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LETTER TO THE EDITOR

K $\rightarrow \pi\pi\pi\gamma$ DECAY IN A CHIRAL LAGRANGIAN APPROACH*

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We analyse K$\rightarrow \pi\pi\pi\gamma$ decay in the framework of a chiral Lagrangian supplemented by the $1/N$ expansion. This process is dominated by the inner bremsstrahlung emission. The basic process K $\rightarrow \pi\pi\pi$ is well described within the chiral Lagrangian approach. It is necessary to introduce the higher-order interactions in order to obtain result close to the experimental one. In addition to the dominant part of the amplitude we determine direct emission part. For the calculation of decay width we make the cut in photon energy and obtain result in agreement with the experiment.

The $K^+ \rightarrow \pi^+\pi^+\pi^-\gamma$ decay has been considered well known for a long time[1]. We find it useful to reexamine this decay using a simple chiral Lagrangian approach. Namely, the truncated chiral Lagrangian approach to the low-energy processes became more applicable when the connection with large N limit was clarified[2,3,4]. In the large N limit only single trace couplings in the strong chiral Lagrangian are kept:

$$\mathscr{L}_s = \frac{f^2}{4}\{\mathrm{Tr}\,[\partial_\mu U \partial^\mu U^+ + rM\,(U + U^+) - \frac{r}{\Lambda_0^2}(M\partial^2 U + \mathrm{h.\,c.}) +$$

$$+ \text{non-derivative terms}] + \frac{1}{\Lambda_1^2}\,\mathrm{Tr}\,(\partial_\mu U^+ \partial^\mu U \partial_\nu U^+ \partial^\nu U) - \qquad (1)$$

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$$-\frac{1}{\Lambda_2^2}\,\mathrm{Tr}\,(\partial_\mu U^+ \partial_\nu U \partial^\mu U^+ \partial\, U)\}$$

where

$$U = \exp\left(\frac{\mathrm{i}}{f}\Pi\right), \quad \Pi = \sum_a \lambda_a \pi^a, \quad f = 0.093\ \mathrm{GeV}.$$

Introducing the weak and electromagnetic interactions by standard gauge procedure we obtain »bosonized« chiral quark current

$$J^\mu = \bar{q}_1 \gamma^\mu (1-\gamma_5) q_2 \equiv (\bar{q}_1 q_2)_{V-A} = \mathrm{i} f^2 (\partial^\mu U U^+)_{21} +$$

$$+ \mathrm{i} f^2 \left\{\frac{r}{2\Lambda_0^2}(\partial^\mu U M - M \partial^\mu U^+) - \frac{1}{\Lambda_1^2}(U\partial\, U^+ \partial_\nu U \partial^\mu U^+ - \right. \tag{2}$$

$$\left. - \partial^\mu U \partial^\nu U^+ \partial_\nu U U^+) + \frac{2}{\Lambda_2^2}(U \partial^\nu U^+ \partial_\mu U \partial_\nu U^+)\right\}_{21} .$$

The dimensionful parameters Λ_0, Λ_1 and Λ_2 are expected to be of the order of the chiral symmetry-breaking scale, i. e. 1 GeV.

The introduction of the SU(3) breaking term determines the physically observed f coupling constant as:

$$f_\pi = f\left(1 + \frac{m_\pi^2}{\Lambda_0^2}\right). \tag{3}$$

The weak decay constants are related by[2,3]:

$$\frac{f_k}{f_\pi} = 1 + \frac{m_K^2 - m_\pi^2}{\Lambda_0^2}. \tag{4}$$

In the large N limit the $S = 1$ weak Hamiltonian is simply given by

$$H_w = g\,\{(Q_2 - Q_1) + \frac{\omega}{\sqrt{2}}(Q_2 + 2Q_1)\} \tag{5}$$

where Q_2 is the four-quark operator

$$Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}. \tag{6}$$

The strong interactions induce the Q_1 operator:

