

## STATISTICAL PROPERTIES OF MULTIPIION PRODUCTION

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A unitary eikonal model for multipion production is analyzed from a statistical point of view. We show how the knowledge of the square of the elastic amplitude may be used to find the form of the pion distribution if the two-nucleon source is coherent, chaotic, squeezed or a combination of them.

### *1. Introduction*

It is generally accepted that any realistic model of multiparticle production in hadron physics should satisfy the constraint imposed by unitarity. There are a number of solvable unitary models in which multiparticle production is dominated by the multiperipheral mechanism<sup>1-4)</sup>. These models are mainly based on the eikonal approximation, which offers a useful framework for discussion of high-energy phenomena. Usually, the results obtained within the QED relativistic eikonal model are accepted as a guide assuming that at high energies multiple exchanges between incident hadrons, in our case protons, are also uncorrelated in the impact-parameter space. This, however, is not valid if the constraints imposed by energy-momentum conservation<sup>3,6)</sup> and other additive quantum numbers, such as charge and isospin<sup>7)</sup>, are introduced for the production amplitude.

The eikonal model is therefore a good approximation if it is possible to neglect the fragmentation of incident particles (leading-particle effect). In this case, the production of particles is predominantly in the central region (pionization

region) of the rapidity plot. The S matrix is then diagonal in the rapidity difference and the relative impact parameter of the two incident protons. It is also an operator in the space of pions.

In the present paper we discuss an eikonal model with a unitary S matrix at high energy in which the two-nucleon system is treated as a stochastic c-number source of pions. We derive several classical pion distribution functions in the impact-parameter space.

In Section 2 we present the derivation of the model and define the pion-density matrix operator using the coherent-state representation.

In Section 3 we discuss the multiplicity-distribution function that follows from a particular choice of the pion-density operator.

## 2. The model

It is often possible to understand many results in hadron-hadron collisions in terms of the simple idea that the outgoing particles may have three origins:

beam + target  $\rightarrow$  (beam fragments) + (target fragments) + (central production),

i. e. fragments of beam particles, fragments of target particles and the central production.

To isolate the central production, we adopt high-energy longitudinally dominated kinematics, with two outgoing leading particles approximately saturating the energy and beam-direction momentum conservation laws. The essential restrictions are as follows:

- (i) the leading-particle effect,
- (ii)  $|y| \leq (1 - \epsilon) Y$ ,  $Y = \ln \frac{s}{m^2}$ ,  $\epsilon > 0$ , (1)
- (iii)  $|\vec{q}_T|$  small,
- (iv)  $\langle n \rangle < s^{\frac{1}{2} \epsilon}$ ,  $\langle n \rangle =$  particle multiplicity.

Here,  $y$  and  $\vec{q}_T$  denote, respectively, the rapidity and the transverse variable of a secondary particle, and  $Y$  is the (relative) rapidity of the incident two-proton system.

Using the following set of  $3n + 2$  independent variables  $Y, \vec{p}, (\vec{q}_{iT}, y_i)$ ,  $i = 1, 2, \dots, n$ , the  $n$ -particle contribution to the s-channel unitarity is

$$A_n(Y, \vec{p}) = \frac{1}{4s} \int d^2B e^{i\vec{p}\vec{B}} \prod_{i=1}^n d\hat{q}_i |T_n(Y, \vec{B}; \hat{q}_1 \dots \hat{q}_n)|^2, \quad (2)$$

where

$$\hat{q}_i \equiv (\vec{q}_{iT}, y_i); \quad d\hat{q}_i = \frac{d^2q_{iT}}{(2\pi)^2} \frac{dy_i}{4\pi}$$

and the normalization is such that

$$A_n(Y, O) = s\sigma_n(Y), \tag{3}$$

$$\sigma_{inel}(Y) = \sum_{n=1}^{\infty} \sigma_n(Y).$$

The connection between  $T_n(Y, \vec{B}; \hat{q}_1 \dots \hat{q}_n)$  and the elastic amplitude is obtained through unitarity. At high energy, the elastic amplitude may be written as

$$T(Y, \vec{p}) = 2is \int d^2B (1 - S(Y, \vec{B})) \exp(i\vec{p}\vec{B}) = \int d^2B \exp(i\vec{p}\vec{B}) T(Y, \vec{B}), \tag{4}$$

where

$$T(Y, \vec{B}) = 2is(1 - S(Y, \vec{B})). \tag{5}$$

The two-particle unitarity constraint on  $T(Y, \vec{B})$  is

$$\text{Im } T(Y, \vec{B}) = \frac{1}{4s} |T(Y, \vec{B})|^2 + s(1 - |S(Y, \vec{B})|^2). \tag{6}$$

