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RESEARCH BRIDGING THEORY AND PRACTICE: COLLABORATIVE ACTION RESEARCH IN GEOMETRY EDUCATION

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ABSTRACT

This study explores the potential of the Collaborative Action Research (CAR) model to bridge the gap between research and practice in geometry education. Using the Development of Geometrical Thinking (DGT) program, CAR facilitated a collaborative process among teachers and researchers through iterative cycles of planning, action, observation, and reflection. Conducted with 88 seventh-grade students and four mathematics teachers in Croatia, the study aimed to enhance geometric understanding and problem-solving skills.

The CAR model translated theoretical insights into practical teaching strategies, leading to significant improvements in student learning outcomes and teacher attitudes. Teachers became more reflective practitioners, adapting methods to meet students' needs effectively. These findings affirm CAR's value as a framework for professional growth and sustained instructional innovation in mathematics education.

Keywords: action research, collaboration, geometry education, geometrical thinking, teacher attitudes

INTRODUCTION

Research plays a crucial role in shaping teaching practices, improving student outcomes, and informing educational policies. However, a gap remains between research and its classroom application. Teachers, often introduced to research-based frameworks through professional development, face challenges in effectively implementing these insights due to time constraints, complex theories, and resistance to change. This gap is particularly evident in geometry education. Despite valuable models like van Hiele's levels of geometric thinking and Duval's cognitive apprehension, they are often underutilized in practice due to insufficient support for their integration. This study explores Collaborative Action Research (CAR) as a model to bridge this gap. Rooted in the Development of Geometrical Thinking (DGT) program, CAR emphasizes both student growth in geometric thinking and teacher professional development through reflective practices. By addressing barriers such as time constraints and resistance to change, the research demonstrates how CAR fosters collaboration, reflection, and innovation in geometry education, aligning research with practice to enhance teacher efficacy and student learning.

THEORETICAL BACKGROUND

The development of geometric thinking is a multifaceted process, encompassing various cognitive, affective, and instructional components. Understanding how students develop their geometric reasoning skills, and how teachers can support this development, is crucial for improving geometry education. This theoretical background draws on key frameworks, particularly those of Duval's cognitive apprehension and van Hiele's levels of geometric thinking, to explain the stages of geometric understanding and the role of teachers in guiding students through these stages. Additionally, the importance of teacher attitudes and CAR in fostering effective instructional practices is also discussed.

Theories of geometrical thinking: Duval and van Hiele

Van Hiele's Levels of Geometric Thinking (1986) offer a hierarchical model of geometric reasoning. Van Hiele identified five stages of geometric thought: (1) Recognition, where students identify shapes visually; (2) Analysis, where they explore and describe the properties of shapes, though they may not relate these properties to broader geometric principles; (3) Informal Deduction, where students start making logical inferences; (4) Formal Deduction, where formal logical structures and proofs are applied; and (5) Rigor, where students engage with abstract and formal proof systems. These levels highlight how geometric thinking evolves from visual recognition to logical deduction and formal reasoning. A key challenge in geometry education is guiding students through these developmental stages, ensuring that they transition smoothly from one level to the next. Effective teaching requires a recognition of where students are in their geometric development and the provision of appropriate scaffolding to support their advancement through these levels (De Villiers, 1994).

Duval's Framework of Cognitive Apprehension (1995) builds upon Van Hiele's model by identifying four interrelated types of cognitive apprehension that are crucial for geometric understanding. Perceptual apprehension enables the recognition and identification of geometric figures through their visual attributes. Sequential apprehension involves understanding and articulating the step-by-step construction or transformation of geometric figures. Discursive apprehension pertains to the ability to make logical deductions and draw inferences using definitions, theorems, and provided information. Operative apprehension facilitates the dynamic manipulation of geometric figures to solve problems or investigate proofs.

Duval asserts that geometric understanding extends beyond mere visual recognition. It requires the integration of these apprehensions to enable robust mathematical reasoning and deeper engagement with geometric concepts. A drawing serves as a true geometric figure only when it activates perceptual apprehension in conjunction with at least one other type of apprehension, as perceptual apprehension alone is insufficient for facilitating mathematical reasoning (Duval, 1995, p. 143).

Duval's structured teaching sequences (2017) offer an effective approach to helping students navigate from informal, perceptual engagement with geometric concepts to more formal, abstract reasoning. These sequences begin with physical manipulation of geometric objects, progressing to abstract reasoning and formal mathematical language. Teachers guide students through activities that foster sequential, discursive and operative apprehension, allowing them to explore geometric ideas through hands-on manipulation, reconfiguration, construction, and logical reasoning. This gradual shift from informal exploration to formal reasoning is critical in mastering complex geometric concepts. As Duval states „Geometry, more than other areas of mathematics, can be used to discover and develop different ways of thinking. That must be an essential goal for the teaching of geometry." (Duval, 1998, p.51).

