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SU (6) — BOSON EXTENDED RANDOM PHASE APPROXIMATION: NEW APPROACH TO THE MICROSCOPIC SUBSTANTIATION OF INTERACTING BOSON MODEL

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Lie algebraic approach to the microscopic foundation of Interacting Boson Model — 1 is developed, treating on equal footing the SU (6) governed dynamics and the accompanying SU (6) constraints. In this approach the collective RPA phonon operators are used as »preferred pairs« with subsequent enforcement of the relevant SU (6) algebra. The »decoupled« phonon subspace is identified as carrier space of the totally symmetric irreducible representation of SU (6) group. From the microscopic Hamiltonian the fragment with the ensuing IBM-1 boson form is extracted.

The Interacting Boson Model (IBM)[1], under the influence of which the area of low energy excitations of nuclei has blossomed into an active and exciting field[2–4], was advanced by Iachello and Arima at first as purely phenomenological theory. Much of the appeal of the original version of the model, referred to as IBM-1 (IBM stands for IBM-1 and all its derivatives), stems from its overtly Lie algebraic structure, which enables classification (according to the IReps of SU (6) group) of the boson basis states, to be exploited in conjunction with dynamical symmetries of the SU (6) boson Hamiltonian[1,3,4].

The impressed impact of the »new phenomenology« on the studies in nuclear structure and spectroscopy[3] has naturally raised the question about the link of IBM to the admittedly more fundamental nuclear Shell Model (SM). In particular, the microscopic substantiation of IBM in the context of spherical SM, has indeed been put on the agenda and, despite the formidable work done in this direction[5] has not been removed eversince[2,4,6]. This problem has been investigated by several authors utilizing a variety of methods (Cf., e. g.: Table 1 in Ref. 6; Ch. 15 of Ref. 4 and vol. 2, part III, Sec. VIII of Ref. 2). A few problems encounteres in the implementation of these microscopic approaches have also been remarked[2,6,7]:

(i) The mapping of the SM fermionic space into the small IBM boson subspace is, understandably, in no way unique.

(ii) The mapping procedure can be carried out explicitly in schematic cases only. In realistic cases the mapping procedures yield very complicated expressions. This is the crux of the matter, since the mapping lies in the heart of any attempt to derive IBM-like Hamiltonian starting from the SM.

(iii) The construction of the boson images of the IBM Hamiltonian and physical operators (which is based on equating matrix elements of the appropriate operators in the fermion space with those in the boson space for lowest generalized seniority states) can be effected in a closed form again only in particular cases.

(iv) Attempts to provide microscopic foundation of IBM-1 assume that the total number of s- and d-bosons, N, equals to the number of valence pairs (»IBM counting rule«). Such being the case, this is a phenomenological step, which can be avoided if one makes the best use of the underlying SU (6) symmetry. Moreover, fuller enlisting of symmetry considerations helps to resolve some of the problems mentioned above (Cf. Sec. II).

This is by far not a complete list of predicaments of present-day IBM microscopy. They are only being mentioned as a rationals for the claim that additional efforts be directed to the synthesis of a theoretical framework which, exploiting maximally the inherent algebraic properties, accommodates sound microscopic support for the IBM-1.

In this connection we note that an alternative approach to arrive from the Shell Model to collective SU (6)-model (referred to as Truncated Quadrupole Phonon Model (TQM)[*]) has been chosen by Janssen, Jolos and Dönau as early as in Refs. 9—11. The TQM Hamiltonian has been constructed at the first set out on the basis of a sufficiently generic microscopic Hamiltonian[9-11]. With their seminal work[9] the authors of TQM have set the method for microscopic treatment of the collective quadrupole degree of freedom to enroll approximate bifermion SU (6) algebra, referred to as Quadrupole Collective Algebra (QCA)[12]. Since exact boson realizations of QCA are available (Cf., e. g. Ref. 12), the passage to bosons is straightforward which made it possible to obtain the TQM Hamiltonian and physical operators directly, without having to resort to mapping of matrix elements. However, this approach encounters problem too. In particular, since the SU (6) symmetry has been enforced[9-11], this necessarily lays down conditions on the amplitudes of the collective pairs, composing QCA. The authors of TQM have pointed out that the relevant constraints, to be referred to as SU (6)-enforcing conditions (SU (6)-EC), can be inferred from the Jacobi identities what is, as a matter of fact, an intricate problem to solve. This fact might have been the reason that further development of the approach, based on enforced symmetries, remained dormant for some time.