$$Q_1 = (\bar{s}d)_{V-A}(\bar{u}u)_{V-A}. \tag{7}$$

The famous penguin operator becomes in the large N limit by the bosonization procedure proportional to $Q_2 - Q_1$. The hadronic matrix elements of the $Q_{1,2}$ weak operators have to be evaluated in the large N approximation, since higher order (in the $1/N$ expansion) corrections associated with quark-gluon and light meson loops are effectivelly included in the measured g and ω. Their theoretical values[3,4] are indeed close to the experimental ones ($g^{exp} = 8.8\ 10^{-6}$ GeV^{-2} and $\omega^{exp} = 1/22$).

The K → $\pi\pi\pi\gamma$ decays exhibit two parts of the amplitude: the internal bremsstrahlung and the direct emission terms.

Both parts of the amplitude can be obtained replacing the derivatives in our basic strong Lagrangian by the covariant ones:

$$D_\mu U = \partial_\mu U + \mathrm{i}eA_\mu [Q, U], \quad Q = \mathrm{diag}\left[\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right]. \tag{8}$$

The dominant process is the emission of low-frequency photons (internal bremsstrahlung). The corresponding amplitude depends on the calculation of the K → → $\pi\pi\pi$ decay[5]. Using the already mentioned technique we find:

$$A^{\mathrm{IB}}(\mathrm{K}^+ \to \pi^+\pi^+\pi^-\gamma) = eA_0\varepsilon^\mu M_\mu$$

$$A_0 \equiv A(\mathrm{K}^+ \to \pi^+\pi^+\pi^-) \tag{9}$$

$$M_\mu = \frac{P_\mu}{P\cdot k} - \frac{p_{1\mu}}{p_1\cdot k} - \frac{p_{2\mu}}{p_2\cdot k} + \frac{p_{3\mu}}{p_3\cdot k}$$

where in the center of Dalitz plot we calculate:

$$A_0 = \frac{2}{3} g m_k^\gamma \left\{1 + 6\frac{m_\pi}{\Lambda_0^2} + \frac{4}{3}(m_k^2 - 3m_\pi^2)\left(\frac{1}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right\}. \tag{10}$$

For the direct emission terms, which can be determined as electromagnetic transition of the following types:

$$A_1^{\mathrm{DV}} = \langle \pi^+ | \mathscr{L}_{em} | \pi^+\pi^+\pi^-\gamma\rangle \frac{-1}{m_K^2 - m_\pi^2} \langle \mathrm{K}^+ | \mathscr{L}_w | \pi^+\rangle \tag{11}$$

and

$$A_2^{\mathrm{DV}} = \langle \mathrm{K}^+ | \mathscr{L}_{em} | \mathrm{K}^+\pi^+\pi^-\gamma\rangle \frac{-1}{m_\pi^2 - m_K^2} \langle \mathrm{K}^+ | \mathscr{L}_w | \pi^+\rangle \tag{12}$$

where

$$\mathscr{L}_{em} = \frac{\mathrm{i}}{4} e \frac{1}{f^2} A_\mu \mathrm{Tr}\left\{\frac{1}{3}[Q, \Pi^3]\,\partial_\mu \Pi - \frac{1}{2}[Q, \Pi^2]\,\partial_\mu \Pi^2 + \frac{1}{3}[Q, \Pi]\,\partial_\mu \Pi^3\right\} \tag{13}$$

we find:

$$A_1^{DB}(K^+ \to \pi^+\pi^+\pi^-\gamma) = \frac{16}{3} eg \frac{m_K^2}{m_K^2 - m_\pi^V} (p_1 + p_2) \cdot \varepsilon \tag{14a}$$

$$A_2^{DB}(K^+ \to \pi^+\pi^+\pi^-\gamma) = \frac{16}{3} eg \frac{m_\pi^2}{m_\pi^2 - m_K^2} (p_1 + p_2) \cdot \varepsilon. \tag{14b}$$

