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GIANT RESONANCES: NEW GLIMPSE AT COLLECTIVE DYNAMICS IN THE CONTINUUM

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From the general theory of resonance nuclear reactions we derive the approach for the description of giant resonances as emerging out of the interplay of two types of collectivity: internal coupling by the residual interaction and external coupling via common decay channels. Implications for the observable giant resonance pattern are discussed. In particular, the structure at the unperturbed 1p-1h energy is predicted with the presumable enhancement for resonances built on the excited states. It is shown how the fragmentation over complicated configurations and the spreading width could be incorporated into the theory.

1. Introduction

Giant resonance (GR) in nuclei are considered usually as a typical example of collective excitations in a many-fermion system[1]. Related collective phenomena are well known in macroscopic Fermi-systems where they are referred to as Landau collisionless zero-sound waves[2]. Microscopically, such excitations are described as coherent superpositions of one particle — one hole (1p-1h) states with

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given total quantum numbers. The conventional way to find the right superposition is to solve[3] the equations of the random phase approximation (RPA), i. e. to diagonalize the residual quasiparticle interaction in the 1p-1h space.

Certainly, the RPA does not give an exact solution for the many-body problem. It neglects the corrections due to admixture of complicated many particle — many hole configurations. Such contributions are usually incoherent and give rise to the non-collective background in the proper GR wave function. In the time — dependent picture, it means that the initial 1p-1h wave packet acquires, through the collision-like processes, complicated shell model components. Therefore, it is natural to speak on the damping of GR the corresponding energy uncertaincy being called the spreading width $\Gamma^{\downarrow}$. In a finite system, as a nucleus, with the discrete energy spectrum, one could expect to see in a high resolution experiment the fine structure of GR fragmented over the energy interval $\Delta E \sim \Gamma^{\downarrow}$ where N levels of complex nature are located, $N \sim \Delta E/D$ (D is an average level spacing), and each of them carries a certain amount of the collective strength. The structure of GR is known in light nuclei (see, for example, recent electronuclear experiments[4]). As for heavier nuclei, here one observes, as a rule, only broad bumps enveloping the underlying structure.

In the preceding paragraph, we did not mention the obvious, although extremely important, fact that all excited nuclear levels belong to the continuum. They have a finite lifetime and what we really see is their excitation and deexcitation through various reaction channels. The discrete character of the spectrum is the approximation valid for well separated levels $|n\rangle$ with the spacing D large in comparison with the level width $\Gamma_n^{\uparrow}$ connected with the irreversible decay to the continuum (the escape width for the particle emission channels or the radiation width). But at the GR energy the typical escape widths $\Gamma^{\uparrow}_{1p-1h}$ of 1p-1h excitations are comparable with D_{1p-1h} so that the coupling to the continuum cannot be treated as a perturbation broadening slightly the isolated levels.

A number of practical methods were developed to take into account the continuum effects for the GR. We just mention the most advanced ones: shell model in the continuum[5], RPA in the continuum[6], finite Fermi-system theory[7], resonant RPA[8]. All of them imply elaborate and time-consuming numerical calculations. Therefore, some general features of the dynamics turn out to be hidden by the tedious formalism and various procedures for optimizing parameters.

In what follows, we show, using oversimplified models based on the general dynamical principles, that the strong coupling to the continuum results in the new collective behaviour, namely the synchronization of decaying states through the common decay channels. The analogous cooperative phenomenon is known in optics. The interaction of N two-level atoms (or even classical dipole radiators), confined in a volume of the size small as compared to the radiation wavelength, creates, through the common radiation field, the broad coherent state (the Dicke state[9]) with the width $\Gamma \sim N\gamma$ which is N times larger than the single atom width γ.

We have shown earlier[10,11] that such short-lived quantum states are generated by the overlapped intrinsic levels ($\Gamma^{\uparrow} \gtrsim D$) as a result of their interaction via the continuum. They are not destroyed by the chaotic coupling to the incoherent background although the latter influences the energy behaviour of the cross sections. Thus, the observable pattern of GR is determined by the competition of two types of collectivity: the standard collectivization of strengths (internal coupling) and the collectivization of widths (external coupling) plus the spreading due to the damping into complex configurations.

We start with the general scheme for the description of the resonances embedded in the continuum.

2. *Effective Hamiltonian and collectivity via continuum*

We consider an open quantum system which has N intrinsic states $|n\rangle$, $n = 1, \ldots, N$, which can be excited and deexcited through k open channels $a = 1, \ldots, k$. The general theory of resonance reactions[5,12,13] allows to eliminate the channel variables and to derive the effective Hamiltonian $\mathscr{H}$ describing the unstable intermediate states. The price one has to pay for such an elimination is that the operator $\mathscr{H}$ is to be non-hermitian.

Let intrinsic dynamics be governed by a hermitian Hamiltonian including a mean field and a residual interaction. The diagonalization of this matrix gives stationary eigenvectors and real energy eigenvalues. Now, let these states have an access to the decay channels. The external coupling can be described by the $N \times k$ matrix $\mathbf{A} = \{A^a_n\}$ of the decay amplitudes connecting the intrinsic state $|n\rangle$ to the reaction channel a. For sake of simplicity, we neglect the energy dependence of these amplitudes assuming that it is smooth within the energy interval ΔE and we are far away from the channel thresholds. Both assumptions could be checked and lifted if necessary. To make main ideas more transparent, we don't take into account the channel coupling and the potential scattering.

The scattering matrix for the $a \to b$ process at the energy E can be written down as

$$S^{ba}(E) = \delta^{ba} - \mathrm{i}T^{ba}(E) \tag{2.1a}$$

$$= \delta^{ba} - \mathrm{i} \sum_{mn} A^{b*}_m \mathscr{G}_{mn}(E) A^a_n. \tag{2.1b}$$

Here the energy dependence comes from the propagator $\mathscr{G}(\mathscr{E})$ of the intermediate unstable system, defined as the function of the complex energy variable $\mathscr{E}$,

$$\mathscr{G}(\mathscr{E}) = \frac{1}{\mathscr{E} - \mathscr{H}}, \tag{2.2}$$

with the poles at the points

$$\mathscr{E}_n = E_n - \frac{\mathrm{i}}{2} \Gamma_n \tag{2.3}$$

of the eigenvalues of the effective non-hermitian Hamiltonian

$$\mathscr{H} = H - \frac{\mathrm{i}}{2} W \tag{2.4}$$

where $N \times N$ matrices H and W are both hermitian. The complex eigenvalues (2.3) give energies E_n and widths Γ_n of the unstable states with the pure exponential time dependence $\sim \exp\left(-\frac{i}{\hbar}\mathscr{E}_n t\right)$.