Here,  $s(1 - |S(Y, \vec{B})|^2)$  denotes the overlap function  $F(Y, \vec{B})$  of Van Hove<sup>5)</sup> in the impact-parameter space. It is defined as the contribution of inelastic collisions to the unitarity equation for elastic scattering:

$$s(1 - |S(Y, \vec{B})|^2) = F(Y, \vec{B}) = \frac{1}{4s} \sum_{n=1}^{\infty} \int \prod_{i=1}^n d\hat{q}_i |T_n(Y, \vec{B}; 1 \dots n)|^2. \tag{7}$$

The total inelastic cross section is

$$\sigma_{inel}(Y) = \frac{1}{s} \int d^2B F(Y, \vec{B}) = \int d^2B (1 - |S(Y, \vec{B})|^2). \tag{8}$$

The reference to the leading-nucleon states is in the diagonal variables  $Y$  and  $\vec{B}$ , where  $\vec{B}$  can be interpreted as the transverse distance between nucleons. The discussion of unitarity and other properties of the production amplitude  $T_n(Y, \vec{B}; \hat{q}_1 \dots \hat{q}_n)$  can be made more transparent if we write the S matrix as a unitary operator  $\hat{S}(Y, \vec{B})$  in the space of pions. The initial state vector for the pion field is  $\hat{S}(Y, \vec{B})|0\rangle$ . The «clean» vacuum state  $|0\rangle$  is disturbed by the presence of two nucleons. Because of the leading-particle effect, two nucleons are always present and act as a pion source. Therefore, the vacuum state for pions is in fact a two-

nucleon state. The elastic scattering amplitude is the matrix element of  $\hat{S}(Y, \vec{B})$  between the states with no pions, i. e.

$$S(Y, \vec{B}) = \langle 0 | \hat{S}(Y, \vec{B}) | 0 \rangle. \tag{9}$$

The  $n$ -pion production amplitude is then given by ( $n \neq 0$ )

$$iT_n(Y, \vec{B}; \hat{q}_1 \dots \hat{q}_n) = 2s \langle \hat{q}_1 \dots \hat{q}_n | \hat{S}(Y, \vec{B}) | 0 \rangle \tag{10}$$

and we write

$$|T_n(Y, \vec{B}; q_1 \dots q_n)|^2 = 4s^2 \text{Tr} \{ \varrho(Y, \vec{B}) | \hat{q}_1 \dots \hat{q}_n \rangle \langle \hat{q}_1 \dots \hat{q}_n | \}, \tag{11}$$

where the pion-density operator  $\varrho(Y, \vec{B})$  is defined as

$$\varrho(Y, \vec{B}) = \hat{S}(Y, \vec{B}) | 0 \rangle \langle 0 | \hat{S}^\dagger(Y, \vec{B}), \tag{12}$$

together with the normalization condition

$$\text{Tr} \{ \varrho(Y, \vec{B}) \} = 1 \tag{13}$$

which follows from the unitarity of  $\hat{S}(Y, \vec{B})$ .

Using the pion-number operator

$$N = \int \frac{d^2 q_T dy}{(2\pi)^2 4\pi} a^\dagger(\vec{q}_T, y) a(\vec{q}_T, y), \tag{14}$$

we can write

$$|S(Y, \vec{B})|^2 = \text{Tr} \{ \varrho(Y, \vec{B}) : e^{-N} : \}, \tag{15}$$

where  $: :$  indicates the operation of normal ordering.

### 3. Multiplicity distributions

The pion-multiplicity distribution  $P_n$  for  $n \geq 1$  is given by

$$P_n = \frac{\sigma_n}{\sigma_{inel}} = \frac{1}{\sigma_{inel}} \int d^2 B \text{Tr} \{ \varrho(\vec{B}) : \frac{N^n}{n!} e^{-N} : \}. \tag{16}$$

For simplicity, in (16) we have suppressed the dependence on the variable  $Y$ .

Both the pion-multiplicity distribution and its factorial moments may be derived from a pion-generating function

$$G(z) = \sum_{n=1}^{\infty} (1-z)^n P_n = \frac{1}{\sigma_{1n01}} \int \text{Tr} \{ \varrho(\vec{B}) : (e^{-zN} - e^{-N}) : \} d^2B. \quad (17)$$

A simple calculation shows that for  $n \geq 1$

$$P_n = \frac{(-)^n}{n!} \frac{d^n}{dz^n} G(z)|_{z=1} \quad (18)$$

and

$$\langle n(n-1) \dots (n-r+1) \rangle = (-)^r \frac{d^r}{dz^r} G(z)|_{z=0}, \quad (19)$$

with the normalization

$$\begin{aligned} G(1) &= 0, \\ G(0) &= 1. \end{aligned} \quad (20)$$

Let us now discuss some of the multiplicity distributions that follow from a particular choice of the pion-density operator  $\varrho(Y, \vec{B})$ . A wide class of pion-density operators may be described by means of the »diagonal« coherent-state representation<sup>7,8)</sup>.