Teacher attitudes and their influence on geometry education

Teacher attitudes play a pivotal role in shaping the effectiveness of geometry instruction and influencing students' engagement with the subject. Research underscores that teachers' attitudes toward mathematics, particularly geometry, significantly affect their teaching strategies, which in turn impact students' learning outcomes (Aquilina et al., 2024; Burte et al., 2020). These attitudes encompass cognitive components (beliefs about geometry and its importance), affective components (emotional responses, such as anxiety or enthusiasm), and behavioural components (actions based on their beliefs and emotions) (Reid, 2015). Personal experiences, professional development, and classroom environments further shape these attitudes (Guskey, 2002).

Teachers who approach geometry with enthusiasm and confidence are more likely to implement dynamic, student-centred strategies that foster engagement and promote deeper conceptual understanding (Hannula, 2002; Russo et al., 2020). Conversely, negative attitudes, such as viewing geometry as overly abstract or difficult, may lead to instructional avoidance of challenging concepts, thus limiting opportunities for students to develop robust geometric thinking (Reid & Ali, 2020; Vidić & Đuranović, 2020).

Reflective practices are instrumental in reshaping teacher attitudes, as they encourage educators to critically evaluate their beliefs and instructional approaches in light of their students' needs and learning contexts (Bognar, 2011; McNiff & Whitehead, 2011). Such practices, often integrated into professional development programs, enable teachers to transition toward more effective, research-informed teaching methods, enhancing both their attitudes and instructional efficacy.

Collaborative action research (CAR) and its role in geometry education

Collaborative Action Research (CAR) provides a robust framework for professional development, emphasizing iterative cycles of planning, action, observation, and reflection (McNiff & Whitehead, 2011). Within the domain of geometry education, CAR enables teachers to experiment with research-based teaching strategies, critically examine their practices, and collaboratively tackle instructional challenges (Bruce et al., 2011; Betts et al., 2017; Kostos & Shin, 2010). This process empowers educators to bridge the gap between theory and practice, integrating theoretical frameworks into their teaching while refining instructional approaches based on classroom observations and feedback (Schoenfeld, 2019; Duval, 2017).

CAR fosters collaboration among teachers and researchers, creating a synergistic environment that supports professional growth and improves both teaching practices and student outcomes. Teachers become active participants in the research process, enhancing their instructional techniques and their students' geometric thinking through shared reflection and adaptation (Bonner, 2006; Wright, 2021). This participatory model not only positively influences teacher attitudes but also drives meaningful pedagogical changes, promoting a classroom environment conducive to the development of deeper and more sophisticated geometric reasoning (Hannula, 2002; Vula, 2013).

RESEARCH OBJECTIVE AND QUESTIONS

This research aims to explore the role of CAR in enhancing teaching practices and student learning in geometry education. The study employs a model of CAR tailored to this context, featuring iterative cycles of collaborative planning, implementation of DGT program, classroom observation, and reflective analysis. Grounded in van Hiele's and Duval's frameworks, this model focuses on integrating evidence-based strategies into teaching. Specifically, the study addresses the following research questions:

1. To what extent does Collaborative Action Research influence the geometry teaching practices of elementary school teachers?
2. To what extent does Collaborative Action Research influence the geometry learning outcomes of elementary school students?

By addressing these questions, the study highlights CAR's transformative potential in fostering reflective teaching, professional growth, and innovative instructional practices. Building on recent research emphasizing the importance of collaborative professional development (e.g., Aquilina et al., 2024; Russo et al., 2020), the findings demonstrate the capacity of this CAR model to drive sustainable educational improvement. As Croatia has been advancing its educational research and practice, CAR stands out as a cornerstone for bridging the gap between theory and practice, with particular relevance to teaching and learning geometry.

METHODOLOGY

Research design

This study follows a quasi-experimental mixed-methods design, integrating both qualitative and quantitative approaches to examine the impact of the DGT program. While elements of CAR are incorporated, the study is primarily an intervention-based quasi-experiment, with an experimental group (EG) implementing the DGT program and a control group (CG) following standard instructional practices.

Unlike "traditional" action research models (e.g., McNiff & Whitehead, 2006), which emphasize fully participatory research without controlled comparisons, this study adopts an intervention approach where CAR serves as a professional development tool rather than the primary research methodology. To align with contemporary methodological frameworks that integrate action research within quasi-experimental designs, this study follows a mixed-methods approach (Ivankova & Wingo, 2018; Martí, 2016) which helps to secure an effective and scientifically sound approach, producing credible and valid conclusions about interventions outcomes and enhancing translation of research into practice (Ivankova & Wingo, 2018, p.9). This specific research approach can be graphically presented as follows (See Figure 1):

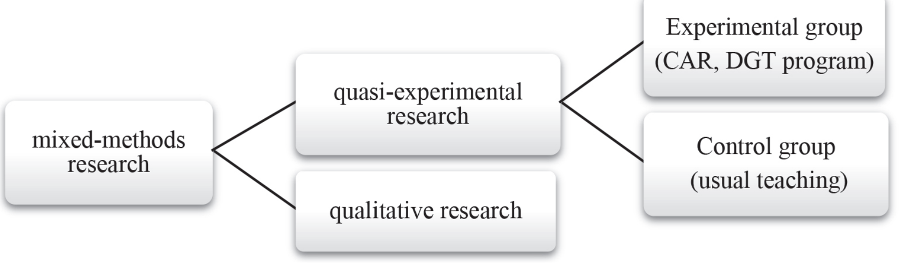


Figure 1. Graphical representation of the specificities of the research approach

Namely, this research design ensures a structured comparison between the two teacher groups while also capturing the reflective and iterative aspects of CAR as a mechanism for professional growth and instructional improvement and consequently change in students geometrical thinking.