The point is that the unitarity condition* $\hat{S}^{+}\hat{S} = \hat{S}\hat{S}^{+} = \hat{1}$ determines uniquely the antihermitian part of the total effective Hamiltonian (2.4) in terms of the same amplitudes which have appeared in the definition (2.1b):

$$W_{mn} = \sum_{a=1}^{k} A_m^a A_n^{a*}. \tag{2.5}$$

For small k, the separable structure of W gives rise to remarkable universal features of resonance dynamics.

At weak external coupling, W is a perturbation for the intrinsic Hamiltonian H. Therefore the off-diagonal matrix elements of (2.5) are of minor importance whereas the diagonal ones, $W_{nn} = \sum_a |A_n^a|^2$, coincide with the small widths Γ_n of the isolated narrow resonances. In the opposite limit, W becomes the part of the total Hamiltonian dominating the dynamics. Then the off-diagonal elements of W couple strongly the intrinsic states through the continuum. The border between the two regimes should lie at $\Gamma \sim D$ when the whole set of intrinsic states is involved in the coupling through common decay channels so that one expects here a sharp percolation — like phase transition. Indeed, it has been observed in the numerical calculations for the nuclear[14] and spin dissipative[15] systems; its general nature was formulated by the authors[10,11].

The rank of the matrix (2.5) coincides with the number k of open channels. Therefore W cannot have more than k nonzero eigenvalues; they are positive and give, in the strong coupling limit, widths of short-lived states. For the one-channel case, the only non-zero eigenvalue Γ_1 of W is equal to its trace,

$$\Gamma_1 = \mathrm{Tr}\, W = \sum_a |A_n|^2 \equiv w. \tag{2.6}$$

The corresponding eigenstate $|1\rangle$ is just the Dicke state mentioned in Introduction. For $k \ll N$ open channels, k states share the total width w (2.6).

Hence, taking into account the full Hamiltonian (2.4), we will have in the strong coupling limit the coexistence of k broad and $N - k$ narrow resonances. The collectivization of widths forms special wave packets which are fitted to the common decay channels and scatter from the system by means of fast »direct« processes; remaining states become more long — lived. The relation of this picture to the conventional concepts of the nuclear reaction theory (compound nucleus, doorway states, Ericson fluctuations, various representations of the S-matrix and so on) was discussed in detail in Ref. 11.

* Here and below the caret marks the matrices in the $k \times k$ channel space; the symbol det stands for the determinant in the same space.

To complete the formal derivation, it is convenient to express the Green function $\mathcal{G}$ (2.2) for the unstable system in terms of the Green function

$$G(\mathcal{E}) = \frac{1}{\mathcal{E} - H} \tag{2.7}$$

for the intrinsic closed system. Using the separable form (2.5) of W one obtains

$$\mathcal{G}(\mathcal{E}) = G(\mathcal{E}) - \frac{\mathrm{i}}{2} G(\mathcal{E}) \mathbf{A} \frac{1}{1 + \frac{\mathrm{i}}{2} \hat{K}(\mathcal{E})} \mathbf{A}^{+} G(\mathcal{E}). \tag{2.8}$$

The matrix in the channel space

$$\hat{K}(\mathcal{E}) = \mathbf{A}^{+} G(\mathcal{E}) \mathbf{A} \tag{2.9}$$

has poles on the real axis at the points $E = \varepsilon_n$ of eigenvalues of H. Complex eigenvalues $\mathcal{E}_n$ (2.3) of the total effective Hamiltonian $\mathcal{H}$ are the roots of the secular equation, see Eq. (2.8),

$$\det \left[\hat{1} + \frac{\mathrm{i}}{2} \hat{K}(\mathcal{E}) \right] = 0. \tag{2.10}$$

The scattering matrix (2.1) takes now a form

$$\hat{S} = \frac{\hat{1} - \frac{\mathrm{i}}{2} \hat{K}}{\hat{1} + \frac{\mathrm{i}}{2} \hat{K}}, \quad \hat{T} = \frac{\hat{K}}{\hat{1} + \frac{\mathrm{i}}{2} \hat{K}}. \tag{2.11}$$

3. *Giant resonances and two types of collectivity*

With the use of an oversimplified solvable model which still maintains main features of the real situation, we show the interplay of the coherent internal coupling creating the collective GR and the external coupling generating the Dicke resonance. Our model contains intrinsic level energies $\varepsilon_1, \ldots, \varepsilon_N$, presumably of the 1p-1h nature, the residual factorizable interaction of the multipole-multipole type (as in the clasical prototype paper[16]), and the coupling through continuum in the form of Eqs. (2.4) and (2.5):

$$\mathcal{H}_{mn} = H_{mn} - \frac{\mathrm{i}}{2} W_{mn} = \varepsilon_n \delta_{mn} + \lambda\, d_m d_n - \frac{\mathrm{i}}{2} A_m A_n. \tag{3.1}$$

Here we consider the simplest one-channel case (»elastic scattering«). More general problem will be discussed in the next section. The »dipole moments« d_m in Eq. (3.1) can be interpreted as multipole matrix elements $\langle m|d|0\rangle$ for the excitation of the state $|m\rangle$ from the ground state $|0\rangle$; λ is the corresponding coupling constant. For the time-reversal invariant system, the amplitudes d_m and A_m can be chosen as real numbers.