Due to recent publications[13-16], the enforced symmetry approach has been increasingly brought to bear on the problem of microscopic foundation of IBM-1. Rigorous derivation of the ensuing SU (6)-EC and explicit microscopic construction of Dyson (DR), Holstein-Primakoff (HPR) and Schwinger (SR) realizations of QCA (each of them interrelated with the SU (6)-EC) have been itemized in Ref. 13. The developments presented in Refs. 13—16 have enabled us to advance a new approach to the microscopic substantiation of IBM-1, to be referred to as SU (6)-Boson Extended Random Phase Approximation (SU (6)-BERPA). The key role in this approach is played by the collective RPA quadrupole phonon operators subjected to SU (6)-EC. Substituting the SR of the *constrained* RPA quadrupole phonon operators and their commutators into the Quasiparticle Phonon Model[17] (QPM) Hamiltonian (which is an established microscopic model), we derive a genuine IBM-1 Hamiltonian with coefficients, depending on known microscopic quantities and on the amplitudes of the constrained phonons. This microscopically deduced IBM-1 Hamiltonian will be referred to as SU (6)-B-ERPA Hamiltonian.

The consistent treatment of the symmetry governed dynamics and of the SU (6)-EC that go with it, constitutes the major innovation of the SU (6)-B-ERPA, being a part and parcel of the entire theoretical edifice.

The subsequent treatment will provide the outline of SU (6)-B-ERPA as one possible way towards meeting the demands for a sound microscopic theory of IBM-1.

The partial contributions to the development of SU (6)-B-ERPA are contained in a series of papers[13-16]. Here we present the full account of the theoretical underpinning of SU (6)-B-ERPA. A detailed expose is presented with the purpose to publicize more widely the basic ideas and the mathematical apparatus of this alternative approach to the microscopic foundation of IBM-1.

Prior to setting up the SU (6)-B-ERPA, we recall a few items pertaining to the IBM-1 Hamiltonian (whose microscopic derivation is the ultimate end of the SU (6)-B-ERPA). The standard form of the later reads[1,4]:

$$\begin{aligned} H_{\mathrm{IBM-1}} &= \bar{h} \sum_{\mu} d_{2\mu}^{+} d_{2\mu} + C_{h1} \left(\frac{1}{2} s^{+} s^{+} s s \right) + C_{h2} \sum_{\mu} d_{\mu}^{+} d_{\mu} s^{+} s + \\ &+ \sum_{L=0,2,4} \bar{C}_L \left(\frac{1}{2} \sqrt{2L+1} \right) [[d_2^{+} \otimes d_2^{+}]_{(L)} \otimes [\tilde{d}_2 \otimes \tilde{d}_2]_{(L)}]_{(00)} + \\ &F \{ [[d_2^{+} \otimes d_2^{+}]_{(2)} \otimes [\tilde{d}_2 \otimes s]_{(2)}]_{(00)} + \mathrm{h.c.} \} + \\ &+ G \{ [[d_2^{+} \otimes d_2^{+}]_{(0)} \otimes [s \otimes s]_{(0)}]_{(00)} + \mathrm{h.c.} \}. \end{aligned} \tag{1}$$

Performing the needed recouplings in the tensor products featuring in (1), one can represent the IBM-1 Hamiltonian as an SO (3)-scalar, built from the generators $d_{2\mu}^{+} s$, $s^{+} d_{2\mu}$, $d_{2\mu}^{+} d_{2\nu}$ ($\mu, \nu = 0, \pm 1 \pm 2$) of the canonical SU (6) algebra in their Schwinger realization. (Cf., e. g. Ref. 12). The Hamiltonian (1) conserves the total number of s- and d-bosons, N, which, for a given nucleus, is *postulated* to be equal to the number of pairs of valence nucleons (»IBM-counting rule«). This is in essence a free parameter. The parameters in IBM-1 Hamiltonian are usually determined by the least square fit to observed nuclear properties[1-4].

