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THE DOUBLE BETA-DECAY AND GRAND UNIFICATION

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The grand unified theories which still have a chance to be true predict preferentially that the neutrino is identical with its antiparticle and therefore that it is a Majorana particle which violates lepton number conservation. Such a neutrino should have a finite mass and also a small right handed weak interaction. If the double neutrinoless beta-decay is observed with the full decay energy in the two electrons, it would establish that the electron neutrino is a Majorana particle. The upper limit of the right handedness of the weak interaction derived from the lower limit of the life-times against the neutrinoless beta decay is $|\langle\eta\rangle| < 1.8 \cdot 10^{-8}$ and the upper limit for the neutrino mass is $|\langle m_\nu\rangle| < 1.9$ [eV].

1. Introduction

The neutrino is the only fermion from which we do not know if it is different from its antiparticle ($\nu \neq \overline{\nu}$) and therefore a Dirac particle or if it is identical with its antiparticle ($\nu = \overline{\nu}$) and therefore a Majorana particle.

In 1955 the physicists did believe that this question was solved in favour of a Dirac neutrino by an experiment of Davis[1]. He was using antineutrinos from the beta decay in a reactor to induce the inverse beta decay in ^{37}Cl for which one needs in the standard model a neutrino:

$$n \rightarrow p + e^- + \overline{\nu} \tag{1}$$

$$\nu + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-.$$

Since he did not observe the formation of ^{37}Ar by the inverse beta decay he assumed that the antineutrino ν is different from the neutrino ν and thus the neutrino is a Dirac particle. But this conclusion was outdated already two years later. In 1956 Lee and Yang[2] proposed to test if in the weak interaction parity is conserved, and in 1957 Wu et al.[3] did find that indeed parity is violated in the beta decay and soon it turned out that it is violated maximaly and the weak interaction is purely left handed. Therefore, the neutrino created in the beta decay of the neutron in Eq. (1) has a positive helicity (is right handed), while the neutroin needed for the inverse beta decay must have a negative helicity and therefore needs to be left handed. Thus, even if the neutrino is a Majorana particle, reaction (1) would be forbidden due to helicity mismatch of the two reactions. Thus, we know already since about 30 years that the problem, if the neutrino is a Dirac or if it is a Majorana particle, is not solved. But why are we just today discussing this question so intensively? The reason is that the grand unified theories from which we think that they are most successful predict that the neutrino is a Majorana particle[4]. Measurements of the lower limit of the proton life-time seem to exclude SU5 and thus one concentrated on SO10, which is also a subgroup of dynamical groups discussed in superstring theories. SO10 which has first been proposed by Fritsch and Minkowski[5] predicts in improved versions[4] not only, that the neutrino is a Majorana particle, but automatically predicts also that the neutrino has a mass and a weak right handed interaction. As we will see below these facts allow the double beta decay without neutrinos. Or inversely: The existence of the double neutrinoless beta decay would establish that the neutrino is a Majorana particle.

Fig. 1 shows the double beta decay with two neutrinos in the standard model.

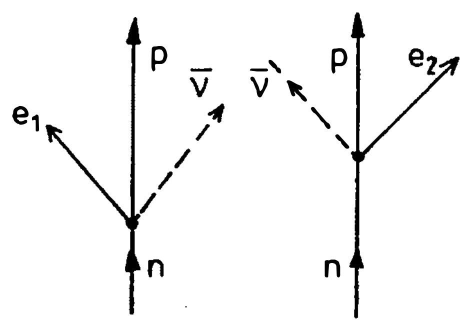

Fig. 1. Double beta-decay with two neutrinos in the standard model. Two neutrions inside the nucleus are transformed by two beta decays into protons. Two electrons and two neutrinos are emitted into the continuum. The kinetic energy available from the Q value of the reaction has to be distributed over the four particles emitted into the continuum. The phase space is proportional the product of the momentum squared of all the particles in the continuum. Thus, the fact that the available kinetic energy has to be distributed over four particles, reduces the phase space drastically.