Let us introduce notations

$$\{P(\mathscr{E}), Q(\mathscr{E}), R(\mathscr{E})\} = \sum_n \frac{\{d_n^2, A_n d_n, A_n^2\}}{\mathscr{E} - \varepsilon_n} \tag{3.2}$$

for the polarization loops generated by the two types of interaction (3.1). The Green function (2.7) for the intrinsic system is easily found to be

$$G_{mn}(\mathscr{E}) = \frac{\delta_{mn}}{\mathscr{E} - \varepsilon_m} + \frac{\lambda}{1 - \lambda P(\mathscr{E})} \frac{d_m d_n}{(\mathscr{E} - \varepsilon_m)(\mathscr{E} - \varepsilon_n)}. \tag{3.3}$$

Using Eqs. (2.9), (3.2) and (3.3) one gets

$$K(\mathscr{E}) = \sum_{mn} A_m G_{mn}(\mathscr{E}) A_n = R(\mathscr{E}) + \frac{\lambda}{1 - \lambda P(\mathscr{E})} Q^2(\mathscr{E}) \tag{3.4}$$

so that the secular equation (2.10) reads

$$\Lambda(\mathscr{E}) \equiv [1 - \lambda P(\mathscr{E})] \left[1 + \frac{\mathrm{i}}{2} R(\mathscr{E})\right] + \frac{\mathrm{i}}{2} \lambda Q^2(\mathscr{E}) = 0, \tag{3.5}$$

and the scattering amplitude (2.11) is

$$T(E) = \Lambda^{-1}(E)\{[1 - \lambda P(E)] R(E) + \lambda Q^2(E)\}. \tag{3.6}$$

For a stable system ($A_m \to 0$), Eq. (3.5) converts into the textbook RPA equation

$$1 - \lambda P(E) = 0 \tag{3.7}$$

which gives one collective root at $E = E_{coll}$ shifted from the interval ΔE, where the original levels ε_n were located, up or down for positive or negative values of λ, respectively. The shifted collective state absorbs almost the total dipole strengths but it is stationary and has no decay width. Remaining $N - 1$ levels carry no collectivity. Their energies are slightly displaced within the spacing between two neighbouring energies ε_n.

In an open system without intrinsic collectivity ($\lambda \to 0$), the secular equation takes the form

$$1 + \frac{\mathrm{i}}{2} R(\mathscr{E}) = 0, \tag{3.8}$$

in agreement with Eq. (2.10). The collective solution of Eq. (3.8) is the Dicke resonance shifted, similar to the collective root of Eq. (3.7) but along the imaginary axis. In the case of strong external coupling, this shift is almost equal to the total summarized width w, Eq. (2.6).

For the general case (3.5) the result depends on the N-dimensional vectors $\mathbf{A} = \{A_n\}$ and $\mathbf{d} = \{d_n\}$. If internal and external coupling are both strong as compared to the energy splitting, one can consider the degenerate problem ($\varepsilon_n = \varepsilon$). Then the usual GR (3.7) is located at

$$E_{coll} = \varepsilon + \lambda \mathbf{d}^2 \tag{3.9}$$

whereas the Dicke resonance (3.8) is displaced to the complex point

$$\mathscr{E}_D = \varepsilon - \frac{i}{2}\mathbf{A}^2 = \varepsilon - \frac{i}{2} w. \tag{3.10}$$

The scattering amplitude (3.6) contains two resonances and their interference is dependent on the mutual orientation of $\mathbf{A}$ and $\mathbf{d}$,

$$T(E) = \frac{(E - E_{coll})\, w + \lambda (\mathbf{A} \cdot \mathbf{d})^2}{(E - E_{coll})(E - \mathscr{E}_D) + \frac{i}{2} \lambda (\mathbf{A} \cdot \mathbf{d})^2}. \tag{3.11}$$

If the excitation from the ground state ($\sim\mathbf{d}$) and the decay ($\sim\mathbf{A}$) correspond to completely »orthogonal« mechanisms, $(\mathbf{A} \cdot \mathbf{d}) = 0$, an experiment shows the non-shifted Dicke resonance only,

$$T_{\perp}(E) = \frac{w}{E - \mathscr{E}_D} = \frac{w}{E - \varepsilon + \frac{i}{2} w}, \tag{3.12}$$

the GR at $E = E_{coll}$ is decoupled from the continuum. In the opposite, »parallel« situation, $\mathbf{A} = \text{const} \cdot \mathbf{d}$, the Dicke GR combining both types of collectivity (shift $\lambda \mathbf{d}^2$ and width w) arises,

$$T_{\parallel}(E) = \frac{w}{E - E_{coll} + \frac{i}{2} w}. \tag{3.13}$$

The rest of states at $E = \varepsilon$ have neither shift nor width. Both limiting cases (3.12) and (3.13) lead to the broad ($\Gamma = w$) Breit-Wigner resonance at $E = E_{coll}$ or at $E = \varepsilon$ for the parallel or perpendicular cases, respectively.

At intermediate values of the angle Θ between $\mathbf{A}$ and $\mathbf{d}$ the scattering amplitude has two resonance peaks sharing the total width. Eq. (3.4) gives

$$K(E) = w \left(\frac{\sin^2 \Theta}{E - \varepsilon} + \frac{\cos^2 \Theta}{E - E_{coll}} \right). \tag{3.14}$$

The specific feature of the one-channel case is that the function (3.14) vanishes at the energy

$$E_0 = \varepsilon + \lambda d^2 \sin^2 \Theta \tag{3.15}$$

between ε and E_{coll}. The corresponding scattering amplitude (2.11),

$$T(E) = \frac{(E - E_0)\, w}{(E - \varepsilon)(E - E_{coll}) + \frac{i}{2}(E - E_0)\, w}, \tag{3.16}$$

has two complex poles. The elastic cross section $\sigma = |T|^2$ reveals two equally high maxima, $\sigma = 4$, at $E = \varepsilon$ and $E = E_{coll}$ and drops to zero at $E = E_0$. In the limit of very strong internal coupling, $w \ll \lambda \mathbf{d}^2$, the maxima have Breit-Wigner shapes and widths $w \sin^2 \Theta$ and $w \cos^2 \Theta$ for the non-shifted peak and for the shifted one, respectively.

