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Comparative Analysis of Linear Deformations of Conformal Map Projections for the Territory of Republic of Serbia

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Abstract: In this paper we analyzed absolute linear deformations of conformal map projections for geographic area containing the territory of Serbia. The main goal was to determine optimal conformal projection for analyzed area, based on obtained values of linear scale in various projection variations. Four variants of cylindrical, three variants of conic and two variants of planar projections are included in the analysis. All selected projections were or still are in use for the analyzed area. In most of the previous studies deformations were analyzed for an area bounded by two parallels and two meridians, but in this paper we singled out and analyzed projection deformations that occurred within the boundaries of the mapping area. Based on the conducted analyses we concluded which conformal projection would be optimal choice for analyzed geographic territory, as well as for some other similar areas.

Keywords: conformal map projections; geographic area of Serbia; linear deformations; measures of quality

1 INTRODUCTION

According to type of deformation map projections are divided into conformal, equivalent and equidistant, while, according to position of the projection plane to the Earth's surface they are divided into direct, transversal and oblique. In this paper conformal projections are analyzed, that is the projections which maintain the angles. They are especially important since they are widely adopted and used for land survey, topographic, aerial and marine maps, military applications, etc. The main contributor to such widespread usage is the fact that linear scale in a point does not depend on azimuth, unlike non-conformal projections where linear scale depends not only on the position of the point but on the observed direction (azimuth) in a certain point [1].

Over the years, tens of different conformal map projections have been developed. Most commonly used are cylindrical and conic [2]. Cylindrical projections are mostly used in land survey and large-scale maps, like geodetic plans and topographic maps, while conic projections are used for small-scale maps such as maps for aerial navigation [3]. Also, in order to reduce the number of linear deformations in wider geographic area secant surfaces are used. For instance, Lambert found many variants of projections using his procedure based on differential equations of mapping one surface to the other, but, as he hypothesized, solving general problem leads to infinite number of different map projections [4]. Also, Halley gave geometric proof of conformity of stereographic projections, while Lambert derived this proof using calculus [4, 5].

Other scientists studied conformal projections, especially optimal map projections. Contemporary cartography gives two definitions of optimal map projection: ideal and best projection [6]. The term optimal map projection of some closed domain is not unique and mathematically is not strictly defined by a single function [7]. Concept of ideal map projection is related to the task of finding the projection provided that the greatest linear deformation within the area of mapping has to be minimal [8]. However, this formulation is considered incomplete. For instance, the definition of minimal deformation is not given. Because of this and other difficulties, it is reasonable to determine the best projection from the given class, instead of finding the unique ideal map projection of area of interest [9].

Since each map projection is the best in some sense, it is necessary to specify what the best projection is. Since conformal projections are under consideration, that is the ones that do not distort angles and where the scale of area is equal to linear scale squared, it is sufficient to adopt a criterion on the linear deformation [9]. One possibility is to minimize extreme linear deformations. In other words, that is the projection which yields minimal greatest linear deformations [10]. Namely, general quality measure of map projections has to contain linear deformations on entire mapping area [11]. Also, one of assessment criteria of map projections is based on the comparison of maximal deformation values in all points of cartographic grid scales in main directions, that is, linear scales along the meridians and parallels of conformal cartographic grid [12].

Different systems of conformal map projections are or were in use in the Republic of Serbia. In Autonomous province of Vojvodina, for example, two types of topographic survey were used, so called old and new survey. The graphic survey conducted by Austro-Hungarian monarchy in stereographic projection at the end of 19th and the start of 20th century is referred to as old survey [13]. Numerical survey, conducted partially before and mostly after the World War 2, in Gauss-Krüger projection, is called the new survey [14]. Nowadays, Lambert conical and Universal Transverse Mercator (UTM) projections are in use. Each of these cartographic systems has its own advantages and drawbacks. Within the analysis of deformations in major map projections, statistical data as well as numerical and visual interpretations of the usage of each system can be obtained.

The study presented in this paper focuses on the comparative analysis of several types of conformal projections in terms of linear deformations. Analyzed projections (variants of conic, cylindrical and planar) are chosen based on their characteristics and usage in Serbia.

The goal is to determine the optimal projection for the territory of Serbia.

2 AREA OF INTEREST

Geographic area of the Republic of the Serbia is situated north of the Equator and east of the Greenwich. It is between latitudes of 41°53' and 46°11' north and longitudes 18°49' and 23°00' east (Fig. 1). 45th parallel is crossing this territory (Ruma - Stara Pazova - Bela Crkva) which puts it in the middle of the Northern hemisphere, equally distant from the Equator and the North Pole (Fig. 1).