Participants

The participants in this study included four in-service mathematics teachers and their 88 students from four 7th-grade classrooms in urban schools in Istria County, Croatia. Two teachers (ET, i.e., ET1 and ET2) were assigned to the experimental group (EG, 40 students), where they participated in CAR. These teachers used Duval's framework to deliver lessons on geometric topics such as polygons, circles, and discs, engaging students in hands-on activities aimed at deepening their geometric understanding and problem-solving skills. Their instructional approach followed the structured content of the DGT program, emphasizing visualization, dimensional deconstruction, and cognitive apprehensions. The other two teachers (CT, i.e., CT1 and CT2) formed the control group (CG, 38 students) and taught using their usual methods. This division allowed for a robust evaluation of the impact of the CAR model.

Data collection and analysis

This study employed a mixed-methods approach, combining qualitative and quantitative methodologies to comprehensively evaluate the impact of the CAR process, implemented through the DGT program, on teacher attitudes and student learning outcomes. In alignment with the school curriculum, the research was conducted during the second term of the school year, from January to July 2022.

The qualitative component focused on capturing changes in the experimental teachers' attitudes and instructional practices. Data collection involved teacher questionnaires, semi-structured interviews, classroom observations, and additional resources such as research notes, student notebooks, video recordings, and photographs. These instruments provided a multifaceted perspective on the CAR process and the DGT program's influence.

Quantitative data were gathered through pre- and post-tests designed to measure changes in students' geometric thinking and their ability to coordinate Geometric Figure Apprehension (GFA) as per Duval's framework and van Hiele's levels of geometric reasoning (VH). The pre-test, developed by the researcher, and the post-test, designed collaboratively with the ETs, assessed students' progress across defined sublevels of VH. Each VH level was further divided into sublevels: one indicating a lower or intermediate mastery and another signifying higher or complete mastery. For example, VH Level 1 (Recognition) included sublevels 10 (lower) and 11 (higher), while VH Level 2 (Analysis) included sublevels

20 and 21. GFA categories were assessed based on the interaction between perceptual apprehension and other types, such as discursive, sequential, and operative apprehensions. These categories were ranked from 2 (lower interplay) to 5 (high interplay) (Antunović, 2024). The pre-test underwent four piloting stages, initially comprising separate assessments for VH and GFA. However, due to pandemic-related constraints, a single, integrated test format was ultimately adopted.

The pre-intervention phase established baseline data through a pre-test administered to students and pre-questionnaires and interviews with ETs. These instruments evaluated students' initial geometric reasoning levels, teachers' familiarity with geometric theories, and their attitudes toward teaching geometry. This data informed the design of the DGT program, aligning its content and activities with curriculum requirements and the specific needs of students and teachers. The researcher also developed resources, such as scientific articles, instructional materials on polygons and circles, and activity ideas, to support content delivery and address classroom challenges identified during this phase.

Post-intervention questionnaires and interviews captured changes in teacher attitudes, instructional strategies, and perceptions of the CAR process and its effects on students. These tools assessed shifts in teaching practices and engagement levels, providing insights into the effectiveness of the DGT program in enhancing geometric understanding.

Classroom observations conducted throughout the study offered rich qualitative data, documenting how teachers implemented theoretical concepts and strategies developed during the CAR sessions. Observations focused on teacher-student interactions, activity types, and levels of student engagement (Schoenfeld, 2013). Supplementary sources, such as research notes, student notebooks, and video recordings, provided additional evidence of changes in instructional practices and classroom dynamics.

Qualitative data were analysed thematically, identifying patterns and themes in teacher attitudes, instructional strategies, and the CAR process's impact. Responses from questionnaires, interviews, and observational data were integrated with insights from pre- and post-tests, creating a cohesive understanding of the study's outcomes. Triangulation of multiple data sources enhanced the reliability and validity of findings, offering a robust evaluation of the DGT program's influence.

Quantitative data analysis involved statistical tests using SPSS to assess changes in student performance. Within-group changes were analysed using the Wilcoxon Signed Rank Test, while between-group differences were examined with the Mann-Whitney U Test. These analyses highlighted the significance of the CAR process in fostering improvements in students' geometric reasoning and problem-solving skills, providing valuable evidence of the DGT program's effectiveness.