Thus, the interplay of two types of collectivity redistributes significantly observed strengths and widths in the GR region as compared with the picture based exclusively on the internal coupling. Even in the degenerate case (for example, in the spherical harmonic oscillator field) there should be, apart from the shifted GR, the non-shifted peak in the vicinity of the unperturbed 1p-1h energy for a given multipolarity (1 $\hbar\omega$ for the electric dipole). But the cross section pattern will be different as observed in different reaction channels. It will be illustrated in the following section.

4. *Two — channel model*

Here we consider the model of Sect. 3 with two open channels: »nucleon« (N) and »gamma«. Correspondingly, we can observe (N, N), (γ, γ), (γ, N) and inverse (N, γ) reactions. We define the γ-channel assuming that the amplitude vector $\mathbf{A}^{(\gamma)} = \{A_m^{(\gamma)}\}$ is generated by the radiation of the same multipolarity as the internal dipole vector $\mathbf{d}$ so that these two vectors are parallel,

$$\mathbf{A}^{(\gamma)} = \sqrt{a}\, \mathbf{d}, \tag{4.1}$$

with the coupling constant a measuring the strength of the electromagnetic interaction.

For the degenerate case we put the origin of the energy scale in the point $E = \varepsilon$ of intrinsic levels. Using the dimensionless variables (x is real and z is complex)

$$x = \frac{E}{w}, \quad z = \frac{\mathscr{E}}{w}, \qquad x_c = \frac{\lambda \mathbf{d}^2}{w} \tag{4.2}$$

we rewrite the one-channel secular equation (see the denominator of Eq. (3.11)) as

$$(z - x_c)\left(z + \frac{i}{2}\right) + \frac{i}{2} x_c \cos^2 \Theta = 0. \tag{4.3}$$

As it is clear from (3.1) and (4.1), the whole two-channel problem reduces to the one-channel one by substituting $\lambda \to \lambda - \frac{i}{2} a$ or $x_c \to x_c - \frac{i}{2} \delta$, $\quad \delta = a\, \mathbf{d}^2/w$. Then one gets for the scattering amplitudes, Eqs. (2.9) and (2.11),

$$\begin{pmatrix} T^{NN}(x) \\ xT^{\gamma N}(x) \\ xT^{\gamma\gamma}(x) \end{pmatrix} = \frac{1}{\Lambda(x)} \begin{Bmatrix} x - \left(x_c - \frac{i}{2}\delta\right) \sin^2 \Theta \\ \sqrt{\delta}\, x \cos \Theta \\ \delta \left(x + \frac{i}{2} \sin^2 \Theta\right) \end{Bmatrix}, \tag{4.4a}$$

$$\Lambda(x) = x(x - x_c) - \frac{1}{4} \delta \sin^2 \Theta + \frac{i}{2} [(1 + \delta)\, x - x_c \sin^2 \Theta]. \tag{4.4b}$$

Figs. 1—3 show the (N, N),(γ, N) and (γ, γ) cross sections as functions of energy for $\cos^2 \Theta = 0.9$, 0.5 and 0.1. Here we took equal strengths of two coup-

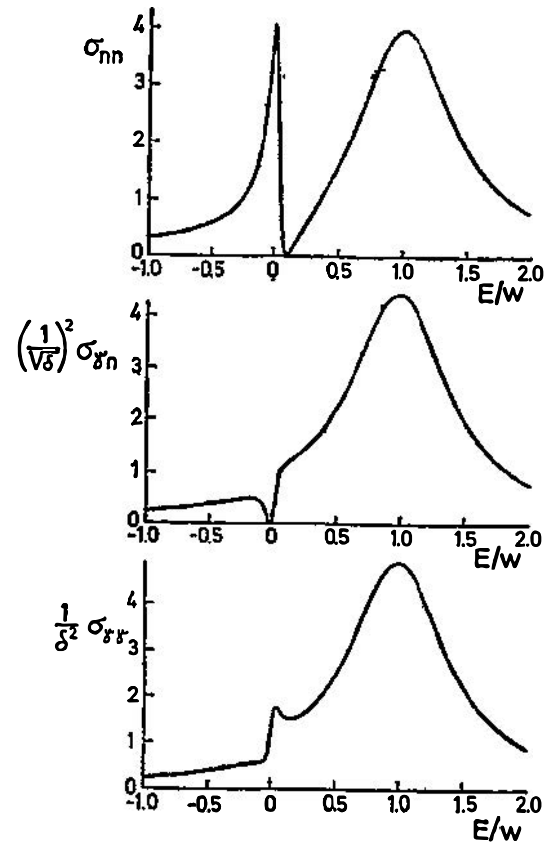


Fig. 1. Cross sections for different reaction channels as functions of energy for the degenerate intrinsic levels at $\varepsilon = 0$; $\cos^2 \Theta = 0.9$.

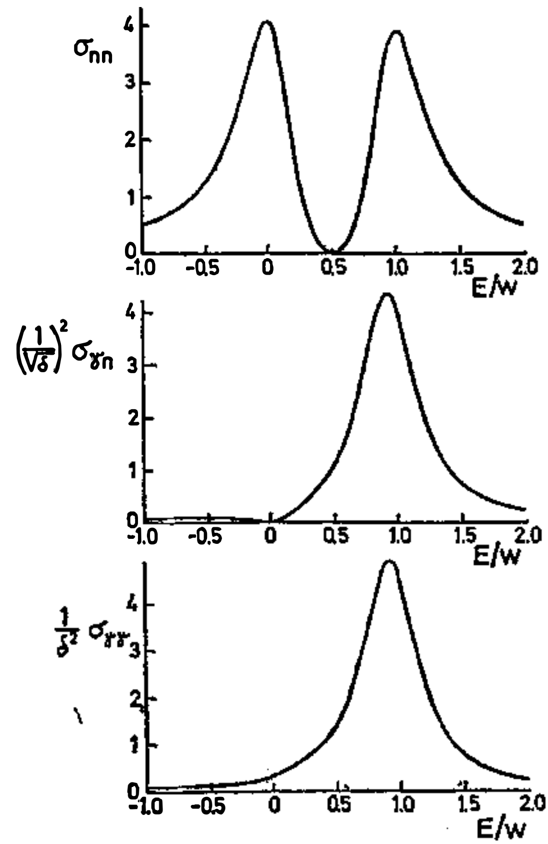


Fig. 2. The same as Fig. 1; $\cos^2 \Theta = 0.5$.

lings, $x_c = 1$. In the (N, N) scattering one can see the redistribution of widths between two peaks determined by the angle Θ. In the photonuclear reactions, with Θ increasing, the dipole strength is accumulated by the GR peak and its width decreases up to zero at $\Theta = \frac{\pi}{2}$. Since

$$K^{\gamma N} = \sqrt{a} \frac{Q}{1 - \lambda P} \overset{\text{deg.}}{\rightarrow} \sqrt{\delta} \frac{\cos \Theta}{x - x_c}, \tag{4.5a}$$

$$K^{\gamma\gamma} = a \frac{P}{1 - \lambda P} \overset{\text{deg.}}{\rightarrow} \frac{\delta}{x - x_c}, \tag{4.5b}$$

in the degenerate case there is no strong resonances in photonuclear processes at the non-shifted energy ($x \approx 0$).

