

WILSON LOOP WITH LIGHT-LIKE SIDES IN THE FEYNMAN AND LIGHT-CONE GAUGES

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We show the equivalence of the Wilson loop with two light-like sides in the Feynman and light-cone gauges to order g^2 in perturbation theory. This loop shows doubly logarithmic ultraviolet divergence already at order g^2 (in the sense that in dimensional regularization there is a $(d - 4)^{-2}$ term).

1. Introduction

Non-covariant gauges found applications in a wide range of fields, such as QCD¹⁾, supersymmetric Yang-Mills²⁾ and string theories³⁾.

However, consistent Feynman rules (especially the presence or absence of ghosts) and the correct prescription for spurious axial-gauge singularities of the propagator still remain in dispute. One test to single out possible sets of Feynman rules and propagator prescriptions can be performed by verifying in SU(N) gauge theory that the Wilson loop gives the same result as in the Feynman gauge. This is required by gauge invariance.

A Wilson loop is a path- and time-ordered contour integral:

$$W = N^{-1} \langle 0 | \text{Tr} PT \exp \left(ig \oint A_\mu dx^\mu \right) | 0 \rangle. \quad (1)$$

We do not choose the usual rectangular Wilson loop, with one pair of sides in the t -direction, but the loop with sides in the directions of two light-like vectors, n and n^* , where

$$\begin{aligned} n^2 = n^{*2} &= 0, \\ n &= (1, 0, 0, -1), \\ n^* &= (1, 0, 0, 1). \end{aligned} \tag{2}$$

The sides in the direction of n^* are of length L , and the pair of sides in the direction of n is of length T .

2. Light-cone gauge

Fig. 1 shows the only diagram contributing to order g^2

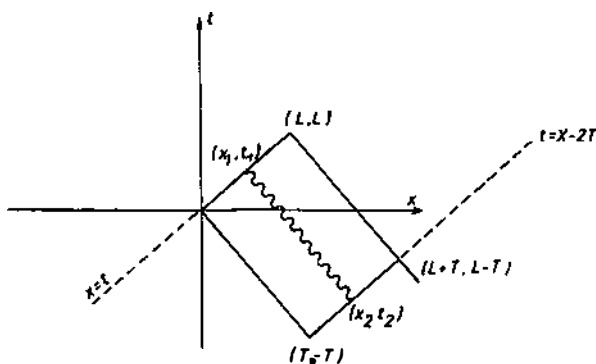


Fig. 1. Wilson loop with light-like sides in the light-cone gauge.

$$\begin{aligned} W &= -ig^2 \text{Tr} (t_a t_a) \int_0^L dt_1 \int_{L-T}^{-T} dt_2 \int \frac{d^n k}{(2\pi)^n} e^{i[k_0(t_1-t_2) - k_3(x_1-x_2)]} \times \\ &\times \frac{n_\nu^* n_\mu^*}{k^2} \left(\delta_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n \cdot k} \right). \end{aligned} \tag{3}$$

Introducing Mandelstam's prescription for spurious singularities $(n \cdot k)^{-1}$ and variables:

$$\begin{aligned} n \cdot k &= k_0 + k_3 = k_+, \\ n^* \cdot k &= k_0 - k_3 = k_-, \end{aligned} \tag{4}$$

and dimensional regularization, the vacuum expectation value becomes

$$W = -8ig^2 C_R \int \frac{d^n k}{(2\pi)^n} \frac{1}{k_+ k_- - K^2 + i\eta} \frac{1}{k_+ + i\epsilon k_-} \frac{1}{k_-} (1 - \cos k_- L) e^{-i\tau k_+} \quad (5)$$

Other diagrams, like self-energy and cusp, vanish owing to the identity

$$\frac{n_\mu}{k^2 + i\eta} \left(\delta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k} \right) = 0. \quad (6)$$

Integrating over dk_+ in the complex plane closing the contour in the lower half-plane, we get

$$W = -8\pi g^2 C_R \int_{-\infty}^{\infty} dk \int d^{2-\omega} K \Theta(k) \frac{1}{k} (1 - \cos kL) \frac{1}{K^2 - i\eta + i\epsilon k^2} \times \\ \times \left(e^{-i\tau \frac{K^2 - i\eta}{k}} - e^{-\epsilon\tau k} \right) (2\pi)^{-4+\omega}. \quad (7)$$

Here we have denoted $k_- \equiv k$ and the poles are shown in Fig. 2.