Figure 1 Geographic location of Republic of Serbia [15, 16]

3 RESEARCH OBJECTIVES

The goal of the analysis presented in this paper is to determine optimal conformal projection with minimal absolute linear deformations within the area of interest. Several map projections are included in the analysis, with various parameters (Tab. 1). For the territory of Serbia the most important projections among the cylindrical ones are Gauss-Krüger and UTM. Historically, Kingdom of Yugoslavia (which Serbia was part of) adopted Gauss-Krüger projection as the official state projection in 1924, along with Bessel ellipsoid. Territory of Yugoslavia was divided into three zones with central meridians at 15°, 18° and 21° east. Value for X-axis (Northing) was distance from the Equator, while 500000 was added to the distance from the central meridian for Y-axis (Easting) value [17].

After the breakup of Yugoslavia, Serbia continued to use Gauss-Krüger projection and Bessel ellipsoid until the year 2011, when UTM projection and GRS80 ellipsoid were introduced. Point positions are represented in either geographic coordinates on the surface of the ellipsoid or in projected coordinates [18]. Besides these, for cartographic purposes several conic projections can be used, mostly normal conical and polyconic projections with certain mapping conditions or for modifications of linear scales [4]. Normal conical conformal projection in scales 1:500000 and 1:1000000 [19, 20] is particularly widespread. Gauss-Krüger and UTM projections are analyzed with linear scales of 0.9999 and 0.9996. Two variations of both conic and planar projections are also included in the analysis (Tab. 1).

Table 1 Analyzed variants of map projections

Type of projecting surface	Variant	Linear scale
Cylindrical	Gauss-Krüger	0.9999 0.9996
	UTM	0.9999 0.9996
Conic	Lambert V	variable
	Kavrayskiy I	variable
	Kavrayskiy II	variable
Planar	Stereographic	0.9999 0.9996

3.1 Cylindrical Projections

Cylindrical map projections use a cylinder as a surface onto which the surface of Earth's ellipsoid or sphere is mapped. Cylinder is split along one generatrice and the surface is developed into a plane. In case of normal cylindrical projections, the cylinder axis coincides with Earth's axis of rotation and cartographic grid consists of meridians and parallels. These projections are suitable for mapping of areas that are elongated along the parallels, while for areas along the meridians transverse cylindrical projections are more suitable.

If the cylinder touches the central meridian it is called tangent cylinder. If the cylinder and ellipsoid intersect along two lines (called standard lines) the cylinder is secant. Standard lines are positioned symmetrical to central meridian and they provide wider mapping zone, while retaining allowed values of absolute linear deformations. Positioning of secant cylinder is done indirectly based on linear scale factor along the central meridian. For UTM projection, with mapping zone 6° wide, the value of scale factor is 0.9996, while for Gauss-Krüger, with 3° wide zone, it is 0.9999. These values provide the linear deformations along standard lines to be 0.

Gauss-Krüger and UTM are transverse projections that are often used for mapping all over the world and many countries adopted them as state projections. The main differences between these two projections are the linear scale along the central meridian and the mapping zone width, as explained earlier. To calculate surface and linear scale and orthogonal coordinates following expressions are used [21]:

$$U = \frac{tg(45° + \frac{\phi}{2})}{tg(45° + \frac{\psi}{2})}, \tag{1}$$

$$\sin\psi = e\sin\varphi, \tag{2}$$

$$x = k\ln U, \tag{3}$$

$$y = k(\lambda - \lambda_0), \tag{4}$$

$$m = n = \frac{k}{r}, \tag{5}$$

$$r = N\cos\varphi, \tag{6}$$

$$p = m^2, \tag{7}$$

$$\omega = 0, \tag{8}$$

$$k = N_0\cos\varphi_0, \tag{9}$$

3.2 Conic Projections

Conic map projections use a cone to project points from the surface of the Earth's ellipsoid or sphere. The cone is split along one generatrix and developed into a plane. In the case of normal conic projections the lateral surface touches the ellipsoid/sphere along one (tangent) or two parallels (secant). There are no deformations along these parallels therefore they are called standard parallels. For relatively narrow areas (6°-7°) tangent cone is used with standard parallel in the middle of the area. For wider areas a secant cone is used with standard parallels chosen to provide even distribution of deformations.