Fig. 2 shows the diagram for the double neutrinoless beta decay. This is naturally possible if the neutrino is identical with its antiparticle since at the first vertex, one would emit an antineutrino and at the second vertex one needs to absorb a neutrino in the standard model. Since we have only two particles in the continuum in the process in Fig. 2, the phase space is bigger by a factor 10^6 compared to Fig. 1. Thus, even if the matrix element squared is reduced by a factor 10^{-6},

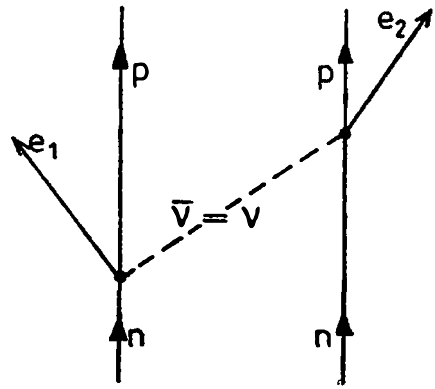

Fig. 2. Diagrams for the double neutrinoless beta decay with a Majorana neutrino. By having only two particles in the final states in the continuum, the phase space is increased by a factor of about 10^6 compared to Fig. 1. Even with a Majorana neutrino this process is only possible if the neutrino has a finite mass and/or also a right handed weak interaction.

one has the same transition probability for the neutrinoless double beta decay as for the double beta decay with two neutrinos shown in Fig. 1.

But even if the neutrino is a Majorana particle, the process in Fig. 2 cannot happen since for a pure left handed weak interaction theory, the emitted neutrino must be right handed (positive helicity), while the absorbed neutrino must be left handed (negative helicity). But grand unified theories predict also that the neutrino has a mass and a slight right handed weak interaction. With a finite mass the neutrino has not any more a good helicity and the interference term between the leading helicity and the small admixtures allows a double neutrinoless beta decay. The same happens if we have also right handed vector bosons which allow a right handed weak interaction. If the first vertex is a left handed V-A interaction and the second vertex is a right handed $V + A$ interaction, the process in Fig. 2 is also allowed. Thus, the double neutrinoless beta decay can either happen due to the finite mass of the neutrino or due to a small admixture of right handed leptonic currents. Thus the matrix element for the double neutrinoless β-decay consists of two parts: One proportional to the neutrino mass and the other to the small right handedness η of the weak interaction. η is equal to the mass ratio of the left and right vector bosons squared and to $\sin^2 \vartheta$ of a mixing angle of the two vector bosons.

2. Description of the two neutrino duoble beta decay

Since there are measurements available for the two neutrino double beta decay with the geochemical method[11,12], and also one in the laboratory[13,14], one could try to calculate for a test of the theory the double beta decay with two neutrinos and compare them with the data. If one performs this calculation, one finds that the calculated transition probability for the double beta decay with two neutrinos ($2\nu\beta\beta$) is by a factor 10 to 100 too large[5]. This discrepancy typical for the $2\nu\beta\beta$ decay naturally casts doubt on the reliability of the calculations of the $0\nu\beta\beta$ transition probabilities.

The nuclear many-body methods for calculating the double beta decay are either shell model calculations[6)] or extended shell model calculations based on the Hartree-Fock-Bogoliubov approach called MONSTER[7)] or they are based on the random phase approach (RPA). Let's try to analyse the $2\nu\beta\beta$ decay in the RPA approximation.