Description of the DGT program

The Development of Geometrical Thinking (DGT) program was designed as a Collaborative Action Research (CAR) initiative, emphasizing iterative development and continuous refinement to align teaching practices with the evolving needs of both teachers and students. The program focused on the content of polygons, circles, and disks for 7th-grade students, integrating theoretical frameworks such as van Hiele's levels of geometric thinking and Duval's cognitive apprehension model with practical applications. The CAR process, characterized by collaboration, reflection, and adjustment, provided a structured framework for enhancing instructional strategies while fostering professional growth among teachers.

The DGT program unfolded through cycles, involving seven meetings and 18 classroom observations, which allowed for the systematic application and refinement of the program. In the first cycle, the groundwork for the program was prepared. Initial meetings introduced the research plan, outlined the objectives, and defined roles and ethical guidelines. Work schedules were coordinated for classroom observations and program implementation, ensuring a clear structure for collaborative efforts between the researcher and the experimental group teachers (ET). During the second cycle, ETs engaged with theoretical foundations through lectures on van Hiele's and Duval's frameworks, delivered by the researcher. The researcher presented findings from the pre-test, which highlighted gaps in students' visualization skills and understanding of fundamental geometric concepts. These insights informed refinements to the DGT program's activities and content, in line with CAR principles of iterative adjustment. To further support ETs, the researcher provided a curated selection of scientific articles on geometric thinking, emphasizing geometrical figure apprehension, teaching sequences, qualitative, non-numerical tasks and tools such as grids, geoboards, and GeoGebra. These resources, identified through teacher surveys and interviews, helped bridge theory and practice, guiding teachers in developing tasks that encouraged visualization, mathematical communication, and conceptual understanding. In the third cycle, the focus shifted to workshops through two modules, one on polygons and the second on circles. The researcher and ETs explored curriculum outcomes and analysed textbooks to design tasks and activities. The workshops emphasized hands-on learning and critical thinking, providing teachers with strategies to address student misconceptions and enhance engagement. Teachers also prepared materials for student project tasks, aligning activities with curriculum goals while fostering creativity and exploration in geometry. Additional content, such as tiling the plane and Thales' theorem of the inscribed angle, was incorporated to further deepen students' understanding.

The CAR process allowed the roles of both the researcher and ETs to evolve. Initially, the researcher led the program, developed materials, and structured the instructional strategies. However, as the program progressed, the researcher's role gradually shifted to that of an observer and critical friend. Through collaborative discussions and joint decision-making, the researcher provided feedback while encouraging the teachers to take greater ownership of the program. As the program advanced, the ETs transitioned from being passive receivers of knowledge to active contributors and co-creators of instructional content. Over time, they developed greater confidence and autonomy, leading lessons and designing activities with minimal support from the researcher. This shift empowered the ETs and significantly enhanced their teaching practices. By the third cycle, the researcher's role had shifted primarily to facilitating reflective practice, guiding teachers in evaluating and refining their teaching methods based on classroom observations and real-time feedback. This collaborative approach fostered a learning environment where both the researcher and ETs contributed to the continuous improvement of the program.

The iterative nature of the CAR process ensured the program remained adaptable and responsive to both teacher and student needs. Data from classroom observations informed adjustments to teaching approaches, while regular reflective meetings provided opportunities for real-time problem-solving and collaboration. At the conclusion of the program, the researcher and ETs collaboratively designed the post-test, which was a revised version of the pre-test. This collaboration ensured that the assessment was aligned with the program's objectives and accurately reflected the experiences and growth of both the teachers and students throughout the CAR process.

Overview of conducted lessons and content

Based on lesson reports from teachers in both groups, Table 1 provides a summary of the conducted lessons.

Table 1. Overview of conducted lessons and content

Groups	Polygons	Circle	Total lessons	Project tasks	Additional content	Textbooks
EG1	17	14	31	YES (Polygons – Rouses; Polygons in the coordinate system; Circles -Area of a disk and a circular segment)	YES Central and inscribed angle; Thale's theorem; Chordal quadrilateral	Šikić et.al., 2020
EG2	22	12	34	YES (Tiling the plane; non-numerical tasks; Relation two circle, line and circle)	YES Relation two circle, line and circle	Antunović-Piton, et.al, 2020
CG1	16	23	39	NO	YES Central and inscribed angle; Thales's theorem; Relation two circle, line and circle	Šikić et.al., 2020
CG2	20	10	30	YES Diagonals of polygons; native teaching - perimeter of the disc and length of circular arc	NO	Paić et al., 2021

The total number of lessons was relatively balanced across all groups, ensuring that all students were exposed to the required geometry content, following the national curriculum. The teachers of EG followed the DGT program focusing on cognitive apprehension, and they included more project tasks and additional content. Regarding the teachers in the control group (CG), CT1 incorporated additional content, while CT2 implemented project-based learning.

As part of the DGT program implementation, a total of 18 classroom observations were conducted, equally distributed between the two experimental groups. The impact of these instructional approaches will be further analysed in the results section.

RESULTS

The results are presented in both qualitative and quantitative formats, with qualitative data drawn from teachers of experimental group questionnaires, and interviews conducted before and after the DGT program and classroom observations. Moreover, the data from control group teachers' questionnaire answers obtained after their teachings were analysed. Quantitative data were derived from pre- and post-tests assessing students' van Hiele's levels of geometric thinking and Duval's geometric figure apprehension.