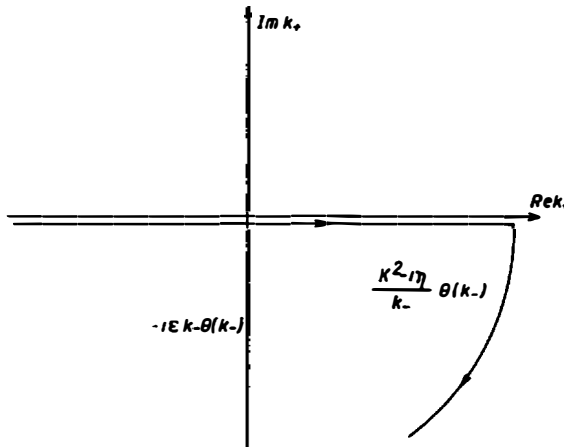


Fig. 2. Position of the poles in the complex k_+ plane with Mandelstam's prescription for spurious singularities.

To perform integration over $d^{2-\omega} K$, we need only two integrals:

$$B = \int d^{2-\omega} K \frac{1}{K^2 - i\eta + i\epsilon k^2} = \pi^{1-\frac{\omega}{2}} (i\epsilon k^2 - i\eta)^{-\frac{\omega}{2}} \Gamma\left(\frac{\omega}{2}\right), \quad (8)$$

$$A = \int d^{2-\omega} K \frac{1}{K^2 - i\eta + i\epsilon k^2} e^{-i\frac{T}{k}(K^2 - i\eta)} \tag{9}$$

In Eq. (9) we introduce the Schwinger parameter

$$\frac{1}{K^2 - i\eta + i\epsilon k^2} = -i \int_0^\infty dx e^{ix(K^2 - i\eta + i\epsilon k^2)} \tag{10}$$

and using

$$\int_{-\infty}^\infty dt e^{k(at^2 + 2bt) - \eta t^2} = \begin{cases} \frac{1+i}{\sqrt{2}} \sqrt{\pi/a} e^{-ib^2/a} & \text{for } a > 0 \\ \frac{1-i}{\sqrt{2}} \sqrt{\pi/|a|} e^{-ib^2/a} & \text{for } a < 0 \end{cases} \tag{11}$$

we obtain

$$A = -i \int_0^\infty dx e^{x(\eta - \epsilon k^2) - \eta \frac{T}{k}} \left(\left| x - \frac{T}{k} \right| \right)^{1 - \frac{\omega}{2}} \times \\ \times \left\{ \Theta \left(x - \frac{T}{k} \right) \left(\frac{1+i}{\sqrt{2}} \right)^{2-\omega} + \Theta \left(-x + \frac{T}{k} \right) \left(\frac{1-i}{\sqrt{2}} \right)^{2-\omega} \right\}. \tag{12}$$

The first part is

$$\int_{T/k}^\infty dx e^{-x(\epsilon k^2 - \eta)} \left(x - \frac{T}{k} \right)^{\frac{\omega}{2} - 1} = \Gamma \left(\frac{\omega}{2} \right) (\epsilon k^2 - \eta)^{-\frac{\omega}{2}} e^{-\frac{T}{k}(\epsilon k^2 - \eta)}. \tag{13}$$

The second part is more complicated:

$$\int_0^{T/k} dx e^{\eta \left(x - \frac{T}{k} \right) - x\epsilon k^2} \left(\frac{T}{k} - x \right)^{1 - \frac{\omega}{2}} = \pi^{1 - \frac{\omega}{2}} (-\epsilon k^2 + \eta)^{-\frac{\omega}{2}} \times \\ \times e^{-\frac{T}{k}(\epsilon k^2 - \eta)} \gamma \left(\frac{\omega}{2}, -\frac{T}{k}(\epsilon k^2 - \eta) \right), \tag{14}$$

which, on expansion of the incomplete gamma function, becomes

$$\pi^{1 - \frac{\omega}{2}} \frac{2}{\omega} e^{-\frac{T}{k}(\epsilon k^2 - \eta)} \left(\frac{T}{k} \right)^{\frac{\eta}{2}} + O(\epsilon). \tag{15}$$

The problematic parts $(\varepsilon k^2 - \eta)^{-\omega/2}$ cancel in W and we get

$$W = 16i \pi^{2-\frac{\omega}{2}} g^2 C_R \frac{1}{\omega} \left(\frac{1-i}{\sqrt{2}} \right)^{2-\omega} T^{\frac{\omega}{2}} \int_0^\infty dk k^{-1-\frac{\omega}{2}} (1 - \cos Lk) (2\pi)^{-4+\omega}. \quad (16)$$

The final integration gives

$$W = -16 g^2 C_R \pi^{2-\frac{\omega}{2}} e^{\frac{i\pi\omega}{4}} \frac{1}{\omega} \Gamma\left(-\frac{\omega}{2}\right) \cos \frac{\omega\pi}{4} (TL)^{\frac{\omega}{2}} (2\pi)^{-4+\omega}. \quad (17)$$

This Wilson loop is doubly logarithmically divergent (in the sense that in dimensional regularization there is an ω^{-2} term) and behaves like the power of the area $(TL)^{\omega/2}$.