Positioning of standard parallels depends on the mapping area, mostly its location, shape and size. For instance, for territories elongated along the parallel they are positioned near northern and southern edge of the territory, while for the territories elongated along the meridian normal conic projection is not the most adequate choice. Cylindrical or transverse conic are better suited in this case. For irregular shapes standard parallels are chosen empirically. In general, standard parallels are chosen to provide minimal absolute and average linear deformations. Equations of normal conic projections are as follows [22]:

$$U = \frac{tg(45° + \frac{\varphi}{2})}{tg(45° + \frac{\psi}{2})}, \tag{10}$$

$$\sin\psi = e\sin\varphi, \tag{11}$$

$$x = q - \rho\cos\delta, \tag{12}$$

$$y = \rho\sin\delta, \tag{13}$$

$$\delta = k(\lambda - \lambda_0), \tag{14}$$

$$\rho = \frac{K}{U^k}, \tag{15}$$

$$m = n = \frac{kK}{rU^k}, \tag{16}$$

$$r = N\cos\varphi, \tag{17}$$

$$p = m^2, \tag{18}$$

$$\omega = 0, \tag{19}$$

In given equations parameters *k* and *K* are determined in different ways, under different conditions. For these calculations Kavrayskiy proposed two procedures [22]:

I procedure – scales at edging parallels should be equal and greater than 1 that many times that 1 is greater than the minimal scale. Based on this condition following expressions to calculate *k* and *K* are derived:

$$k = \frac{\lg r_s - \lg r_n}{\lg U_N - \lg U_S}, \tag{20}$$

$$K = \frac{1}{k}\sqrt{r_0 r_S U_0^k U_S^k} = \frac{1}{k}\sqrt{r_o r_N U_o^k U_N^k}, \tag{21}$$

II procedure - scales at edging parallels should be equal and to differ from 1 for the same amount the scale on middle parallel differs from 1. This condition yields following expressions to calculate *k* and *K*:

$$k = \frac{\lg r_S - \lg r_N}{\lg U_N - \lg U_S}, \tag{22}$$

$$K = \frac{2r_S r_m U_S^k U_m^k}{k(r_S U_S^k + r_m U_m^k)} = \frac{2r_N r_m U_N^k U_m^k}{k(r_N U_N^k + r_m U_m^k)}, \tag{23}$$

Work of German scientist Johann Heinrich Lambert had significant influence to development of conic projections, especially conformal projections, where, for the first time, their more complete mathematical explanation can be found [22, 23]. Therefore, in literature these projections are usually called Lambert projections, which relates to Lambert conformal conic projection with two standard parallels (variant V) in particular. Values of linear deformations, as well as their symmetry, provide particular reason to adopt V variant of Lambert projection. Namely, linear scales are symmetrical in direction of meridians and parallels. Amplitudes of linear modules are smaller than for other projections, which consequences smaller areal deformations. Meridians are mapped as straight lines converging into one point (the apex of a cone). Parallels are concentric circles with their center in cone's apex. Constants k and K are calculated using following expressions:

$$k = \frac{\lg r_s - \lg r_N}{\lg U_N - \lg U_S}, \tag{24}$$

$$K = \frac{2U_N^k r_N U_0^k r_0}{k(U_0^k r_0 + U_N^k r_N)} = \frac{2U_S^k r_S U_0^k r_0}{k(U_0^k r_0 + U_S^k r_S)}, \tag{25}$$

3.3 Planar Projections

If the surface of Earth's ellipsoid/sphere is mapped onto a plane map, the map projection is called planar projection. For all planar projections (besides normal ones) it is usual that the Earth is approximated with the sphere. In case of normal planar projections, depending on a map scale, Earth's ellipsoid or sphere of radius R is mapped [24]. In this paper oblique stereographic projection is analyzed. Here the surface of ellipsoid or sphere is projected onto a plane from the point on the surface of the Earth that is opposite to the side of the planes center [23, 24]. Besides it is conformal, this projection maps a circle on a sphere onto a circle in a plane [22, 24]. General equations of oblique conformal planar projections are as follows:

$$\delta = a, \tag{26}$$

$$z = a\sin\left(\frac{\sin\lambda\cos\varphi}{\sin z}\right), \tag{27}$$

$$z = a\cos(\sin\varphi_0\sin\varphi + \cos\varphi_0\cos\varphi\cos\lambda), \tag{28}$$

$$\rho = 2Rtg\frac{z}{2}, \tag{29}$$

$$x = 2R\frac{\sin\varphi\cos\varphi_0 - \cos\varphi\sin\varphi_0\cos\lambda}{1 + (\sin\varphi\sin\varphi_0 + \cos\varphi\cos\varphi_0\cos\lambda)}, \tag{30}$$

$$y = 2R\frac{\cos\varphi\sin\lambda}{1 + (\sin\varphi\sin\varphi_0 + \cos\varphi\cos\varphi_0\cos\lambda)}, \tag{31}$$

$$\mu_z = \mu_a = \mu = \sec^2\frac{z}{2}, \tag{32}$$

$$p = \sec^4\frac{z}{2}, \tag{33}$$

$$\omega = 0, \tag{34}$$

$$d = |1 - \mu|, \tag{35}$$

3 METHODOLOGY

In order to conduct a comparative analysis it is necessary to establish a uniform methodology and criteria. This mostly relates to unique criterion for deformation assessment and specification of cartographic area.