Coherent states  $|Y, \vec{B}; a\rangle$  are defined as eigenstates of the pion annihilation operator  $a(\hat{q})$ , that is

$$a(\hat{q}) |Y, \vec{B}; a\rangle = a(Y, \vec{B}; \hat{q}) |Y, \vec{B}; a\rangle. \quad (21)$$

For any complex function  $a$ , we have

$$|Y, \vec{B}; a\rangle = \exp \left[ -\frac{1}{2} n(Y, \vec{B}, a) \right] \exp [c^\dagger(Y, \vec{B})] |0\rangle, \quad (22)$$

where

$$c(Y, \vec{B}) = \int d\hat{q} a^*(Y, \vec{B}; \hat{q}) a(\hat{q}), \quad (23)$$

and

$$[c(Y, \vec{B}), c^\dagger(Y, \vec{B})] = n(Y, \vec{B}; a) = \int d\hat{q} |a(Y, \vec{B}; \hat{q})|^2. \quad (24)$$

We write

$$\varrho(Y, \vec{B}) = \int \mathcal{D}[a] \Phi[a] |Y, \vec{B}; a\rangle \langle Y, \vec{B}; a|, \quad (25)$$

where  $\int \mathcal{D} [\alpha]$  indicates a functional integration over all possible forms for the function  $\alpha (\dots)$ , and  $\Phi [\alpha]$  is an arbitrary functional of  $\alpha$ . The condition

$$\text{Tr} \{ \rho (Y, \vec{B}) \} = 1 \tag{26}$$

implies

$$\int \mathcal{D} [\alpha] \Phi [\alpha] = 1. \tag{27}$$

Real physics enters the problem when we specify the form of  $\Phi [\alpha]$ . Ideally, one would like to derive  $\Phi [\alpha]$  from the knowledge of the underlying dynamics. Lacking such knowledge, one is usually tempted to follow an analogy with the generalized Ginsburg-Landau theory of superconductivity<sup>9)</sup> in retaining only the first few terms in a series expansion of  $-\ln \Phi [\alpha]$  in powers of  $\alpha$  and its derivatives. The derivatives of  $\alpha$  either with respect to the rapidity  $y$  or with respect to the transverse momentum  $\vec{q}_\perp$  give rise to short-range correlations in the rapidity or the transverse momentum space, respectively.

By making an appropriate choice for  $\Phi [\alpha]$ , we can obtain a wide class of pion-emission distributions ranging from chaotic to highly coherent. The pion-distribution functions in the impact-parameter space  $\vec{B}$  are most easily studied using the generating function (Eq. (17)) which we write in the coherent-state representation as

$$G (z) = \frac{1}{\sigma_{inel}} \int d^2 B g (\vec{B}; z) \tag{28}$$

where

$$g (\vec{B}; z) = \psi (z) - \psi (1),$$

$$\psi (z) = \int \mathcal{D} [n] \Phi [n] e^{-zn(\vec{B})} = \sum_{n=0}^{\infty} (1-z)^n p_n (\vec{B}) \tag{29}$$

with

$$p_n (\vec{B}) = \int \mathcal{D} [n] \Phi [n] \frac{[n(\vec{B})]^n}{n!} e^{-n(\vec{B})} \tag{30}$$

and

$$p_0 (\vec{B}) \equiv |S(\vec{B})|^2 = \int \mathcal{D} [n] \Phi [n] e^{-n(\vec{B})}.$$

Note that the average number of produced pions is

$$\langle n \rangle = \frac{1}{\sigma_{inel}} \int d^2 B \langle n(\vec{B}) \rangle, \tag{31}$$

where

$$\langle n(\vec{B}) \rangle = \sum n p_n (\vec{B}) = \int \mathcal{D} [n] \Phi [n] n(\vec{B}).$$

For a given  $p_n(\vec{B})$  it is sometimes possible to find the unknown functional  $\Phi [n]$ . The formal solution is

$$\Phi [n] = e^{n\langle\vec{B}\rangle} \sum (-1)^n p_n(\vec{B}) \delta^{(n)}(n\langle\vec{B}\rangle), \tag{32}$$

where  $\delta^{(n)}$  is the  $n$  derivative of the functional  $\delta$ -function. For a coherent or Poissonian distribution where

$$p_n(\vec{B}) = \frac{\langle n(\vec{B}) \rangle^n}{n!} \exp[-\langle n(\vec{B}) \rangle],$$

$$\Phi_P [n] = \delta(n\langle\vec{B}\rangle - \langle n(\vec{B}) \rangle), \tag{33}$$

and for a chaotic or Gaussian distribution where

$$p_n(\vec{B}) = \frac{\langle n(\vec{B}) \rangle^n}{(1 + \langle n(\vec{B}) \rangle)^{1+n}}, \tag{34}$$

we find

$$\Phi_G [n] = \frac{1}{\langle n(\vec{B}) \rangle} \exp\left(-\frac{n\langle\vec{B}\rangle}{\langle n(\vec{B}) \rangle}\right).$$