The variation of these parameters with mass number is smooth and fits are indeed remarkable.

The main aim of any microscopic approach to the »SU (6) phenomenology« is to relate the free parameters of H_{IBM-1}, featuring in (1), to the microscopic quantities of an established fermionic Hamiltonian. Developing the SU (6)-B-ERPA, we have utilized as reference the Quasiparticle-Phonon Model Hamiltonian for spherical nuclei. Detailed presentation of QPM and its applications can be found in Ref. 17. (The QPM has been successfully used for description of the fragmentation of quasiparticle and collective (phonon) states in wide excitation energy interval). Within the QPM, Bogolyubov's quasiparticles and phonons are being used as simple modes of excitations, instead of the nucleon degrees of freedom (hence the name QPM has come into being). More specifically, we employ a particular QPM Hamiltonian including an average nuclear field as the Saxon-Woods potential, superconducting pairing interactions and isoscalar quadrupole-quadrupole forces (Cf. Eq. (1) of Ref. 17):

$$H_{micr} = \sum_{\tau} \{\sum_{jm}^{\tau} (E_j - \lambda)\, a_{jm}^+ a_{jm} - G_\tau/4\, (P_0^+ P_0)^\tau - \varkappa/2 : (M_2^+ \cdot M_2)^\tau :\}. \quad (2)$$

The notation $\tau = (\mathrm{n}, \mathrm{p})$ is used; the summation $\sum^\tau$ for $\tau = \mathrm{n}$ is over the neutron and for $\tau = \mathrm{p}$ over the proton states. The single-particle states are specified (if there is not ambiguity) by the quantum numbers jm; E_j are the single-particle energies; λ is the chemical potential; G and $\varkappa$ are the respective strengths of the monopole pairing and quadrupole-quadrupole interactions. The pair creation and quadrupole operations entering in the scalar products in (2) are defined in a standard fashion:

$$P_0^+ = \sum_{jm} (-1)^{j-m}\, a_{jm}^+ a_{j-m}^+ \quad (3)$$

$$M_{2\mu}^+ = \frac{1}{\sqrt{5}} \sum_{jj'mm'} f_{jj'} \langle jmj'm' | 2\mu \rangle\, a_{jm}^+ a_{jm'} \quad (4)$$

where $f_{jj'}$ stand for the reduced single particle matrix elements of the operator $(ir)^2\, Y_{2\mu}(\Omega)$[17].

In reality H_{micr} given by (2) is nothing else, but the schematic spherical single particle pairing-quadrupole Hamiltonian, which constitutes the main part of the QPM Hamiltonian (Cf. Eq. (1) of Ref. 17).

By performing the canonical Bogolyubov transformation

$$a_{jm}^+ = u_j \alpha_{jm}^+ + (-1)^{j-m} v_j \alpha_{j-m} \quad (5)$$

and introducing subsequently multipole phonon operators[17]

$$Q_{\lambda\mu i}^+ = \frac{1}{2} \sum_{jj'} [\Psi_{jj'}^{\lambda i} \langle jmj'm' | \lambda\mu \rangle\, \alpha_{jm}^+ \alpha_{j'm'}^+ -$$

$$- (-1)^{\lambda-\mu} \Phi^{\lambda i}_{jj'} \langle jmj'm' | \lambda - \mu \rangle a_{j'm'} a_{jm}] \tag{6}$$

$$Q_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} [\Psi^{\lambda i}_{jj'} \langle jmj'm' | \lambda\mu \rangle a_{j'm'} a_{jm} -$$
$$- (-1)^{\lambda-\mu} \Phi^{\lambda i}_{jj'} \langle jmj'm' | \lambda - \mu \rangle a^{+}_{jm} a_{j'm'}] \tag{7}$$

we cast H_{micr} into the usual quasiparticle-phonon representation:

$$H'_{micr} = \sum_{j} \varepsilon_j (2j+1)^{1/2} B(jj;00) - \frac{\varkappa}{20} \sum_{\mu=-2}^{2} \sum_{i,i'=1}^{i_{max}} Z_{1i} Z_{1i'} Q^{+}_{2\mu i} Q_{2\mu i'} -$$
$$- \frac{\varkappa}{40} \sum_{\mu=-2}^{2} (-1)^{\mu} \sum_{i,i'=1}^{i_{max}} Z_{1i} Z_{1i'} (Q^{+}_{2\mu i} Q^{+}_{2-\mu i'} + \text{h.c.}) -$$
$$- \frac{\varkappa}{20} \sum_{\mu=-2}^{2} (\sum_{i,i'=1}^{i_{max}} Z_{1i} [Q^{+}_{2\mu i} + (-1)^{\mu} Q^{+}_{2-\mu i'}]) \cdot$$
$$\cdot \sum_{j_1 j_2} [f_{j_1 j_2} v^{(-)}_{j_1 j_2} B(j_1 j_2; 2-\mu) + \text{h.c.}] -$$
$$- \frac{\varkappa}{10} \sum_{\mu=-2}^{2} (-1)^{\mu} \sum_{j_1 j_2 j_3 j_4} f_{j_1 j_2} v^{(-)}_{j_1 j_2} f_{j_3 j_4} v^{(-)}_{j_3 j_4} B(j_1 j_2; 2\mu) B(j_3 j_4; 2-\mu) \tag{8}$$

where in the definition of $Q^{+}_{\lambda\mu i}$ the index λ denotes multipolarity, μ denotes Z-projection in the laboratory system, and i is the label of the solution of RPA dynamical equation:

$$B(jj'; \lambda\mu) \equiv \sum_{mm'} \langle jmj'm' | \lambda\mu \rangle a^{+}_{jm} (-1)^{j'+m'} a_{j'-m'} \tag{9a}$$

$$Z_{1i} \equiv \sum_{jj'} f_{jj'} u^{(+)}_{jj'} (\Psi^{2i}_{jj'} + \Phi^{2i}_{jj'}) \tag{9b}$$

$$u^{(+)}_{jj'} \equiv u_j v_{j'} + u_{j'} v_j, \quad v^{(-)}_{jj'} = u_j u_{j'} - v_j v_{j'}, \varepsilon_j = [(E_j - \lambda_\tau)^2 + \Delta^2_\tau]^{1/2},$$
$$\Delta_\tau = G_\tau \sum_{j} u_j v_j. \tag{9c}$$

The quantities λ_τ and Δ_τ are calculated according to the well known equation[20]. We note that the fragment of the monopole pairing interaction that does not contribute to the formation of ε_j, featuring in H_{micr}, has been cast aside. Short of the first term in Eq. (8), the rest originates from the quadrupole-quadrupole force, involving the normal product of $(M^{+}_{2} \cdot M_2)$. As to the quadrupole operator, its explicit form in the quasiparticle-phonon representation reads:

$$M^{+}_{2\mu} = \frac{(-1)^{\mu}}{\sqrt{5}} \{\sum_{i=1}^{i_{max}} Z_{1i} [Q^{+}_{2\mu i} + (-1)^{\mu} Q_{2-\mu i}] + \sum_{jj'} f_{jj'} v^{(-)}_{jj'} B(jj'; 2\mu)\}. \tag{10}$$

Note that the first term includes summation over collective ($i = 1$) and noncollective phonons ($i = 2, \ldots, i_{max}$).

It is evident from Eqs. (1) and (8) that the phenomenological and reference Hamiltonians are in different representations, which makes it difficult to compare them. Since direct construction of the boson image of H'_{mlcr} (employing standard boson expansion techniques) does not automatically yield H_{IBM-1} one has to elaborate a special procedure which allows to single out from H'_{mlcr} the fragment with a boson structure identical to that of the H_{IBM-1} defined by (1). The essence of this procedure, which was formulated in principle and used by the creators of TQM there exists a subspace of quadrupole collective states, which are weakly connected with the rest of states. Such being the case, the set of operators, which generate this class of states, are bound to constitute, at least approximately, a closed algebra which has turned out to be isomorphic to SU (6)[9,10]. The possibility to use known boson realization (Dyson, Holstein-Primakoff and Schwinger, see Ref. 12 of the latter) greatly facilitates the bosonization of the reference Hamiltonian and produces a genuine SU (6) boson Hamiltonian, if SR of QCA is used.