Table 2 Sequence of tasks to determine optimal conformal map projection

Number	Task description
1	Definition and adoption of input data for the area of interest, including ellipsoid parameters
2	Calculation of constant values
3	Calculation of the greatest/smallest absolute linear deformation
4	Coordinates calculation and graphical representation of the results
5	Analysis of the results and conclusions

Optimal criterion assumes the analysis of the smallest and the greatest absolute linear deformations in the area of interest. To determine the optimal conformal map projection for the area of interest several tasks are established (Tab. 2).

Definition and adoption of input data assumes definition of parallels and meridians that are boundaries of the area of interest, determination of real boundary of the area (in this case state boundary of the Republic of Serbia), as well as ellipsoid parameters. Calculation of constants relates to projection constant k and integration constant K found in equations for conformal cylindrical and conic projections. The highest value of linear deformations in the area of interest A is calculated using following expression [26, 27]:

$$d_{\max} = \max A|c - 1|, \tag{36}$$

where c is the linear scale in projection.

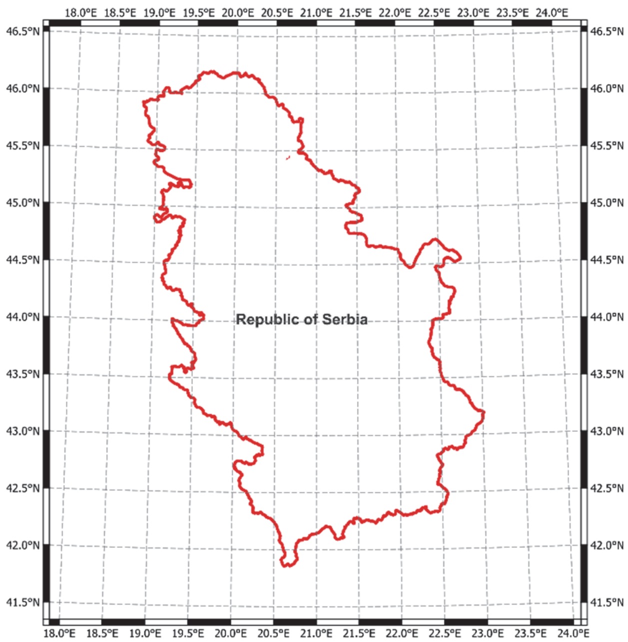


Figure 2 Analyzed frame and state boundaries of Republic of Serbia

However, for complex areas and irregular deformation distributions it is not always possible to calculate these values. Therefore, the calculation and the highest value of linear deformation are found in a finite number of selected points. In this paper, the calculation is done for points within the frame bounded with meridians at longitudes of 18° and 24° east and at latitudes of 41.5° and 46.5° north, with 0.5° increment along both axes. This area, as well as state boundaries of the Republic of Serbia, is represented in Fig. 2. State boundary is in red color, while the frame and the grid of parallels and meridians are in black.

4 RESULTS

Calculation results are represented numerically and graphically. Tables contain measures of quality values for the greatest (Tab. 3), average (Tab. 4) and the smallest (Tab. 5) absolute linear deformation for analyzed geographical area (i.e. within the frame) and within the state boundaries of Serbia expressed in dm /km.

Table 3 The greatest absolute linear deformation in different projections

Projection	Linear scale	Within the frame dm/km	Within the state boundaries dm/km
Gauss-Krüger	0.9999	6.70	2.80
	0.9996	4.00	4.00
UTM	0.9999	6.70	2.80
	0.9996	4.00	4.00
Lambert V	Variable	4.75	4.75
Kavrayskiy I	Variable	4.74	4.74
Kavrayskiy II	Variable	4.75	4.75
Stereographic	0.9999	7.41	4.20
	0.9996	4.45	4.10

Table 4 Average absolute linear deformation in different projections

Projection	Linear scale	Within the frame dm/km	Within the state boundaries dm/km
Gauss-Krüger	0.9999	2.20	0.80
	0.9996	2.40	3.10
UTM	0.9999	2.20	0.80
	0.9996	2.40	3.10
Lambert V	Variable	3.10	3.20
Kavrayskiy I	Variable	3.10	3.20
Kavrayskiy II	Variable	3.10	3.20
Stereographic	0.9999	2.50	0.90
	0.9996	1.90	2.60

Table 5 The smallest absolute linear deformation in different projections

Projection	Linear scale	Within the frame dm/km	Within the state boundaries dm/km
Gauss-Krüger	0.9999	0.00	0.00
	0.9996	0.00	0.50
UTM	0.9999	0.00	0.00
	0.9996	0.00	0.50
Lambert V	Variable	0.00	0.00
Kavrayskiy I	Variable	0.00	0.00
Kavrayskiy II	Variable	0.00	0.00
Stereographic	0.9999	0.00	0.00
	0.9996	0.00	0.00

Each table also lists the linear scale that was analyzed for the corresponding projection. The meridians and parallels where linear deformation equals zero are in green, while meridians and parallels where the linear deformation is maximum are in red (Fig. 3, Fig. 4 and Fig. 5).