Qualitative results

Analyses on Experimental group teachers

1. *Analysis of Pre-Implementation Questionnaire and Interviews*

Before implementing the Development of Geometrical Thinking (DGT) program, a comprehensive analysis of teachers' initial perspectives, challenges, and instructional approaches was conducted through questionnaires and interviews.

This analysis was essential in tailoring the DGT program to address the specific needs of teachers and students. The findings highlighted several key aspects:

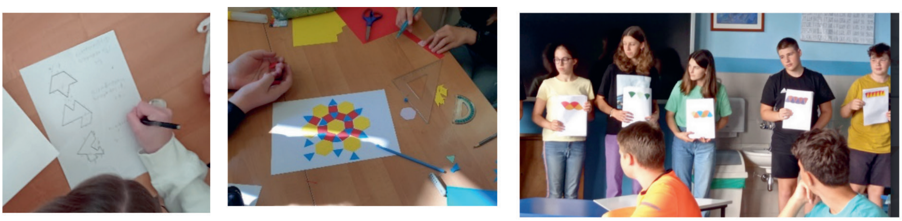
- *Challenges in teaching geometry* – Both teachers acknowledged the role of geometry in cognitive development but faced different obstacles. ET1 used differentiated instruction but lacked theoretical grounding in geometric thinking. ET2 struggled with abstract concepts like congruence and similarity, relying on intuition rather than structured frameworks. These differences highlighted the need for stronger pedagogical foundations.
- *Difficulties in student engagement* – Both teachers noted that students found it challenging to engage with tasks requiring conceptual understanding and visualization. ET1 observed better engagement among older students, while ET2 found students depended heavily on numerical values and measurements, limiting their ability to think abstractly.
- *Curriculum gaps in geometric thinking* – Teachers identified gaps in content sequencing, particularly in introducing complex geometric concepts. ET2 emphasized the need for a more structured approach to topics like similarity and congruence, but neither teacher had extensive knowledge of frameworks such as van Hiele's model, indicating a need for targeted professional development.
- *Need for professional development* – Both teachers expressed a strong need for further training in geometric reasoning. ET1, though effective in adapting instruction, lacked a theoretical foundation, while ET2's difficulties with abstraction and engagement reinforced the need for improved teaching strategies.

2. *Observations of Lessons*

While the pre-implementation analysis provided insights into teachers' perspectives and initial challenges, the classroom observations allowed us to examine how these insights translated into practice through the DGT program and how the teaching strategies evolved during the intervention.

Classroom observations highlighted a rich variety of activities aimed at fostering deeper understanding of geometric concepts among students. Lessons were structured to emphasize hands-on approaches, encouraging students to actively engage with concrete models and visualize geometric properties. Teachers employed structured learning sequences that integrated language use and diverse representations to support conceptual understanding. A significant focus was placed on orchestrating classroom communication, fostering argumentation and discussion, and posing high-order questions to stimulate critical thinking. Collaborative learning and problem-solving were central to the activities, creating an environment where students worked together to explore and resolve geometric challenges (See Figure 2).

The lessons demonstrated strong mathematical-task knowledge for teaching, as educators designed tasks that incorporated all types of cognitive apprehension—perceptual, sequential, discursive, and operative—within a single activity. Moreover, additional content not typically included in the 7th-grade curriculum was introduced, enriching the learning experience and challenging students to extend their understanding of geometric concepts beyond the standard syllabus.



Activity of reconfiguring *Activity -Tilling the plane* *Activity - area of the disc*

Figure 2. Teaching activities designed by the experimental group teacher

3. *Analysis of Post-Implementation Questionnaire and Interviews*

Building upon the classroom observations, the post-implementation analysis conducted through post questionnaire and semi-structured interview, explores how the intervention influenced teachers, i.e., teachers' perspectives on teaching strategies, student engagement, and overall learning outcomes.

After the DGT program, both ET demonstrated significant shifts in teaching practices, attitudes toward geometry, and student engagement. What follows are several noteworthy conclusions regarding its effects:

- *Changes in teaching practices* - Both teachers (ET1 and ET2) shifted from their usually, calculation-based lessons to hands-on, exploratory tasks, promoting active, student-centred learning. ET1 noted, "I gave them tasks to explore basic figures through different situations, to learn to watch," highlighting a focus on observation and exploration. This approach increased teacher motivation and student understanding. ET2 mentioned, "I did a lot of tasks without concrete measures... And the students accepted it, too," reflecting the shift to more conceptual, hands-on methods.
- *Increased student engagement* - Both teachers saw a rise in student engagement, especially with collaborative and dynamic tasks. ET1 observed, "I encouraged them to physically cut out shapes and manipulate them to gain a better understanding," which helped students build confidence and engage more deeply with the material.
- *Usefulness of theoretical frameworks* - Both teachers valued the theoretical foundations (Van Hiele's and Duval's frameworks) for deepening their understanding of geometric thinking. ET1 commented, "The materials were inspiring... I was focused on working with students, so I was more inspired by these materials," showing that the frameworks provided essential context for their teaching.
- *Looking forward* - Both teachers plan to continue using the strategies from the program. ET2 stated, "I will definitely use that in my future work," indicating a strong commitment to applying these hands-on, exploratory tasks in future teaching.