We now turn to the choice of the collective subspace in SU (6)-B ERPA. From Eqs. (8) and (10), it is apparent that the Hamiltonian H'_{mlcr} and the quadrupole operator $M^+_{2\mu}$ are built out of the set of operators ($Q_{2\mu i'}$, $Q^+_{2\mu i'}$, $B\,(jj'; 2\mu)$, $\mu = 0, \pm 1, \pm 2$; $i = 1, 2, \ldots, i_{max}$). The latter set of operators constitutes the building blocks of the reference Hamiltonian. In addition, by acting with the ($Q^+_{2\mu i}$) operators or the phonon vacuum one generates quadrupole collective ($i = 1$) and noncollective ($i = 2, \ldots, i_{max}$) phonon states. It is thus natural to choose as collective subspace the set of states generated by the $Q^+_{2\mu i}$-operators acting on the vacuum. Such a choice is further supported by the observation (established in Ref. 13) that if we assume, according to Janssen, Jolos and Dönau[9-11], that this collective subspace is weakly coupled with the states of another nature, then the set of operators $\{Q_{2\mu i}, Q^+_{2\mu i}, [Q_{2\mu i}, Q_{2\mu i}]\}$ approximately closes the ensuing SU (6) algebra QCA. Indeed it has been shown[13] that:

$$[Q^+_{2\mu i}, [Q_{2\nu i}, Q^+_{2\sigma i}]] = 2 \sum_{\varrho=-2}^{2} (Q^+_{2\varrho i}\, C_{\mu\nu\tau\varrho} + Q_{\varrho i}\, D_{\mu\nu\sigma\varrho}) \tag{11}$$

plus sums involving $\{(\lambda = 2, i \geq 2)$ and $(\lambda \neq 2, i \geq 1)\}$ »scattering« terms

$$[Q_{2\mu i}, [Q_{2\nu i}, Q^+_{2\sigma i}]] = -2 \sum_{\varrho=-2}^{2} (Q_{2\varrho i}\, C_{\mu\sigma\tau\varrho} + Q^+_{2\varrho i}\, D_{\mu\sigma\tau\varrho}) \tag{12}$$

plus sums involving ($\lambda = 1$, $i \geq 2$) and ($\lambda \neq 2$, $i \geq 1$) »scattering« terms.

The explicit forms of $C_{\mu\nu\tau\varrho}$ and $D_{\mu\nu\tau\varrho}$ is given in Ref. 13 (cf. Eqs. (22) and (23)) and will be of no use further. We stress that if the »scattering« terms in commutation relations (11) and (12) are cast aside, the set of operators

$$\{Q_{2\mu i}, Q^+_{2\mu i}, [Q_{2\mu i}, Q^+_{2\nu i}], [Q_{2\mu i}, Q_{2\nu i}], [Q_{2\mu i}, Q_{2\nu i}]\} \tag{13}$$

will compose a closed algebra, but not necessarily a Lie algebra. It has been proved[13], that the necessary and sufficient conditions that the above set of operators forms a Lie algebra, are given by:

$$W^{(K)}_{j_1 j_2} \equiv \sum_{j_3} (-1)^{j_1+j_3} \begin{Bmatrix} 2 & 2 & K \\ j_1 & j_2 & j_3 \end{Bmatrix} \Phi^{21}_{j_1 j_2} \Psi^{21}_{j_2 j_3} = 0 \tag{14}$$

for $k = 0, 1, 2, 3, 4$ and any $(j_1 j_2)$,

$$D_K \equiv \frac{25}{2} \sum_{j_1 j_2 j_3 j_4} (-1)^{j_3 - j_1} \begin{Bmatrix} j_4 & j_3 & 2 \\ j_2 & j_1 & 2 \\ 2 & 2 & K \end{Bmatrix} (\Psi^{21}_{j_1 j_2} \Psi^{21}_{j_3 j_1} \Psi^{21}_{j_2 j_4} \Phi^{21}_{j_3 j_4} -$$