Taking into account the energy spread of intrinsic levels brings an additional structure in the non-shifted region. It is particularly pronounced for photonuclear reactions.

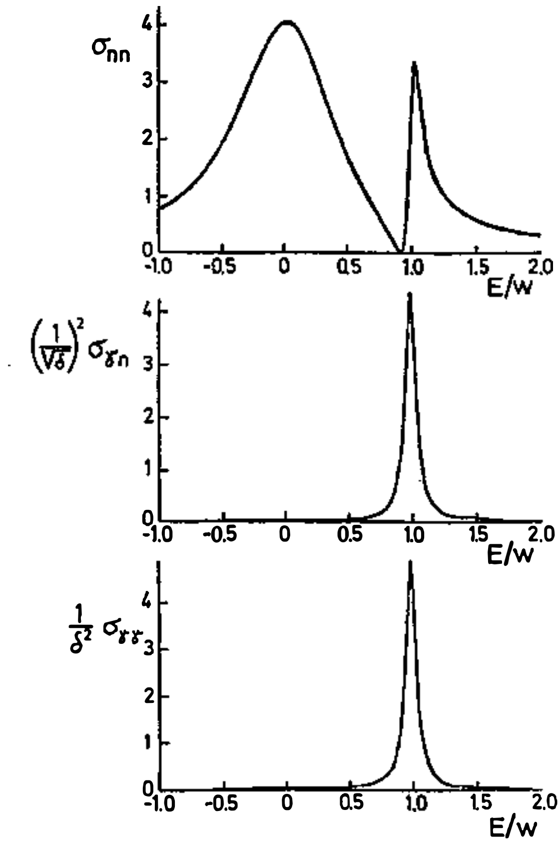


Fig. 3. The same as Fig 1; $\cos^2 \Theta = 0.1$.

Fig. 4 presents the cross sections for the case of five equidistant levels equally coupled to the nucleon continuum (the summarized nucleon width is w). The angle Θ is chosen to be 45°.

Our schematic analysis is based on very general grounds. Therefore, any realistic calculation taking into account continuum effects is expected to reveal the predicted structure. The low-lying component of the GR structure is well known in light nuclei, with a small number of significant open channels. Fig. 5 shows such a structure in the $^{16}O(\gamma, p_0)$ reaction[17] at the vicinity of the only shell model transition $1p_{1/2} \rightarrow 1d_{5/2}$ contributing to the (γ, p_0) channel. The similar structure is present in the (γ, n_0) channel as well as in the electroexcitation[4]. Qualitatively, the data agree with the RPA calculations in the continuum (see, for example, Ref. 18). We believe that it can be treated as a manifestation of the coupling via continuum.

Heavy ion reactions give a possibility to study collective excitations, including GR, built on excited (»hot«) nuclear states. For a parent state in the region of high level density, we can expect the more pronounced role of the external coupling.

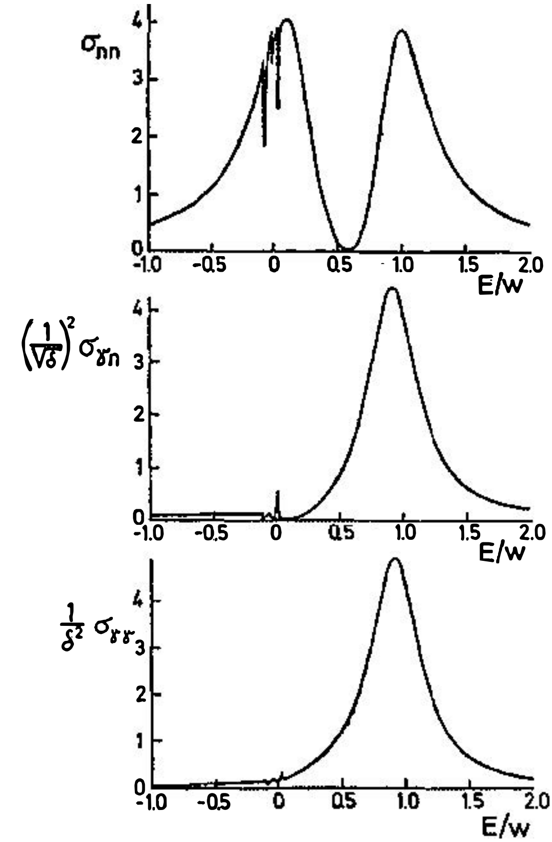


Fig. 4. The same as Fig. 1 for 5 intrinsic levels ε_n distributed equidistantly from $E/w = -0.5$ to $E/w = 0.5$; $\cos^2 \Theta = 0.5$.