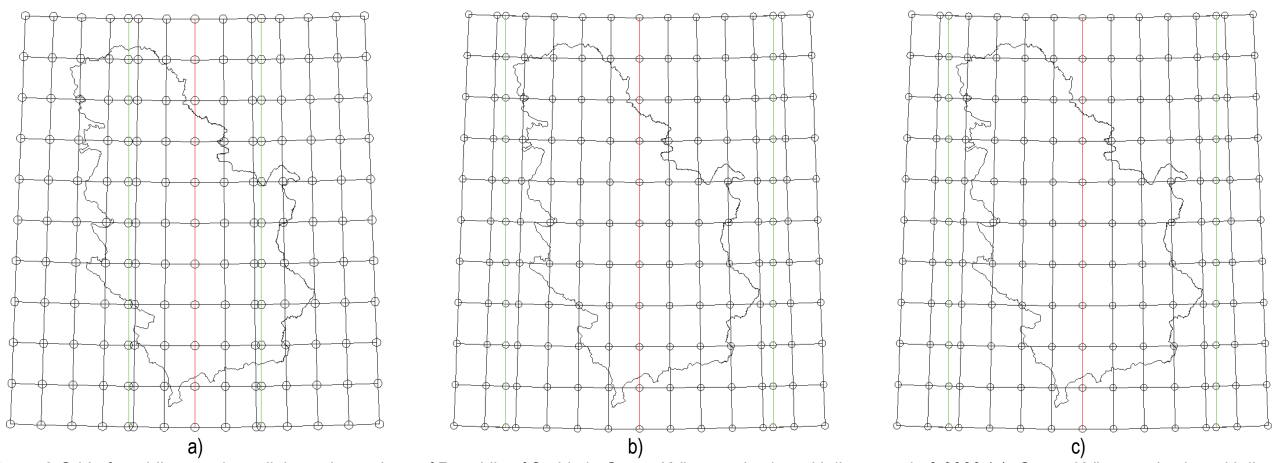


Figure 3 Grid of meridians and parallels on the territory of Republic of Serbia in Gauss-Krüger projection with linear scale 0.9999 (a), Gauss-Krüger projection with linear scale 0.9996 (b) and UTM projection with linear scale 0.9999 (c)