Analysis of Control Group Teachers

The control group teachers (CT1 and CT2) completed a 28-question survey after teaching polygons and circles, providing insights into their teaching practices, challenges, and perceptions. The findings highlighted several key aspects:

- *Perceptions of Teaching Geometry* – CT1 found teaching geometry challenging, but engaging, whereas CT2 considered it less complex and only moderately engaging. Despite these differences, both teachers acknowledged the importance of geometry in developing visual-spatial reasoning, communication, and mathematical thinking.

- *Instructional approaches* - CT1 prioritized content and mathematical processes, ensuring that students followed structured learning sequences. CT2 placed greater emphasis on student motivation, aiming to keep students engaged rather than focusing heavily on mathematical rigor.
- *Classroom challenges* – The teachers faced different obstacles in their teaching. CT1 struggled with classroom dynamics, student differences, and time constraints. CT2 encountered difficulties with teaching materials and administrative duties, which impacted her ability to implement engaging lessons.
- *Gaps in theoretical knowledge* – Neither teacher was familiar with the Van Hiele or Duval's framework for understanding the development of students geometric thinking. This gap in theoretical understanding may have limited their ability to structure lessons effectively and adapt their teaching strategies to students' developmental needs.
- *Reliance on textbook* – Both teachers relied heavily on textbooks and had limited use of interactive learning methods, reducing opportunities for dynamic student engagement. This underscores the need for professional development and the adoption of innovative teaching strategies to enhance the learning experience.

Connecting the above analyses regarding teachers from both groups, with the section on conducted lessons, it can be concluded that while both experimental (ET) and control (CT) teachers covered the required geometry content, their instructional approaches differed significantly. ET teachers, guided by the DGT program, integrated project-based learning, conceptual exploration, and theoretical frameworks like Duval's and van Hiele's models. This resulted in more dynamic, inquiry-driven lessons that deepened student engagement and understanding. In contrast, CT teachers relied mainly on textbook-based instruction, with one emphasizing structured mathematical process and the other prioritizing student motivation. The results indicate that the DGT program played a key role in transforming ET teachers' pedagogical practices, enhancing their ability to foster higher-order thinking and geometric reasoning.

Quantitative results

To complement these qualitative findings, the following section presents a quantitative analysis of students' progress, highlighting measurable changes in geometric thinking, interplay of GFA i.e., their geometrical problem-solving abilities. Quantitative data were collected through pre- and post-tests administered to students. Only students who completed both tests were included in the analysis, resulting in a final sample size of 78.

The pre-test results presented in Table 2 showed no statistically significant differences between the EG and CG, suggesting that both groups had comparable levels of understanding before the intervention. For van Hiele levels (VH), the p-value was 0.094, and for GFA, the p-value was 0.107. Most students in both groups were positioned between van Hiele levels 1 and 2, corresponding to the "Recognition" and "Analysis" stages. In terms of GFA, the majority of students demonstrated an intermediate understanding, indicating weaknesses in their ability to visualize and engage with geometric figures conceptually.

After the intervention the results of post-test (Table 2) demonstrated significant progress in the experimental group. Many students adopted van Hiele level 2 and advanced to van Hiele level 3, indicating a deeper understanding of geometric properties and the ability to engage in deductive reasoning. Similarly, students showed stronger coordination of Duval's types of apprehension, particularly operative and discursive apprehension, indicating a more sophisticated interpretation of geometric figures. In contrast, the control group showed no statistically significant changes from pre-test to post-test, with median scores remaining stable. This suggests that the instructional methods used by the control group were less effective in fostering geometric understanding and problem-solving abilities.

The Wilcoxon test revealed significant within-group improvements for the experimental group in both van Hiele levels ($p=0.030$) and GFA ($p=0.020$), confirming that the DGT program contributed to meaningful growth in geometric thinking. In contrast, the control group showed no significant within-group changes ($p=0.108$ for VH and $p=0.593$ for GFA).

Table 2. Change of students' Van Hiele levels and interplay of geometrical figure apprehensions – results of pre- and post-test

	VH levels			GFA		
Groups	Pre-test Median (Rang)	Post-test Median (Rang)	Within the group - p*	Pre-test Median (Rang)	Post-test Median (Rang)	Within the group - p*
Experimental group (N=40)	20 (10-30)	21 (10-31)	**0.030**	3 (2-5)	4 (2-5)	**0.020**
Control group (N=38)	20 (10-30)	20 (11-30)	0.108	3 (2-4)	3 (2-4)	0.593
Between group - p**	0.094	**0.022**		0.107	**0.020**	

**Wilcoxon Test, **Mann-Whitney test*

Mann-Whitney tests further confirmed significant between-group differences in post-test scores, with the experimental group outperforming the control group in both van Hiele levels (p=0.022) and GFA (p=0.020).

