2}\right) - \beta_D I_8 \left(-3, \alpha + \beta_D, \frac{q}{2}\right) +$$

$$+ \beta_D I_8 \left(-3, \beta_D + \beta'_D, \frac{q}{2}\right)\Big] + 9 \Big[I_8 \left(-4, 2\alpha, \frac{q}{2}\right) - I_8 \left(-4, \alpha + \beta'_D, \frac{q}{2}\right) -$$

$$- I_8 \left(-4, \alpha + \beta_D, \frac{q}{2}\right) + I_8 \left(-4, \beta_D + \beta'_D, \frac{q}{2}\right)\Big]\Big\}. \qquad (A.4)$$

The remaining deuteron body form factors can be expressed as a linear combination of the former ones:

$$C_S(q^2) = C_E^{(1)}(q^2) - \frac{1}{2} C_E^{(2)}(q^2) + \frac{1}{\sqrt{2}} [C_Q^{(1)}(q^2) + 2C_Q^{(2)}(q^2)]$$

$$C_L(q^2) = \frac{3}{2} [C_E^{(2)}(q^2) + 2\sqrt{2}\, C_Q^{(2)}(q^2)]. \qquad (A.5)$$

The integrals needed are given below. Only three out of eight integrals listed are used in the preceeding formulae. The remaining ones are useful for simplifying occurring expressions for the form factors and for cross checks.

Finite integrals

$$I_1(a, b) = \int_0^\infty e^{-ar} j_0(br)\, dr = \frac{1}{b} \arctan \frac{b}{a}$$

$$I_2(a, b) = \int_0^\infty e^{-ar} j_2(br)\, dr = \frac{1}{2b^3} \left[(b^2 + 3a^2) \arctan \frac{b}{a} - 3ab\right]$$

$$I_3(a, b) = \int_0^\infty \frac{e^{-ar}}{r} j_2(br)\, dr = \frac{a^2 + b^2}{6b^2} \left[\frac{3a^2 + 2b^2}{a^2 + b^2} - \frac{3a}{b} \arctan \frac{b}{a}\right]$$

$$I_4(a, b) = \int_0^\infty \frac{e^{-ar}}{r^2} j_2(br)\, dr = \frac{a(a^2 + b^2)}{8b^2} \left[\frac{a^2 + b^2}{ab} \arctan \frac{b}{a} - \frac{3a^2 + 5b^2}{3(a^2 + b^2)}\right]. \qquad (A.6)$$

Infinite integrals

For a positive integer n:

$$I_5(-n, a, b) = \int_0^\infty r^{-n} e^{-ar} \sin br \, dr = \frac{(-)^{n-1}}{(n-1)!} [\text{Real}\,(a+ib)^{n-1} \arctan \frac{b}{a} +$$

$$+ \text{Imag}\,(a+ib)^{n-1} \ln \sqrt{a^2+b^2} - \Psi(n)\, \text{Imag}\,(a+ib)^{n-1} + \text{Divergent part}]$$

$$I_6(-n, a, b) = \int_0^\infty r^{-n} e^{-ar} \cos br \, dr = \frac{(-)^{n-1}}{(n-1)!} [\text{Imag}\,(a+ib)^{n-1} \arctan \frac{b}{a} -$$

$$- \text{Real}\,(a+ib)^{n-1} \ln \sqrt{a^2+b^2} + \Psi(n)\, \text{Real}\,(a+ib)^{n-1} + \text{Divergent part}]$$

$$I_7(-n, a, b) = \int_0^\infty r^{-n} e^{-ar} j_0(br) \, dr = \frac{1}{b} I_5(-n-1, a, b)$$

$$I_8(-n, a, b) = \int_0^\infty r^{-n} e^{-ar} j_2(br) \, dr = \frac{3}{b^3} I_5(-n-3, a, b) -$$

$$- \frac{1}{b} I_5(-n-1, a, b) - \frac{3}{b^2} I_6(-n-2, a, b)$$

where

$$\Psi(1) = -\gamma$$

$$\Psi(n) = -\gamma + \sum_{m=1}^{n-1} \frac{1}{m}$$

$\gamma = 0.5772156649$ is the Euler constant

and

i is imaginary unit. (A.7)

We have explicitly checked that the divergent contributions in the body form factors cancel due to the regularity conditions at the origin.

For the finite integrals the following connections among the integrals can be established:

$$I_7(0, a, b) = I_1(a, b)$$

$$I_8(0, a, b) = I_2(a, b)$$

$$I_8(-1, a, b) = I_3(a, b)$$

$$I_8(-2, a, b) = I_4(a, b). \tag{A.8}$$

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DEUTERONSKA VERTEKS FUNKCIJA UZ UKLJUČIVANJE EFEKATA MEZONSKE IZMJENE

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Izloženi su rezultati novih prilagodbi deuteronskih verteks funkcija za S i D stanje korištenjem standardnog relativističkog razvoja po polovima prve vrste. Statička svojstva deuterona te zbir svih svjetskih podataka o *ed* raspršenju su korišteni kao baza za proceduru prilagodbe. Efekti mezonske izmjene za *ed* raspršenje su direktno uključeni. Podaci dozvoljavaju dva rješenja koja se mogu razlikovati mjerenjem tenzorske polarizacije u *ed* raspršenju. Zaključci se ne mijenjaju uključivanjem efekata mezonske izmjene.