The integration over the pions momenta is performed in the pions center of mass system, while for the photon the rest system of kaon is more suitable. After somewhat lengthy but straightforward calculation we determine the following expression for the $K^+ \to \pi^+\pi^+\pi^-\gamma$ decay width:

$$\Gamma = \frac{a}{(2\pi)^4 \cdot 4m_K} \int_{k_0}^{k_{max}} \left\{ A_0^2 \left(\frac{1}{k} - \frac{2}{m_K} \right) f_1(k) + \right.$$

$$\left. + A_0 A_0^{DB} \sqrt{\frac{1}{k} - \frac{2}{m_k}} f_2(k) - (A_0^{DV})^2 \frac{k}{2} f_3(k) \right\} dk \tag{15}$$

with

$$A_0^{DB} = \frac{16}{3} g,$$

$$f_1(k) = \int_{\omega_{min}}^{\omega_{max}} d\omega \left\{ 2q(\omega) \frac{Q^2 - 2m_\pi^2}{Q^2} \operatorname{arth} \sqrt{\frac{Q^2 - 4m_\pi^2}{Q^2}} + \right.$$

$$\left. + \sqrt{\frac{Q^2 - 4m_\pi^2}{Q^2}} \left[\omega \operatorname{arth} \frac{q(\omega)}{\omega} - 2q(\omega) \right] \right\} \tag{16}$$

$$f_2(k) = \int_{\omega_{min}}^{\omega_{max}} d\omega \left\{ 2Q^2 \operatorname{arth} \sqrt{\frac{Q^2 - 4m_\pi^2}{Q^2}} \operatorname{arth} \frac{q(\omega)}{E_k - \omega} - \right.$$

$$\left. - \sqrt{\frac{Q^2 - 4m_\pi^2}{Q^2}} \left[q(\omega) \frac{(m_K - k)(E_k - \omega)}{m_k - 2k} + (\omega E_k - m_\pi^2) \operatorname{arth} \frac{q(\omega)}{\omega} \right] \right\} \tag{17}$$

$$f_3(k) = \int d\omega \, q(\omega) \sqrt{Q^2 (Q^2 - 4m_\pi^2)}, \tag{18}$$

$$\omega_{min} = m_\pi, \qquad \omega_{max} = \frac{E_k^2 - 3m_\pi^2}{2E_k}, \qquad E_k = \sqrt{m_K(m_K - 2k)},$$

$$q(\omega) = \sqrt{\omega^2 - m_\pi^2}, \qquad Q^2 = E_k^2 - 2E_k\omega + m_\pi^2.$$

It is well known that internal bremsstrahlung gives dominant part of the decay width. For the numerical calculation we use experimentally determined region of the photon energy according to the experimental results[6].

For branching ratio we obtain the value

$$Br(K^+ \to \pi^+\pi^+\pi^-\gamma) = 0.48 \cdot 10^{-4} \tag{19}$$

that is somewhat below the experimental result[6].

Our calculation shows that the applicability of the strong Lagrangian (1) can be extended to additional processes. In order to obtain better agreement with experiment, it is necessary to include higher-order terms in the perturbative expansion.

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K → $\pi\pi\pi\gamma$ RASPAD U MODELU KIRALNIH LAGRANGIANA

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Analiziran je raspad K → $\pi\pi\pi\gamma$ u okviru modela sa kiralnim Lagrangianom i $1/N$ razvojem. U procesu je dominantno tzv. unutrašnje zakočno zračenje. Osnovni proces K → $\pi\pi\pi$ je dobro opisan u ovom modelu uz prisustvo članova višeg reda. Pored dominantnog člana dobijen je član koji odgovara direktnoj emisiji. Numerički proračuni širine raspada izvršeni su u rasponu energija određenih u eksperimentu i dobijeno je dobro slaganje.