Fig. 3 explains why the $2\nu\beta\beta$ decay amplitude is so drastically reduced. Therefore the small effects which normally do not play a major role can affect the

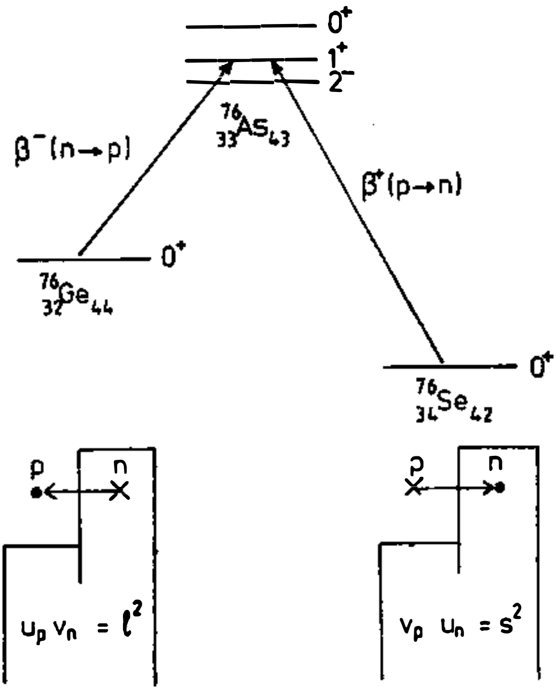

Fig. 3. The upper part shows the way how in the random phase approximation (RPA) the $2\nu\beta\beta$ decay is calculated. One starts with an RPA proton-particle neutron-hole calculation to the intermediate ^{76}As nucleus and also in the same way from the ^{76}Se ground state by neutron-particle proton-hole excitations to the same intermediate nucleus. For the Fermi transitions the β^- $(n \to p)$ amplitude moves just a neutron into the same proton level and the β^+ $(p \to n)$ amplitude moves a proton into the same neutron level. For the Gamow-Teller transitions it can also involve a spin flip, but the orbital part remains the same. One immediately realizes that the occupation and non-occupation amplitudes favour the β^- amplitude, but disfavour the β^+ amplitude. There one has a transition from an unoccupied to an occupied single particle state, which is two-fold small (s^2) first by the fact that the occupation amplitude for the proton v_p and secondly that the unoccupation amplitude for the neutron state u_n are both small. Therefore the $2\nu\beta\beta$ is drastically reduced.

$2\nu\beta\beta$ transition probability. If one looks to the β^+ $(p \to n)$ amplitude one finds that the matrix elements involved in these diagrams are Pauli suppressed by a factor $(u_n v_p)^2 = (small)^4$. The second diagram shown in Fig. 4 shows the quasiparticle-quasiparticle excitations leading due to particle number non-conservation also to the intermediate nucleus ^{76}As. These quasiparticle-quasiparticle matrix elements are not so drastically reduced. They are proportional to $(u_n u_p)^2 =$

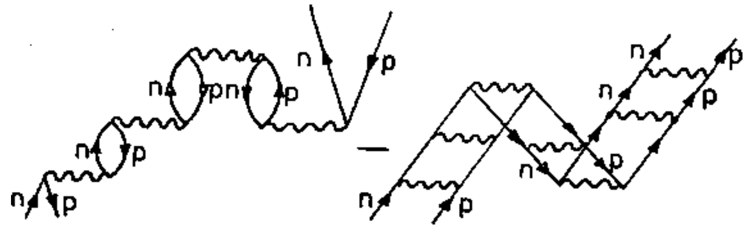

Fig. 4. The left hand side shows the isovector neutron-particle proton-hole excitations leading from ^{76}Se to ^{76}As. The particle-hole matrix elements are proportional to the unoccupation amplitude for the neutrons and the occupation amplitude for the proton squared $(u_p v_n)^2 = (small)^4$. The ground state correlations and the collectivity due to these excitations are therefore extremely weak. The right hand side shows particle-particle excitations where the matrix elements have the Pauli factors $(u_p u_n)^2 = (small)^2 (large)^2$. Normally this particle-particle ground state correlation and excitation are neglected, but due to the weakness of the neutron-particle proton-hole excitations they play a major role in this case. Since the isovector particle-hole force is repulsive and the particle-particle (and the hole-hole) force is attractive the inclusion of the particle-particle (and hole-hole) correlations tend to quench the $2\nu\beta\beta$ transition probability.