$$- \Phi^{21}_{j_1 j_2} \Phi^{21}_{j_3 j_1} \Psi^{21}_{j_2 j_4} \Phi^{21}_{j_3 j_4}) = 0 \tag{15}$$

for $k = 0, 1, 2, 3, 4$,

and

$$C_1 = C_3 = 0 \tag{16}$$

$$C_0 = C_2 = C_9 = C \tag{17}$$

$$C_K \equiv \frac{25}{2} \sum_{j_1 j_2 j_3 j_4} (-1)^{j_3 - j_1} \begin{Bmatrix} j_4 & j_3 & 2 \\ j_2 & j_1 & 2 \\ 2 & 2 & K \end{Bmatrix} (\Psi^{21}_{j_1 j_2} \Psi^{21}_{j_3 j_1} \Psi^{21}_{j_2 j_4} \Psi^{21}_{j_3 j_4} -$$

$$- \Phi^{21}_{j_1 j_2} \Phi^{21}_{j_3 j_1} \Phi^{21}_{j_2 j_4} \Phi^{21}_{j_3 j_4}). \tag{18}$$

Contraints (14)—(17) constitute the so called SU (6)-EC.

In virtue of Eq. (14), the commutators $[Q_{2\mu 1}, Q_{2\nu 1}]$ and $[Q^+_{2\mu 1}, Q^+_{2\nu 1}]$ vanish identically (Cf. Eq. (34) in Ref. 13), thus ensuring the elimination of the redundant operators in the set (13).

While it is by no means a trivial matter to infer constraints (14)—(17) from the condition that the operators set (13) forms a Lie algebra (sufficiency), it is easy to show that, if Eqs. (14)—(17) are satisfied, then the set of operators $\{Q_{2\mu l}, Q^+_{2\mu l}, [Q_{2\mu l}, Q^+_{2\nu l}]\}$ composes a Lie algebra (necessity). The exploit form of the Lie algebra under consideration, the so called QCA reads (Cf. Eqs. (36)—(38) in Ref. 13):

$$[Q^+_{2\mu 1}, [Q_{2\nu 1}, Q^+_{2\sigma 1}]] = C\delta_{\mu\nu} Q^+_{2\sigma 1} + C\delta_{\nu\sigma} Q^+_{2\mu 1} \tag{19}$$

$$[Q_{2\mu 1}, [Q_{2\nu 1}, Q^+_{2\sigma 1}]] = -C\delta_{\mu\sigma} Q_{2\nu 1} - C\delta_{\nu\sigma} Q_{2\mu 1} \tag{20}$$

$$[[Q_{2\mu 1}, Q^+_{2\nu 1}], [Q_{2\sigma 1}, Q^+_{2\varrho 1}]] = C\delta_{\sigma\nu} [Q_{2\mu 1}, Q^+_{2\varrho 1}] - C\delta_{\mu\varrho} [Q_{2\sigma 1}, Q^+_{2\nu 1}]. \tag{21}$$

QCA is isomorphic to the Cartan-Weyl algebra. (Cf. Sec. VII of Ref. 13). The explicit isomorphism QCA $\longleftrightarrow$ SU (6) in conjunction with the SU (6)-EC have permitted to construct microscopic Schwinger boson realization of QCA, which is directly associated[12] with the IBM-1. The SR under consideration can be written as (Cf. Eqs. (51)—(53) in Ref. 13);

$$Q_{\mu}^{+SR} = N^{-1/2} d_{2\mu}^{+} s \tag{22}$$

$$Q_{\mu}^{SR} = N^{-1/2} s^{+} d_{2\mu} \tag{23}$$

$$[Q_{\mu}, Q_{\nu}^{+}]^{SR} = N^{-1} (\delta_{\mu\nu} s^{+} s - d_{2\nu}^{+} d_{2\mu}) \tag{24}$$

with

$$N \equiv \mathrm{Int}\,[C^{-1}] \tag{25}$$

where the quantity C is defined by Eqs. (17) and (18). (26)

This quantity has been shown[12] to measure the deviation of the two-phonon norm from unity. It reflects the fact that the constraint operators $\{Q_{\mu}^{+SR}, Q_{\mu}^{SR}\}$, which are an SU (6)-approximation image of the two-quasiparticle RPA phonon operators (6) and (7), account for Pauli principle in average.