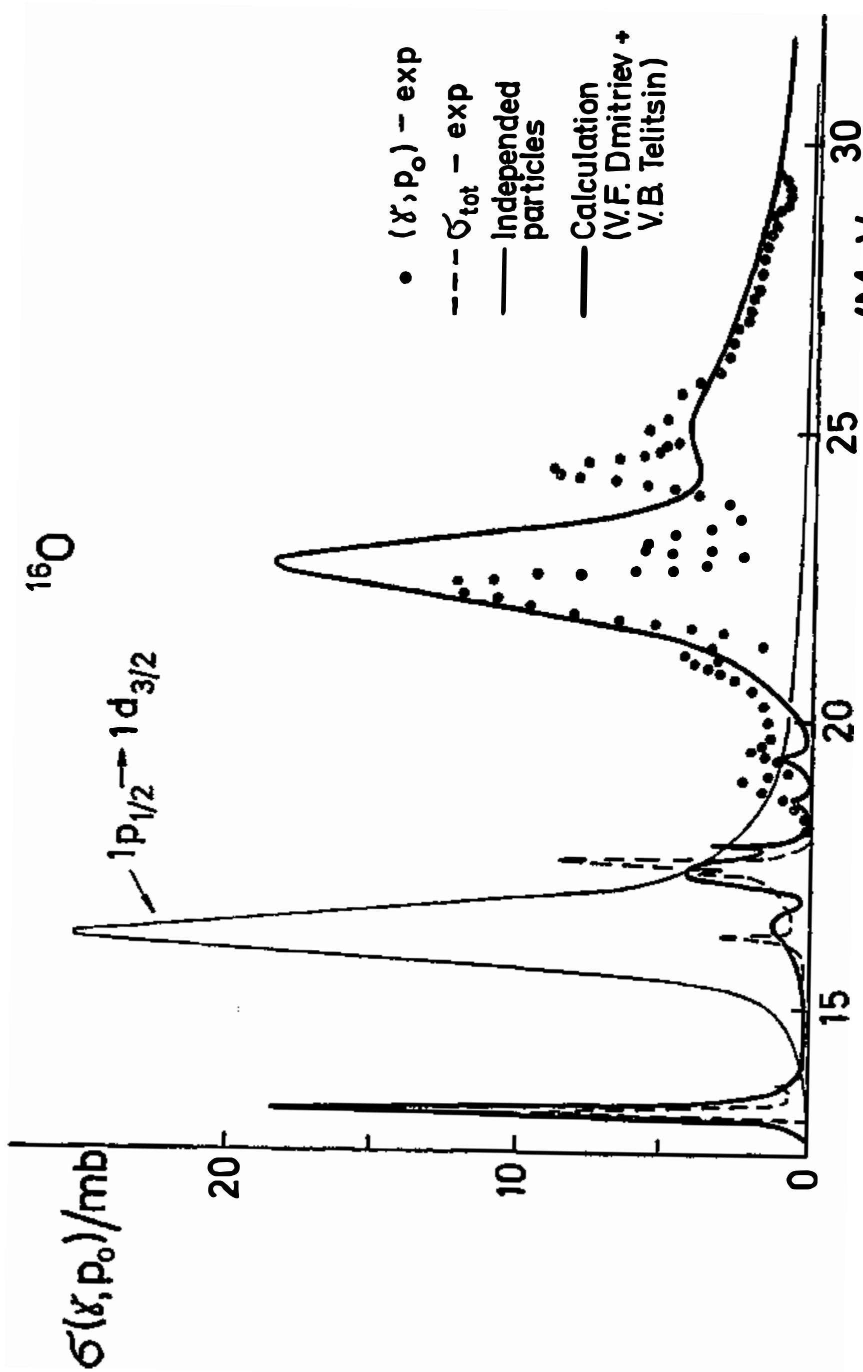
There is some evidence[19] of the restructuring of the GR picture with the temperature increasing, in favour of the low-lying component at $E \approx 1\hbar\omega$.

In our model Hamiltonian (3.1) we neglected the modification of the real part H due to the virtual coupling to the continuum. These corrections depend on the same amplitudes and might give rise to the additional displacement of the resonances along the real axis.

5. *The spreading width*

The approach of Sect. 3 can be generalized to include, in addition to the collective internal coupling imitated by the dipole-dipole term in Eq. (3.1), the incoherent interaction with noncollective degrees of freedom.

In the simplest case, we subdivide the Hilbert space, in the energy range under study, into two classes. States $|n\rangle$ of the class 1 of the dimensionality N_1


^{16}O
$1p_{1/2} \rightarrow 1d_{3/2}$
(γ, p_o) – exp
σ_{tot} – exp
Independed particles
Calculation (V.F. Dmitriev + V.B. Telitsin)
$\sigma(\gamma, p_o)$/mb
ω/MeV
20
10
0
15
20
25
30