The progression data reveals further evidence of the DGT program's effectiveness (See Table 3). The experimental group showed a marked increase in students progressing to higher van Hiele levels, with 52.5% of students making positive progress in van Hiele levels, and 57.5% progressing in Geometric Figure Apprehension (GFA). In contrast, the control group also made some progress, with 34.2% showing improvement in van Hiele levels and 21.1% in GFA, although the majority of students in this group showed no change (63.2%).

Table 3. Progression of van Hiele's sublevels and GFA

	VH			GFA		
Group	Negative	No Changes	Positive	Negative	No Changes	Positive
Experimental Group	12.5%	35%	52.5%	7.5%	35%	57.5%
Control Group	13.2%	52.6%	34.2%	15.8%	63.2%	21.1%

These findings reflect the clear impact of the DGT program on students' geometric understanding. The experimental group made substantial strides, with a significantly higher proportion of students progressing in both van Hiele levels and GFA, demonstrating more effective development of geometric thinking. Meanwhile, the control group showed limited progress, particularly in GFA, where a majority of students showed no improvement. This highlights the value of the CAR model, which facilitated a more hands-on, student-centred learning experience, effectively supporting students' geometric reasoning and problem-solving skills.

DISCUSSION

The results of this study provide compelling evidence of the transformative impact of CAR on both teacher practices and student outcomes in geometry education. Through the DGT program, teachers were able to incorporate theoretical research findings in teaching methods to more dynamic, student-centred approaches. This shift was accompanied by significant improvements in both teacher attitudes and student learning outcomes of the experimental group (EG). These findings emphasize the importance of teacher attitudes in fostering effective learning environments, particularly in subjects like geometry, which require conceptual understanding, linking different representations and problem-solving skills.

Impact of CAR on teacher practices (RQ1)

The first research question examined the influence of CAR on teachers' geometry teaching practices. The results indicate that the DGT program led to significant shifts in teaching practices within the experimental group. Both ET1 and ET2 reported increased motivation, confidence, and enthusiasm for teaching geometry after participating in the CAR process. ET1's comment, *"I gave them such tasks so that they look for the basic figures they know through different situations and activities, to learn to watch,"* reflects this shift toward a more interactive and exploratory approach (Duval 2017, Tumova & Vondrova, 2017) emphasizing the value of hands-on learning. This approach aligns with findings that emphasize the importance of mathematical task knowledge for teaching, where well-designed tasks stimulate exploration and reasoning (Antunović & Baranović, 2023; Duval, 2017; Schoenfeld, 2019).

Similarly, ET2 recognized the potential of action research in professional development, particularly in addressing student difficulties in areas beyond geometry, such as reading literacy. She stated, *"Yes, definitely it would be useful... if there is a problem with how to convey it to them... why the problem is in my methods or is it really a problem that they are not mature or is something else a problem."* This highlights ET2's expanded understanding of how action research can help identify teaching challenges across various subjects and improve overall instructional effectiveness (Yuan & Lee, 2015; Bruce et al., 2011).

The transition from formulaic methods to inquiry-based, student-centred approaches was pivotal in improving both teacher practices and student outcomes. Teachers in the experimental group emphasized the use of hands-on tasks and real-life applications of geometry, fostering a more dynamic classroom environment. Such strategies are supported by research showing that collaborative action research fosters professional growth and the adoption of innovative, evidence-based teaching methods (Wright, 2021; Russo et al., 2020).

In contrast, the control group way of teaching was mostly textbook-driven learning, lecture-based instructions with limited students' interactions and their students did not demonstrate the same level of improvement. This lack of significant progress highlights the importance of implementing research findings and theoretical frameworks to guide innovative teaching strategies, such as those promoted by CAR, which enhance teacher effectiveness, professional reflection, and student engagement (Aquilina et al., 2024; Vula, 2013).

Impact of CAR on student learning outcomes (RQ2)

The second research question explored the influence of CAR on student learning outcomes in geometry. The results revealed that students in the experimental group made substantial progress in their geometric understanding, with 52.5% showing positive changes in Van Hiele levels and 57.5% in Geometric Figure Apprehension (GFA). This positive progression highlights the effectiveness of the DGT program in enhancing students' ability to reason geometrically, as evidenced by both quantitative and qualitative data. These findings align with Duval's (2017) emphasis on the integration of multiple types of cognitive apprehension and van Hiele's (1986) hierarchical model of geometric reasoning, both of which stress the importance of visualization, logical reasoning, and problem-solving in geometry education.

Students in the experimental group showed increased engagement and participation in hands-on, inquiry-based tasks, supporting the notion that active, student-centred learning fosters meaningful engagement (Boaler & Brodie, 2004; Schoenfeld, 2019). The hands-on approach encouraged students to explore geometric problems, reason through solutions, and develop deeper conceptual understanding, reflecting recent findings on the role of inquiry-based and collaborative learning in enhancing mathematical problem-solving skills (Russo et al., 2020; Vula, 2013).