$= (small)^2 (large)^2$. The neutron-particle proton-hole force in the isovector channel is repulsive while the particle-particle force is attractive. Therefore both excitations tend to cancel each other and therefore the amplitude β^+ is drastically reduced. In this way one obtains agreement with the experimental data.

We calculated the $2\nu\beta\beta$ decay half lives for the following $0_i^+ \to 0_f^+$ transitions: $^{76}Ge \to {}^{76}Se$, $^{82}Se \to {}^{82}Kr$, $^{128}Te \to {}^{128}Xe$ and $^{130}Te \to {}^{130}Xe$. We take the full $3\hbar\omega$ and $4\hbar\omega$ major oscillator shells for the description of the nuclei with $A = 76$, 82 and a model space consisting of $1p_{3/2,1/2}$, the full $4\hbar\omega$ shell, $0h_{11/2,9/2}$ and $1f_{7/2,5/2}$ sub-shells for $A = 128$, 130. The single particle energies are calculated with the Coulomb-corrected Woods-Saxon potential. As a realistic two-body interaction we use the nuclear matter G-matrix calculated from the Bonn one boson-exchange potential. In order to take into account the renormalization for the finite basis, we multiply the pairing matrix elements of this force by factors g_{pair}^p and g_{pair}^n. These factors lie close to unity and deviate at most less than 20% from the value obtained from the reaction matrix of the Bonn potential. These factors are adjusted to the odd-even mass differences. We also multiply the particle-hole matrix elements by a factor g_{ph} which we fix to the energy of the Gamow-Teller giant resonance (GTGR) in the intermediate odd-nucleus. This factor is also normally close to unity. The biggest deviation found is 30%. To show the influence of the particle-particle correlations, we multiply also the particle-particle matrix elements by a factor $g_{pp} \cdot g_{pp} = 0$ switches off the particle-particle correlations while $g_{pp} = 1$ gives the values predicted by the theory including these correlations.

3. *The neutrinoless double β-decay*

For the neutrinoless double β-decay we have only lower limits for the lifetimes[8,9,10]. The theoretical transition probability for the zero neutrino double β-decay can be written as

$$P_{0\nu\beta\beta} = const \cdot |m_\nu M_m + \eta M_\eta + \lambda M_\lambda|^2. \quad (2)$$

The experiment yields an upper limit for $P_{0\nu\beta\beta}$. Thus in the parameter space of the electron-neutrino mass m_ν and the right-left (lepton and baryon vertices) handedness $\eta \sim \mathrm{tg}\,\vartheta$ and right-right (lepton and baryon vertices) handedness $\lambda \sim \sim (M_{WL}/M_{WR})^2$ of the weak interaction relative to the left handed one, one finds an ellipsoid, in which all allowed values for the neutrino mass $\langle m_{\nu e}\rangle$ and the right handedness $\langle\eta\rangle$ and $\langle\lambda\rangle$ must lie. Here M_{WL} and M_{WR} are the left and right handed vector boson masses, respectively. ϑ is the mixing angle $W_1 = \cos\vartheta \cdot W_L + \sin\vartheta \cdot W_R$ of the left and right handed vector bosons. The must stringent limits for $m_{\nu e}$ and η are obtained from the lower limit of the ^{128}Te life-time $\tau_{1/2} > 5 \cdot 10^{24}$ years. The results are given in Fig. 5.