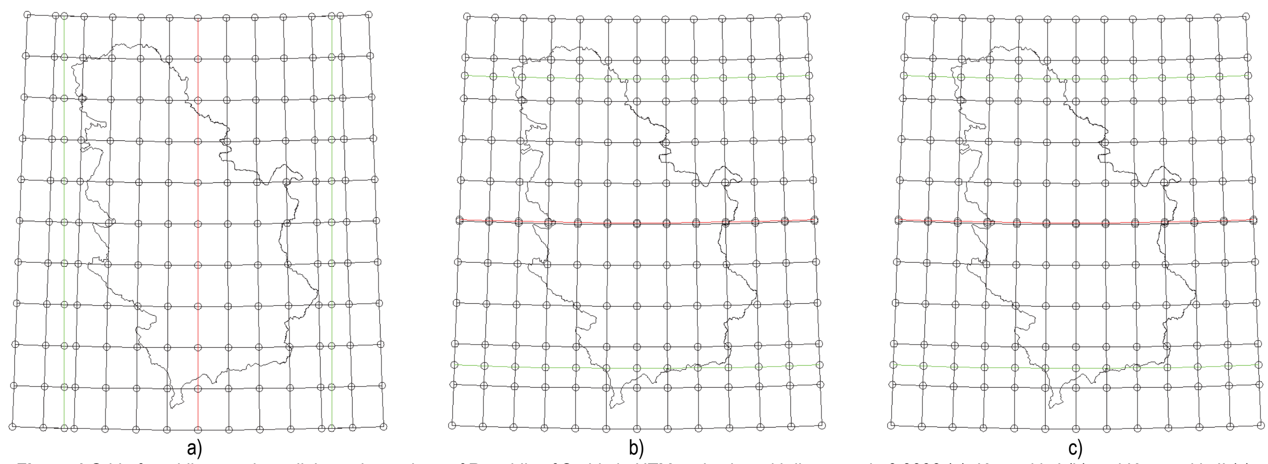
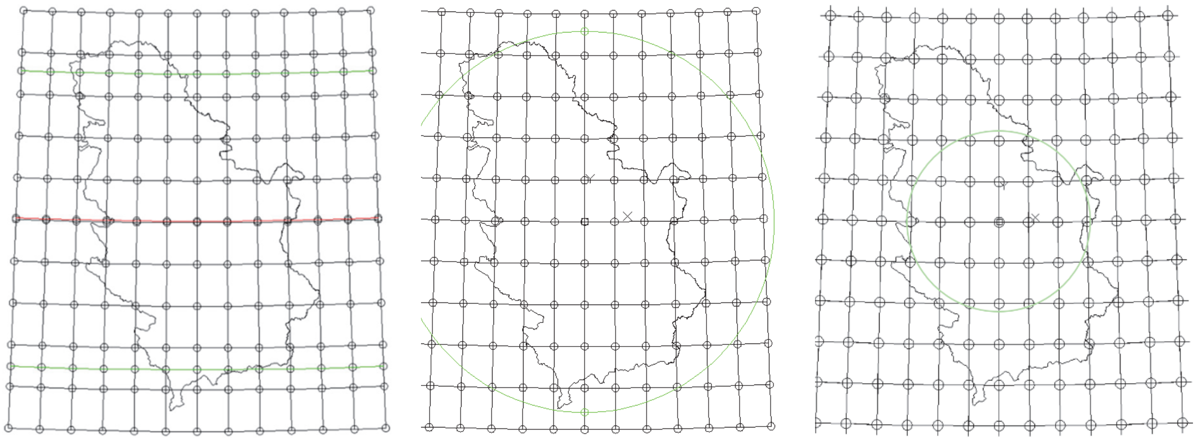


Figure 4 Grid of meridians and parallels on the territory of Republic of Serbia in UTM projection with linear scale 0.9996 (a), Kvrayskiy I (b) and Kavrayskiy II (c)

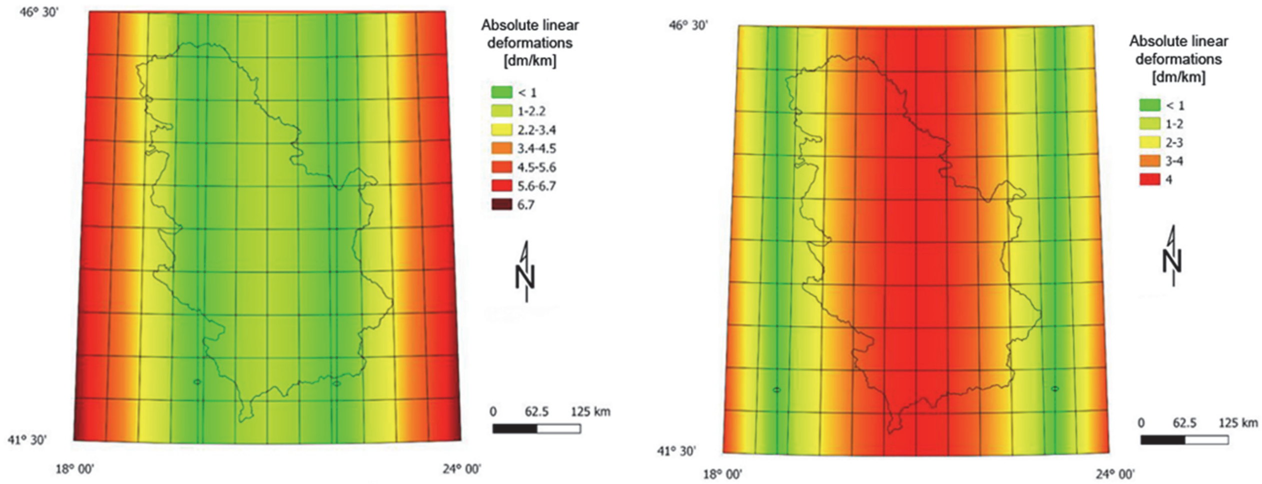


a) b) c)

Figure 5 Grid of meridians and parallels on the territory of Republic of Serbia in Lambert V (a), Stereographic projection with linear scale in the pole 0.9996 (b) and Stereographic projection with linear scale in the pole 0.9999 (c)