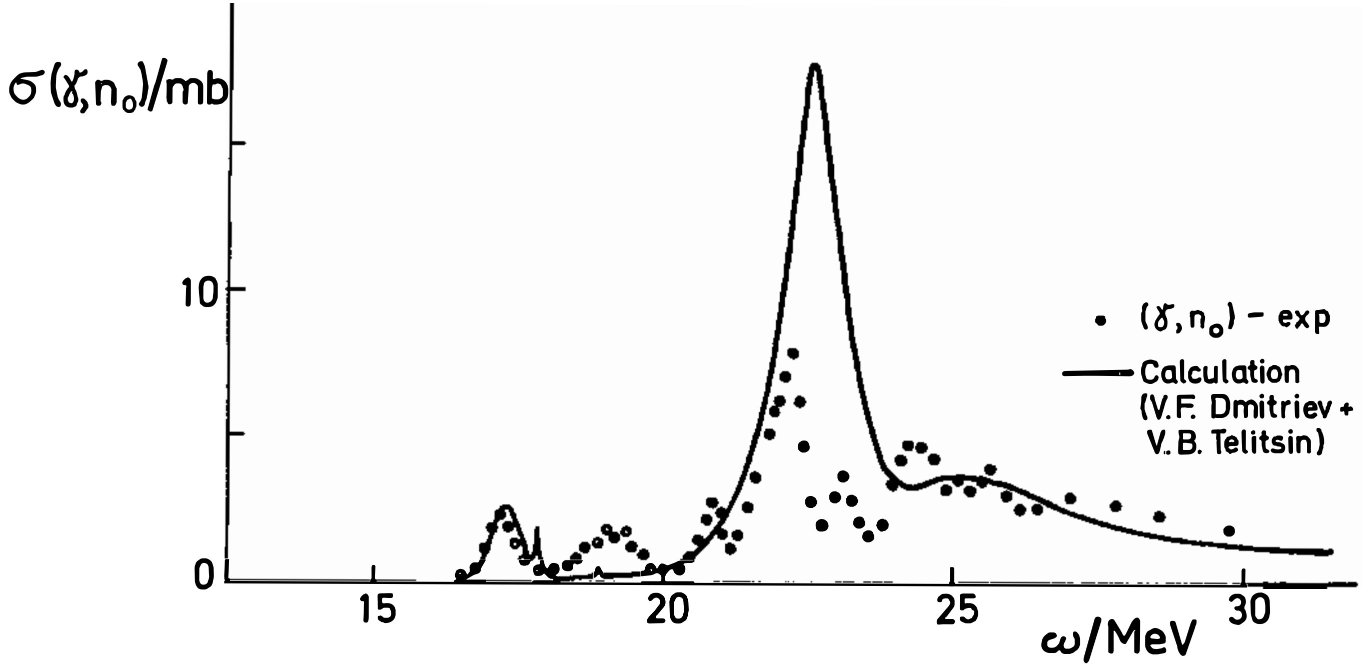


Fig. 5. The $^{16}O\ (\gamma, p_0)$ and $^{16}O\ (\gamma, n_0)$ cross sections (shown by dots) and the total photoabsorption cross section (dashed line); solid line — calculations[18] using RPA in continuum; the peak $1p_{1/2} \rightarrow 1d_{3/2}$ corresponds to the continuum shell model calculations. ($1\ \mathrm{mb} = 10^{-31}\ \mathrm{m}^2$)

are just those considered explicitly in preceding sections. Remaining states $|\mu\rangle$ will be referred to as belonging to the class 2. As a model for such a segregation, one can imagine 1p-1h states (class 1) and many particle — many hole states (class 2). The second class can, in turn, be subdivided further as in the exciton model of preequilibrium processes. One can as well invent alternative ways for constructing the state hierarchy based on other physical grounds.

Independently of details of the subdivision, the total internal hermitian Hamiltonian is now, using obvious notations,

$$H = \begin{pmatrix} H^{(1)} & V \\ V^{+} & H^{(2)} \end{pmatrix} = \begin{pmatrix} \{H^{(1)}_{mn}\} & \{V_{m\nu}\} \\ \{V_{\mu n}\} & \{H^{(2)}_{\mu\nu}\} \end{pmatrix}. \tag{5.1}$$

In accordance with the concept of incoherent interaction, we assume that coupling matrix elements $V_{\mu n}$ are more or less random with the average properties

$$\langle V_{\mu n}\rangle = 0, \quad \langle V_{\mu n} V_{\mu' n'}\rangle = \frac{v^2}{4N_2}\,\delta_{\mu\mu'}\,\delta_{nn'}. \tag{5.2}$$

Here N_2 is the dimensionality of the class 2 (eventually we will go to the limit $N_2 \to \infty$), and the scale factor in Eq. (5.2) is chosen to be consistent with the idea that complicated states $|\mu\rangle$ have, in the simple p-h (or RPA) basis, $N_2 \gg 1$ components with random amplitudes of typical magnitude $\sim 1/\sqrt{N_2}$. According to such a definition, the mean squared matrix element inside the subspace 1,

$$\langle (V^2)_{mn}\rangle = \sum_{\mu} \langle V_{m\mu} V_{\mu n}\rangle = \frac{v^2}{4}\,\delta_{mn}, \tag{5.3}$$

is finite in the limit of large N_2.

For the total Green function of the Hamiltonian (5.1) one has the Dyson equations

$$G_{mn} = G^{(1)}_{mn} + \sum_{pq;\mu\nu} G^{(1)}_{mp} V_{p\mu} G_{\mu\nu} V_{\nu q} G_{qn}, \tag{5.4a}$$

$$G_{\mu\nu} = G^{(2)}_{\mu\nu} + \sum_{pq;\sigma\tau} G^{(2)}_{\mu\sigma} V_{\sigma p} G_{pq} V_{q\tau} G_{\tau\nu} \tag{5.4b}$$

where, obviously,

$$G = \frac{1}{E - H}, \quad G^{(1,2)} = \frac{1}{E - H^{(1,2)}}. \tag{4.5}$$

In the limit of large N_2 the main contribution to the average Green function (5.4a) come from concentrical pairwise contractions of the random matrix elements. Such terms bring in the maximum number of traces over the space 2 which compensates the smallness of the variance (5.2). We denote average Green functions as $g^{(1)}$ and $g^{(2)}$ having in mind the matrices (5.4a) and (5.4b) respectively; matrix elements of G between the two classes vanish after averaging. In addition, we introduce the traces of $g^{(1)}$ and $g^{(2)}$,

$$t_1 = \frac{1}{N_1} \sum_m g_{mm}^{(1)}, \quad t_2 = \frac{1}{N_2} \sum_\mu g_{\mu\mu}^{(2)}. \tag{5.6}$$

Then the average of Eqs. (5.4a) and (5.4b) gives

$$g^{(1)} = G^{(1)} + \frac{v^2}{4} t_2 \, G^{(1)} g^{(1)}, \tag{5.7a}$$

$$g^{(2)} = G^{(2)} + \frac{v^2}{4} t_1 \frac{N_1}{N_2} G^{(2)} g^{(2)}, \tag{5.7b}$$

and, finally, we get a solution in the form of the unperturbed Green function taken at shifted value of the energy argument:

$$g^{(1)} = G^{(1)} \left(E - \frac{v^2}{4} t_2 (E)\right), \quad g^{(2)} = G^{(2)} \left(E - \frac{v^2}{4} \frac{N_1}{N_2} t_1 (E)\right). \tag{5.8}$$

It is very easy to extend this procedure for a hierarchy of states consisting of more than two classes.

If the original energies of the states of two classes are not overlapped, Eqs. (5.8) describe a weak mixing. Each Green function acquires new remote poles with small residues and the level repulsion increases slightly the spacing between classes. In such a way, one can calculate the partial depletion of occupation numbers for the nucleon orbits under the Fermi surface. Part of the single-particle strength goes to complicated configurations which gives rise to the term in the single-particle Green function which is regular near the Fermi surface. Due to the big number N_2, the total depletion could be rather significant. Meanwhile, the distortion of the complicated states is negligible since $N_1/N_2 \ll 1$.