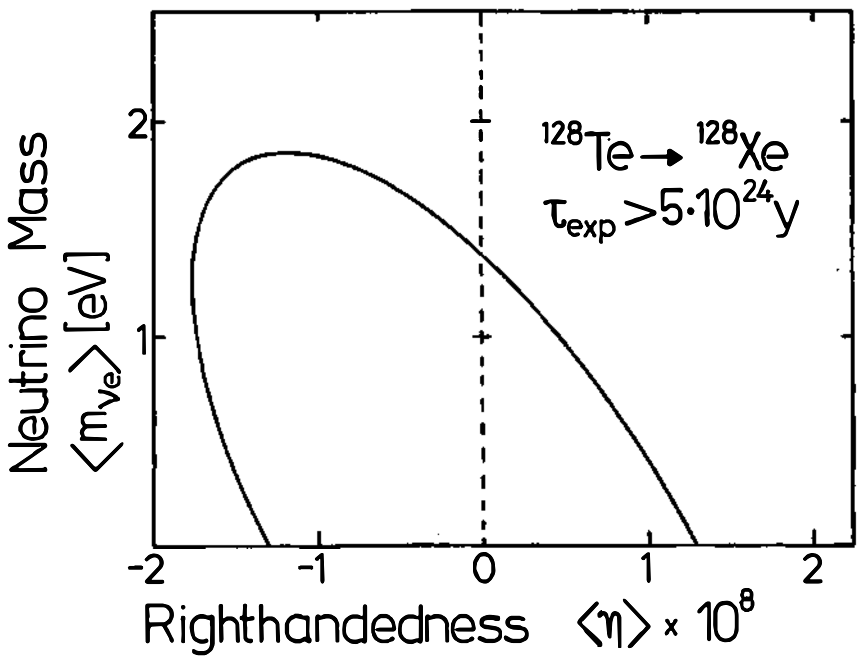

Fig. 5. The allowed regions (inside the ellipses) deduced from the experimental bounds (Caldwell, Berkeley Conference[10] and Refs. 11, 13 and 14) for the neutrinoless double β-decay ^{128}Te → ^{128}Xe with $\tau_{1/2} > 5 \cdot 10^{24}$ years. The upper limit for the neutrino mass is $m \leq 1.9$ eV and for the right handedness $\eta \leq 1.8 \cdot 10^{-8}$ and $\lambda \leq 10^{-6}$.

4. *Particle-particle correlations and the single β^+-decay*

The same reduction which one finds for the β^+-branch of the $2\nu\beta\beta$ decay one should also find for the β^+-decay from neutron deficient nuclei. Indeed it is well known[16,17] that the β^+-decay in neutron deficient nuclei are drastically quenched compared to the theoretical results.

Fig. 6 shows the reduced β^+-decay probability as a function of the particle--particle strength g_{pp} for the β^+-decay of ^{148}Dy → ^{148}Tb. The solid line is cal-