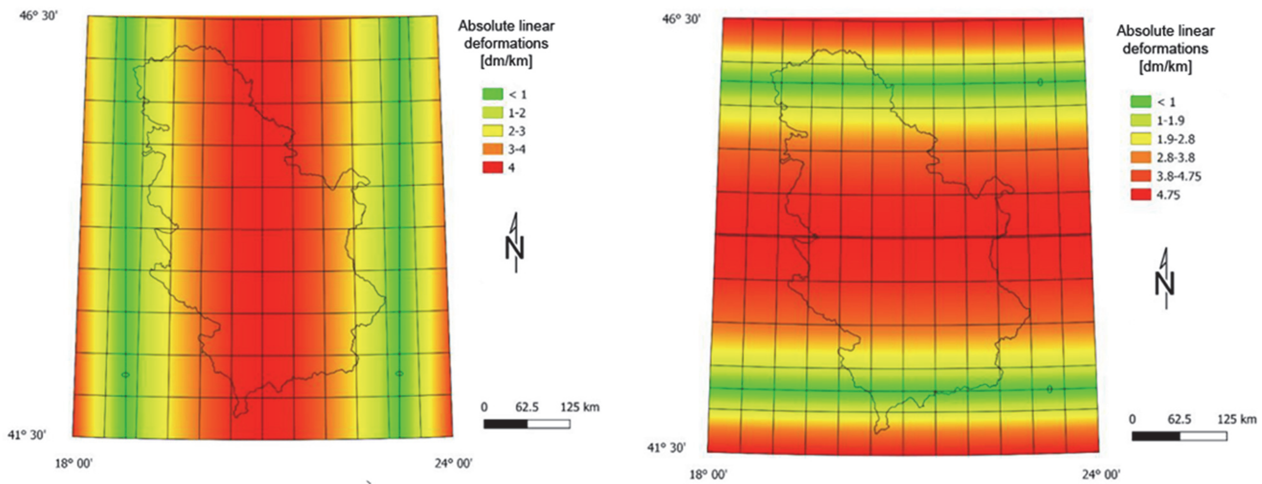
In addition to the state border and the previously mentioned lines, a network of meridians and parallels is also shown in the projection.

Graphical representation of deformation distribution is shown in following figures (Fig. 6, Fig. 7, Fig. 8 and Fig. 9). Values for UTM projection with linear scale 0.9999 are not shown since they are identical to values for Gauss-Krüger projection with the same linear scale. In lighter shades of green, smaller values of absolute linear deformations are shown, yellow represents medium values, while shades of red indicate the highest values.



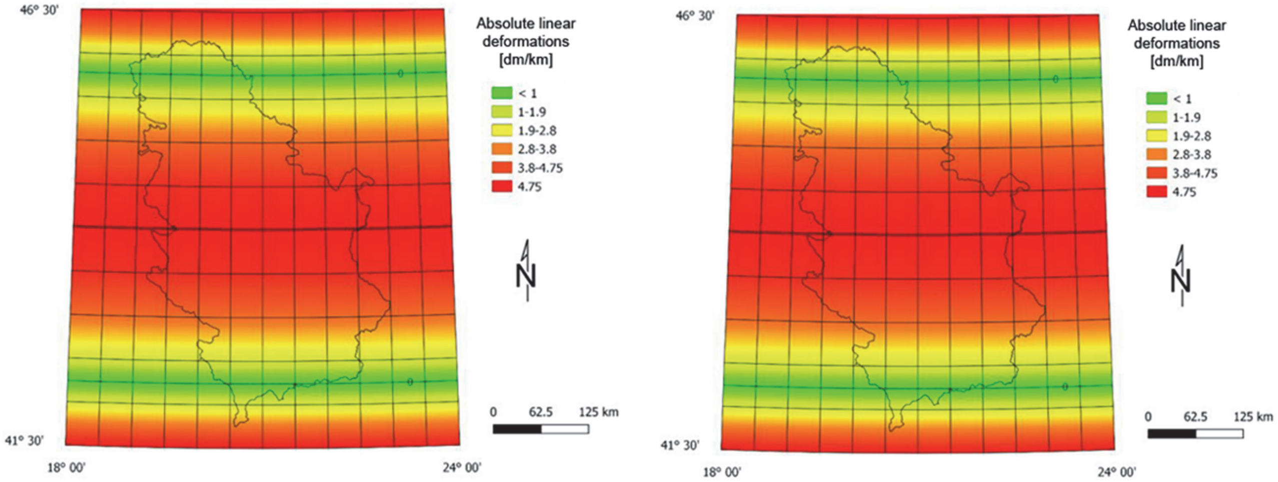
a) b)

Figure 6 Map with color-coded values of absolute linear deformations for Gauss-Krüger projection: linear scale 0.9999 (a), linear scale 0.9996 (b)