We point out the connection to the question of spurious states associated with violation of antisymmetry[21-25] as it stands in the SU (6)-B-ERPA approach.

We see from Eq. (25) that within the SU (6)-B-ERPA the total number of bosons can be associated with the integer number, nearest to C^{-1}.

To emphasize the fact that Eqs. (22)—(24) represent the SR of the enforced QCA, the notation Q_{μ}^{SR}, Q_{μ}^{+SR} has been used for the *constraint* RPA phonon operators. The SU (6) boson images for the operators $B(jj'; LM)$, needed to bosonize the reference Hamiltonian H'_{micr}, have been also constructed in Ref. 13 (Cf. Eqs. (67) and (69)):

$$(2j+1)^{1/2} B(jj; 00) = \sum_{j'} [(\Psi_{jj'}^{21})^2 + (\Phi_{jj'}^{21})] \sum_{\mu=-2}^{2} d_{2\mu}^{+} d_{2\mu} \tag{26}$$

$$B(j_1 j_2; LM) = 5 \sum_{j_3} (-1)^{j_1+j_3} \begin{Bmatrix} 2 & 2 & L \\ j_1 & j_2 & j_3 \end{Bmatrix} (\Psi_{j_1 j_3}^{21} \Psi_{j_2 j_3}^{22} + \Phi_{j_1 j_3}^{21} \Phi_{j_2 j_3}^{21}) \cdot$$
$$\cdot (d_2^{+} \otimes \tilde{d}_2)_{LM}, \; L = 2, 4. \tag{27}$$

We are ready now to deduce from H'_{micr} the fragment with the IBM-1 boson structure. To this end we first separate the *collective* part of H'_{micr}, i. e. the part including the *collective* operators $Q_{2\mu 1}$, $Q_{2\mu_1}^{+}$. This in fact ammounts to restricting the summations in the rhs of Eq. (8) to $i_{max} = 1$. Upon replacement of the operators $\{B(jj; 00), Q_{2\mu_1}^{+}, Q_{2\mu_1}, B(jj'; 2\mu)\}$, featuring in the collective part of H_{micr}, by their boson equivalents (given by (16), (22), (23) and (27), respectively), the collective part of H'_{micr} aquires precisely the form of the IBM-1 Hamiltonian (1).

The term by term comparison of our microscopically derived and the phenomenological IBM-1 Hamiltonians leads to the following microscopic expressions for the IBM-1 Hamiltonian parameters:

$$h = \frac{1}{2} Z_{41} - \frac{\varkappa}{20} Z_{11}^2 - \frac{5}{2} \varkappa Z_{21}^2 \tag{28}$$

$$C_h = C_{h_1} = C_{h_2} = - \frac{\varkappa}{10} Z_{11}^2 / N \tag{29}$$

$$\overline{C}_L = - 25 \varkappa Z_{21}^2 \begin{Bmatrix} 2 & 2 & 2 \\ 2 & 2 & L \end{Bmatrix} \tag{30}$$

$$F = - \frac{\varkappa}{2} (5/N)^{1/2} Z_{11} Z_{21} \tag{31}$$

$$G = - (5)^{1/2} \frac{\varkappa}{40} Z_{11}^2 / N \tag{32}$$

$$Z_{21} \equiv \sum_{j_1 j_2 j_3} (-1)^{j_1 + j_2} f_{j_1 j_2} v_{j_1 j_2}^{(-)} \begin{Bmatrix} 2 & 2 & 2 \\ j_1 & j_2 & j_3 \end{Bmatrix} (\Psi_{j_1 j_3}^{21} \Psi_{j_3 j_2}^{21} + \Phi_{j_1 j_3}^{21} \Phi_{j_3 j_2}) \tag{33}$$

$$Z_{41} \equiv \sum_{j_1 j_2} (\varepsilon_{j_1} + \varepsilon_{j_2}) [(\Psi_{j_1 j_2}^{21})^2 + (\Phi_{j_1 j_2}^{21})^2]. \tag{34}$$