3. Feynman gauge

In the Feynman gauge, the same loop receives contributions from the cusps shown in Fig. 3.

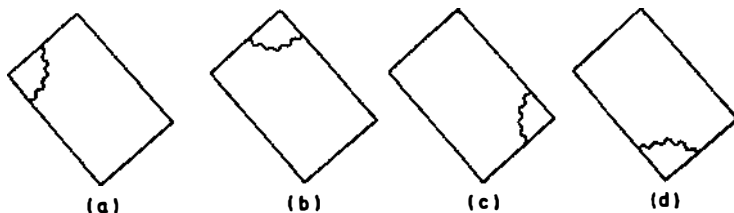


Fig. 3. Wilson loop with light-like sides in the Feynman gauge.

Here we show some details of the evaluation of Fig. 3(b).

$$W_b = ig^2 C_R \int_0^L dt_1 n_\mu^* \int_L^{L-T} dt_2 n_\nu \int \frac{d^4 k}{(2\pi)^4} \frac{\delta_{\mu\nu}}{k^2 + i\eta} \times e^{i[k_0(t_2-t_1) - k_3(x_2-x_1)]}, \quad (18)$$

$$W_b = 2ig^2 C_R \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + i\eta} \frac{1}{(k_0 + k_3)} [e^{-iT(k_0+k_3)} - 1] \times \frac{1}{(k_0 - k_3)} [1 - e^{iL(k_0-k_3)}]. \quad (19)$$

Introducing new variables

$$\begin{aligned} k_0 + k_3 &= k_+, \\ k_0 - k_3 &= k_-, \end{aligned} \tag{20}$$

and dimensional regularization, the expression for the Wilson loop reads

$$\begin{aligned} W_b &= ig^2 C_R \int_{-\infty}^{\infty} dk_+ \int_{-\infty}^{\infty} dk_- \int d^{2-\omega} K \frac{1}{k_+ k_- - K^2 + i\eta} \frac{1}{k_+} (1 - e^{-iT k_+}) \times \\ &\times \frac{1}{k_-} (e^{iL k_-} - 1). \end{aligned} \tag{21}$$

Owing to the factor $e^{-iT k_+}$, we close the contour in the lower half-plane. The poles are shown in Fig. 4.

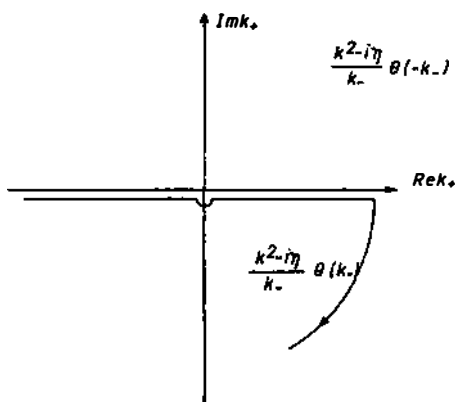


Fig. 4. Poles in the complex k_+ plane in the Feynman gauge.

Integration over dk_+ leads to

$$W_b = 2\pi g^2 C_R \int_0^{\infty} dk_- \int d^{2-\omega} K \frac{1}{K^2 - i\eta} (1 - e^{-iT \frac{K^2 - i\eta}{k_-}}) \frac{1}{k_-} (e^{iL k_-} - 1). \tag{22}$$

Now, we integrate over $d^{2-\omega} K$ using Eqs. (8) and (9) and, finally, over dk_- with the help of the formula

$$\int_0^{\infty} dk k^{-1-\frac{\omega}{2}} (1 - \cos Lk) = -\cos \frac{\omega\pi}{4} \Gamma\left(-\frac{\omega}{2}\right) L^{\frac{\omega}{2}}. \tag{23}$$

The sum of contributions from Fig. 3 gives exactly the same result which we have obtained in the light-cone gauge using Mandelstam's prescription (Eq. (17)).

4. Conclusion

We have checked the equivalence of the Feynman and light-cone gauges for the Wilson loop with light-like sides to order g^2 in perturbation theory.

Although the contributions arise from different graphs, the total sum is the same.

References

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WILSONOVA PETLJA SA SVJETLOLIKIM STRANAMA U FEYNMANOVOM BAŽDARNOM UVJETU I BAŽDARNOM UVJETU SVJETLOSNOG KONUSA

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Pokazana je ekvivalentnost Wilsonove petlje s dvije svjetlolike strane u Feynmanovom baždarnom uvjetu i baždarnom uvjetu svjetlosnog konusa, do reda g^2 u perturbacionoj teoriji. Ova petlja pokazuje dvostruku logaritmičku ultraljubičastu divergenciju već kod reda g^2 (u smislu da u dimenzionalnoj regularizaciji postoji član $(d - 4)^{-2}$).