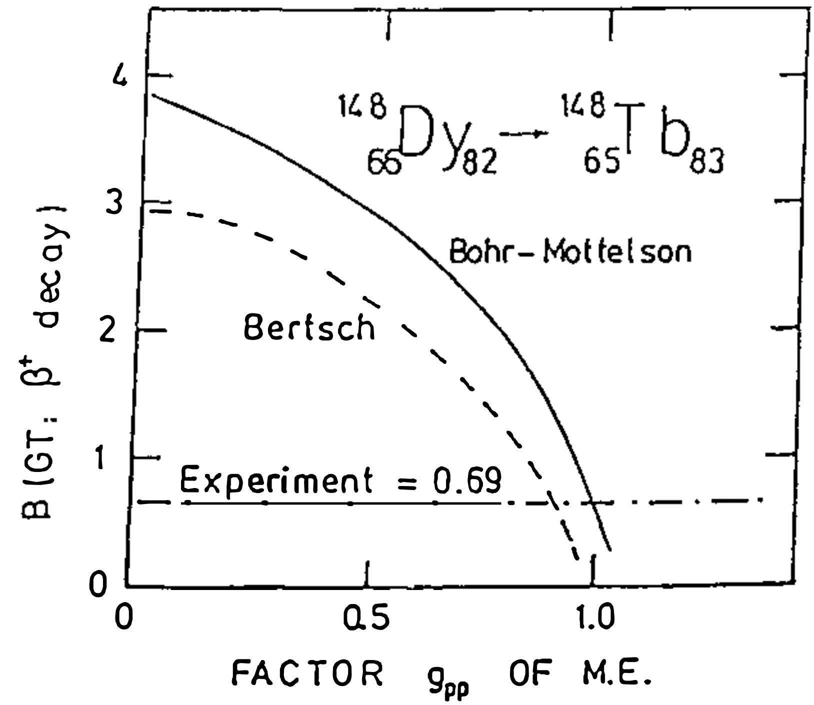

Fig. 6. Reduced β^+-decay probability $B\,(GT)$ as a function of the particle-particle strength g_{pp} with which the particle-particle reaction matrix elements of the Bonn potential are multiplied. The solid line is calculated with single particle wave functions of a Saxon-Woods potential with parameters of Bohr and Mottelson[18] and the dashed line with Saxon-Woods parameters of Bertsch[19]. One finds a drastic reduction for the physical value of the particle-parctile strength $g_{pp} = 1$. Around this value the theory agrees with the experimental data $B\,(GT) = 0.69$. The single-particle basis used in the RPA calculation is the full $N = 4$ and 5 and the $i_{13/2}$ and $i_{11/2}$ shells.

culated using a Saxon-Woods potential with parameters from Bohr and Mottelson[18] and the dashed line using a Saxon-Woods potential with parameters from Bertsch[19]. (The single particle energies given by Bertsch in Ref. 19 are wrong, but the parameters of the Saxon-Woods potential seem to be correct.)

The same effect which yields agreement for the $2\nu\beta\beta$ decay can also explain the quenching in the β^+-decay of neutron deficient nuclei.

5. Conclusions

The double neutrinoless β-decay ($0\nu\beta\beta$) can distinguish if the neutrino is a Dirac particle, that means if the neutrino is different from the antineutrino, or if it is a Majorana particle and therefore identical with its antiparticle. Only in the case of a Majorana particle, the double neutrinoless β-decay ($0\nu\beta\beta$) is possible. Grand unified theories predict that the neutrino is a Majorana particle, especially the ones which are built on the SO(10) group. But being a Majorana particle the neutrino has to have a finite mass and for SO(10) also a slight right handed weak interaction. Since the $0\nu\beta\beta$ decay needs either a finite mass of the neutrino or a right handedness of the weak interaction, one can derive from the lower limit of the half-lives of $0\nu\beta\beta$ decay upper limits for an averaged neutrino mass $\langle m_\nu \rangle \leq$ $\leq 1.8 \cdot 10^{-8}$.

If one tries to test the calculation by the known $2\nu\beta\beta$ decays one finds that the theory is by a factor 10 to 100 larger than the experimental data. But if one includes the particle-particle correlations of protons with neutrons, which are attractive, they cancel by a large part the neutron-particle and proton-hole correlations are cancelled. Including these additional ground state correlations one finds[20-23] a strong quenching of the $2\nu\beta\beta$ transitions in agreement with the experimental data.

For the $0\nu\beta\beta$ transition amplitude this quenching is for the leading recoil term only about a factor 0.7 or 30%. This difference between the $2\nu\beta\beta$ and $0\nu\beta\beta$ decay stems from the fact that in the $0\nu\beta\beta$ one has for the transition operator a dependence on the distance between the two vertices and thus, by a multipole expansion one gets a transition operator which excites also higher multipoles in the intermediate nucleus[20]. These higher multipoles are not strongly quenched and thus the $0\nu\beta\beta$ transitions are not so drastically affected by the particle-particle correlations.

The quenching of the β^+ branch in the $2\nu\beta\beta$ decay can also be tested in the single β^+-decay of neutron deficient nuclei. Including the particle-particle correlations one finds also agreement for these transitions which could not be understood before.

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DVOSTRUKI BETA RASPAD I VELIKO UJEDINJENJE

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Originalni znanstveni rad

Opažanje dvostrukog bezneutrinskog beta raspada nepobitno bi dokazalo da je elektronski neutrino čestica Majorana tipa. Pomoću toga je postavljena gornja granica za masu takvog neutrina.