The quantity Z_{11} has been defined before (Cf. Eq. (9b)). With the aid of the transformation

$$\overline{h}' = \overline{h} + (N - 1)(C_{h_1} - C_{h_2}) = \overline{h} \tag{35}$$

$$\overline{C}_L' = C_L + (C_{h_1} - 2C_{h_2}) = C_L - C_h, \tag{36}$$

the most general IBM-1 Hamiltonian (1) can be transcribed to a six-parameter form. In the sequel we shall deal with the set of six parameters $\{\overline{h}', F, G, \overline{C}_L'\}$.

From Eqs. (28)—(32) it is evident that all the parameters except $\overline{C}_2$ have correct signs ($Z_{11}, Z_{41} > 0$, $Z_{21} < 0$). However, we are not able to obtain a negative-valued $\overline{C}_2'$ in the present approach. The inclusion of the isovector part of the quadrupole-quadrupole force could hopefully resolve this »sign« problem[14].

We have seen that the physical assumption of the collective quadrupole degree of freedom being weakly connected to the other degrees of freedom amounts to a truncation of the shell-model space to the collective subspace generated by the *constraint* quadrupole phonon operators $\{Q_\mu^{+\,SR}\}$, given by Eqs. (22) in conjunction with the SU (6)-EC. Decoupled collective subspace under consideration is, in fact, the totally symmetric IR $[N, O^4]$ of the SU(6) group. Acting repeatedly with the operator Q_u^{+SR} on the highest weight state

$$|hws\rangle \equiv \frac{(s^+)^N}{\sqrt{N!}}|0\rangle \text{ of } [N, O^4], \tag{37}$$

one constructs the entire $(N+5)!/(N!5!)$-dimensional subspace, spanned by the monomials

$$\left\{\frac{(s^+)^{n_s}}{(n_{s!})^{1/2}} \prod_{\mu=-2}^{2} (d_\mu^+)^{n_\mu}|0\rangle\right\}, \tag{38}$$

where

$$n_s + \sum_{\mu=-2}^{2} n_\mu = N \quad \text{(cf. Ref. 12).}$$

It is this decoupled subspace $[N, O^4]$ which has engendered H_{B-ERPA}, the IBM-1 Hamiltonian with microscopic expressions for the parameters given by Eqs. (28)—(32). Once the closure of QCA has been enforced, the problem of the fermions to bosons mapping has a well defined solution, since a full scope exact boson realizations of QCA = SU(6) are known and can be directly used[12].

We emphasize that since the SU(6) based H_{B-ERPA} is an approximation to the QPM Hamiltonian, we do not impose the requirement on the H_{B-ERPA} spectrum to coincide exactly with the spectrum of QPM Hamiltonian. This procedure differs from the ones in Refs. 26, 27.

We have thus succeeded to derive microscopically a Hamiltonian which posseses the ensuing SU(6) sd-boson form (1). We note that the (Ψ, Φ)-amplitudes appearing in the microscopic IBM-1 Hamiltonian can be obtained from the variational principle which ensures simultaneously a minimum of the obtained microscopic Hamiltonian H_{B-ERPA} in the collective one-phonon states and fulfillment of the SU(6)-EC[28].

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SU (6) BOZONSKA PROŠIRENA APROKSIMACIJA SLUČAJNIH FAZA: NOVI PRISTUP MIKROSKOPSKOM OSTVARENJU INTERAKCIJSKO BOZONSKOG MODELA

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Razvijen je algebarski pristup mikroskopskom utemeljenju interakcijsko-bozonskog modela. U tom pristupu kolektivni RPA fononski operatori su korišteni kao »preferirani parovi« uz naknadno nametanje SU (6) algebre.