For the problem of GR, we should take into account that states of the class 2 are located in the same energy region. In the continuous limit, $N_2 \to \infty$, the Green functions $G^{(2)}$ and $g^{(2)}$ have, instead of discrete poles, a branch cut along the real axis, and, according to Eq. (5.6),

$$\operatorname{Im} t_2 (E) = - \frac{\pi}{N_2} \varrho_2 (E) \tag{5.9}$$

where $\varrho_2 (E)$ is the level density for the class 2 states, normalized as $\int \varrho_2 (E) \, \mathrm{d}E = N_2$. Introducing

$$\Delta (E) = \frac{v^2}{4} \operatorname{Re} t_2 (E), \quad \Gamma^{\downarrow} (E) = 2\pi \frac{v^2}{4N_2} \varrho_2 (E), \tag{5.10}$$

we obtain the average Green function (5.8)

$$g^{(1)}(E) = G^{(1)}\left(E - \Delta(E) + \frac{i}{2}\Gamma^{\downarrow}(E)\right) \tag{5.11}$$

which is consistent with the definition of the spreading width $\Gamma^{\downarrow}$ according to the golden rule.

In the degenerate one-channel model of Sect. 3, the scattering amplitude (3.16) becomes

$$T(E) = w\,[E - E_0 - \Delta(E) + \frac{i}{2}\Gamma^{\downarrow}(E)]\,\{[E - \varepsilon - \Delta(E) + \\ + \frac{i}{2}\Gamma^{\downarrow}(E)]\,[E - E_{coll} - \Delta(E) + \frac{i}{2}\Gamma^{\downarrow}(E)] + \\ + \frac{i}{2}w\,[E - E_0 - \Delta(E) + \frac{i}{2}\Gamma^{\downarrow}(E)]\}^{-1}. \tag{5.12}$$

In order to avoid too cumbersome expressions, we neglect here the decay amplitudes from the class 2 states. These amplitudes a_μ also should be treated as random variables,

$$\langle a_\mu \rangle = 0, \quad \langle a_\mu a_\nu \rangle = \frac{u^2}{N_2}\,\delta_{\mu\nu}, \tag{5.13}$$

so that the additional contribution to the K-function (3.4) is

$$\delta K(E) = \sum_\mu \langle a_\mu\, g^{(2)}_{\mu\nu}\, a_\nu \rangle = u^2\, t_2(E) \tag{5.14}$$

and could be included into Eq. (5.12) without difficulties. We will not analyze here the expression (5.12) but note that now there is no real energy with the zero amplitude $T(E)$. In the limit of the spreading width exceeding the collective displacement λd^2, Eq. (5.12) gives

$$T(E) \rightarrow \frac{w}{E - \varepsilon - \Delta(E) + \frac{i}{2}\,[w + \Gamma^{\downarrow}(E)]}. \tag{5.15}$$

Thus, for $\Delta(E)$ and $\Gamma^{\downarrow}(E)$ slowly changing with energy, we have a Breit-Wigner peak with the total width $\Gamma = \Gamma^{\downarrow} + w = \Gamma^{\downarrow} + \Gamma^{\uparrow}$. But it is the case only within the range of the class 2 states. Outside that region, $\Gamma^{\downarrow} = 0$ so that $\Gamma = \Gamma^{\uparrow}$. In general, the average cross section has a shape determined by the energy dependence of $\Gamma^{\downarrow}$, i. e. by the level density of complex states.

As a simple model for the class 2, one can use the Gaussian orthogonal ensemble. Here the statistical properties of matrix elements are[20]

$$\langle H^{(2)}_{\mu\nu} H^{(2)}_{\mu'\nu'} \rangle = \frac{\xi^2}{4N_2} (\delta_{\mu\mu'} \delta_{\nu\nu'} + \delta_{\mu\nu'} \delta_{\nu\mu'}). \tag{5.16}$$

It is well known that for $N_2 \to \infty$ the average Green function of the class 2 is

$$G^{(2)}(E) = \begin{cases} \frac{2}{\xi^2} (E - \sqrt{E^2 - \xi^2}), & E^2 > \xi^2, \\ \frac{2}{\xi^2} (E - \mathrm{i} \sqrt{\xi^2 - E^2}), & E^2 < \xi^2. \end{cases} \tag{5.17}$$

Then, assuming $\frac{N_1}{N_2} \ll 1$, we have from (5.8) $g^{(2)} \approx G^{(2)}$, and

$$\Gamma^{\downarrow}(E) = \frac{v^2}{\xi^2} \sqrt{\xi^2 - E^2}\, \Theta(\xi^2 - E^2).$$

This semicircle law gives the specific shape of the cross sections[11].

6. *Conclusion*

Using very simple models keeping the main features of physical reality we have shown that the observed pattern of the cross sections in the GR region results from the interplay of three dynamical effects:

(i) internal collective coupling of the 1p-1h shell model states by the coherent multipole part of the residual quasiparticle interaction;

(ii) external coupling of the intrinsic states through common decay channels;

(iii) internal incoherent coupling with the sea of complicated configurations. The interaction (i) concentrates the multipole strength at some displaced energy creating the collective vibration. The coupling via continuum (ii) generates broad Dicke-type resonances accumulating the decay width $\Gamma^{\uparrow}$. Depending on the energy spread of the interacting states and on the distribution of multipole strengths and decay amplitudes, various pictures of the intermediate structure arise, different in different reaction channels. Damping phenomena (iii) give rise to the irreversible (in the limit $N_2/N_1 \gg 1$, see Eqs. (5.7)) relaxation processes which lead to the fine structure of the GR, or, after averaging, to the spreading width $\Gamma^{\downarrow}$. This stage of the process goes to the equilibrated compound states and can be described by the random matrix technique.

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GIGANTSKE REZONANCE: NOVI POGLED NA KOLEKTIVNU DINAMIKU U KONTINUUMU

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Pomoću opće teorije rezonantnih nuklearnih reakcija gigantske rezonance se opisuju pomoću dva tipa kolektivnosti: internog vezanja rezidualnim interakcijama i vanjskog vezanja zajedničkim kanalima raspada.