In contrast, students in the control group also showed some progress, but their improvements were less pronounced. With 34.2% showing positive changes in Van Hiele levels and 21.1% in GFA, their learning outcomes were comparatively limited. These results suggest that the teaching methods used by CT, primarily textbook-driven and lecture-based, were less effective in fostering deep geometric understanding. This disparity underscores the potential of CAR, as implemented in the DGT program, to significantly enhance student learning outcomes. By integrating reflective practices and evidence-based strategies, CAR provides a framework for promoting the higher-order thinking and engagement needed for effective geometry education (Aquilina et al., 2024; Yuan & Lee, 2015).

CONCLUSION

The findings of this study demonstrate the transformative impact of CAR on both teacher practices and student learning outcomes in geometry education. The iterative and reflective nature of CAR enabled teachers to transition from lecture-based methods to dynamic, student-centred strategies, leading to significant improvements in student engagement, geometric understanding, and problem-solving skills. Grounded in van Hiele and Duval's frameworks, the DGT program effectively bridged the gap between research and practice, enhancing teacher efficacy and improving students' conceptual grasp of geometry (Antunović, 2024).

This study illustrates CAR as a powerful yet underutilized tool among mathematics educators in Croatia. Despite its proven potential to integrate evidence-based practices into everyday teaching, CAR has not yet gained widespread recognition as a professional development strategy. Geometry, with its emphasis on visualization, logical reasoning, and abstract thinking, offers a particularly effective context for CAR's iterative cycles of planning, action, observation, and reflection. By addressing instructional challenges and fostering teacher growth, CAR advances both professional development and student outcomes, offering a model for bridging the gap between research and classroom practice (Schoenfeld, 2019; McNiff & Whitehead, 2011).

Moreover, the results highlight the broader applicability of the CAR model. Teachers recognized its value in addressing instructional challenges, underscoring its potential for adaptation across other mathematical domains and educational contexts. Building on research that emphasizes collaborative professional development as a driver of educational improvement (Russo et al., 2020; Aquilina et al., 2024), CAR promotes reflective teaching, innovative problem-solving, and sustainable educational progress.

By fostering collaboration, continuous professional development, and evidence-based strategies, CAR provides a robust framework for enhancing educational quality. This process enriches both teachers and students, fostering a supportive and enthusiastic learning environment. Importantly, CAR builds mutual trust not only between teachers and students but also between researchers and teachers, strengthening the critical researcher-teacher-student triangle. This trust creates a positive cycle of collaboration and reflection, where teaching and learning continuously evolve and improve. As Croatia seeks to advance its educational research and practice, CAR emerges as a cornerstone for future initiatives, bridging the divide between theory and practice while inspiring new directions in teaching and learning.

Limitation

This study has limitations related to sampling, test design, and program implementation. The purposive sampling of teachers, particularly in the experimental group, may introduce bias, and limiting generalizability. However, the statistical insignificance between pre-test results of both groups suggests a balanced starting point. Logistical challenges during the pandemic led to the merging of two tests, affecting reliability but also enhancing evaluation by introducing subcategories for van Hiele levels and GFA. Disruptions in the DGT program schedule may have influenced consistency, though they provided opportunities for reflection and adaptation. Reducing teachers' additional workload could improve implementation.

The post-test timing at the end of the school year may have affected student motivation, suggesting future tests be scheduled earlier. Increased teacher collaboration with the researcher may have introduced confounding variables, impacting the experimental group differently. Despite these constraints, the study offers valuable insights into geometry education and cognitive apprehension, informing future research and pedagogical approaches.

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ISTRAŽIVANJE KOJE POVEZUJE TEORIJU I PRAKSU: KOLABORATIVNO AKCIJSKO ISTRAŽIVANJE U POUČAVANJU GEOMETRIJE

Ova studija istražuje potencijal modela kolaborativnog akcijskog istraživanja (CAR) za premošćivanje jaza između istraživanja i prakse u obrazovanju geometrije. Korištenjem programa Razvoj geometrijskog razmišljanja (DGT), CAR je omogućio suradnički proces između učitelja i istraživača kroz iterativne cikluse planiranja, djelovanja, promatranja i refleksije. Istraživanje, provedeno s 88 učenika sedmih razreda i četiri učitelja matematike u Hrvatskoj, imalo je za cilj unaprijediti geometrijsko razmišljanje učenika i njihove vještine rješavanja problema. Model CAR-a preoblikovao je teorijske uvide u praktične strategije poučavanja, što je rezultiralo značajnim poboljšanjima u ishodima učenja učenika i stavovima učitelja. Učitelji su postali refleksivni praktičari, prilagođavajući metode kako bi učinkovito odgovorili na potrebe učenika. Ovi nalazi potvrđuju vrijednost CAR-a kao okvira za profesionalni razvoj, implementaciju rezultata istraživanja te instruktivne inovacije u matematičkom obrazovanju.

Ključne riječi: akcijsko istraživanje, suradnja, geometrijsko obrazovanje, geometrijsko mišljenje, stavovi nastavnika