This gives the following generating function in  $B$  space:

$$\psi_G(z) = (1 - z \langle n(\vec{B}) \rangle)^{-1}. \tag{35}$$

It has been observed experimentally<sup>10)</sup> that hadronic multiplicity distributions show a significant departure from the standard Poisson-like distributions (Eq. (33)). It has also been found that particle multiplicity ( $P_n$ ) may be well fitted with the negative binomial distribution ( $NB$ ) which in our approach corresponds to the generating function

$$\psi_{NB}(z) = \left(1 - z \frac{\langle n(\vec{B}) \rangle}{k}\right)^{-k}. \tag{36}$$

For  $k \rightarrow \infty$ ,

$$\psi_{NB}(z) \rightarrow \psi_P(z) = e^{-z\langle n(\vec{B}) \rangle},$$

that is, it reduces to a Poisson generating function. The parameter  $1/k$  thus measures the deviation from the Poisson, i. e. from the independent particle emission in  $B$  space.

The parameter  $k$  may be related to the correlation length  $L(Y)$  within the considered rapidity range  $Y$ . If  $Y/L$  is large, we may consider  $k \approx Y/L$  as an average number of independent sources of pions acting within the two-nucleon system.

There is also the possibility of constructing more complicated multiplicity distributions by combining two or more generating functions together. For example, the *Poisson-NB* combination is given by

$$\psi_{P-NB}(z) = \left(1 - z \frac{\langle n_{NB}(\vec{B}) \rangle}{k}\right)^{-k} e^{-z \langle n(\vec{B}) \rangle}. \quad (37)$$

Another class of generating functions which also show a strong departure from the Poisson are the one obtained from squeezed coherent states<sup>11)</sup>. These are obtained by a canonical transformation of the pion creation and annihilation operators using a unitary operator  $S(\lambda)$ :

$$S(\lambda) = \exp \left[ \frac{\lambda}{2} A^\dagger - \frac{\lambda^*}{2} A \right], \quad (38)$$

where

$$A = \int d\hat{q} a(\hat{q}) a(\hat{q}).$$

We can show that

$$b(\hat{q}; \lambda) \equiv S^\dagger(\lambda) a(\hat{q}) S(\lambda) = \text{ch} |\lambda| a(\hat{q}) + \frac{\lambda}{|\lambda|} \text{sh} |\lambda| a^\dagger(q) \quad (39)$$

and  $b^\dagger(\hat{q}; \lambda)$  satisfy the same commutation relations as  $a(\hat{q})$ . The squeezed coherent states are then defined as eigenstates of  $b(q; \lambda)$ , that is

$$b(\hat{q}; \lambda) |\alpha; \lambda\rangle = \alpha(\hat{q}) |\alpha; \lambda\rangle. \quad (40)$$

For  $\lambda = 0$ ,  $|\alpha; \lambda\rangle$  becomes the ordinary coherent state of the annihilation operator  $a(q)$ .

If  $|\psi\rangle$  is any state vector of the pion system, then  $S(\lambda)|\psi\rangle$  represents the same system but compressed in position space by a factor  $e^{-|\lambda|}$  and expanded in momentum space by a factor  $e^{|\lambda|}$ .

The pion-multiplicity distribution may then be calculated using the density operator

$$\varrho(\vec{B}) = |\alpha; \lambda\rangle \langle \alpha; \lambda| \quad (41)$$

which gives the following generating function:

$$\psi_\lambda(z) = \langle \alpha; \lambda | : e^{-zN} : | \alpha; \lambda \rangle. \quad (42)$$

#### 4. Conclusions

Within the unitary eikonal model we have studied the pion-multiplicity distributions in pp scattering at high energy.

Using the notion of a pion-density operator we have shown that its coherent-state representation enables one to construct various pion-multiplicity distributions. We have found an explicit form of the pion generating function in the impact-parameter space for coherent, chaotic and squeezed coherent production. A comparison with experimental data is to be presented elsewhere.

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### STATISTIČKA SVOJSTVA MULTIPIONSKE PRODUKCIJE

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Unitarni eikonální model za mnogopionsku produkciju analiziran je sa statističkog stanovišta. Pokazano je kako se poznavanje kvadrata elastične amplitude može upotrijebiti za nalaženje oblika pionske raspodjele ako je dvonukleonski izvor koherentan, kaotičan, stješnjen, ili neka njihova kombinacija.