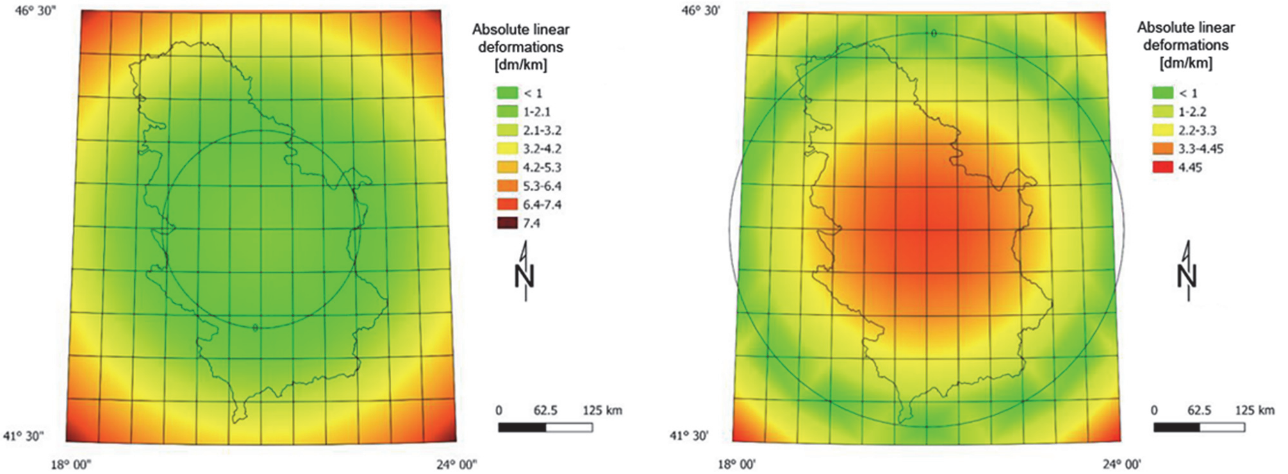
a) b)

Figure 7 Map with color-coded values of absolute linear deformations: UTM projection with linear scale 0.9996 (a), Kavrayskiy I projection (b)



a) b)

Figure 8 Map with color-coded values of absolute linear deformations: Kavrayskiy II (a), Lambert V projection (b)



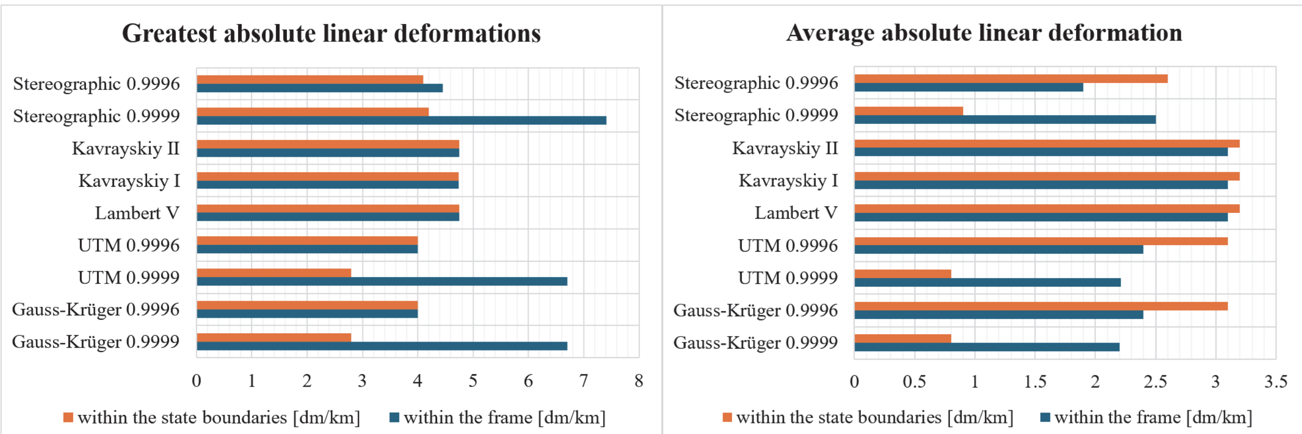
a) b)

Figure 9 Map with color-coded values of absolute linear deformations for Stereographic projection: linear scale in the pole 0.9999 (a), linear scale in the pole 0.9996 (b)

5 DISCUSSION

Besides numerical and graphical results, following diagrams represent the greatest and average absolute linear deformations. Comparative overview of these values is also given for both analyzed frame and the area within the state boundaries (Fig. 10). Given the historical application of the Gauss-Krüger and UTM projections for the area of the Republic of Serbia (chapter 3. Research Objectives), the focus of the analysis below is placed on these two projections.

For the majority of analyzed projections obtained values of greatest absolute linear deformations are the same for analyzed frame and within state boundaries of Serbia. However, for some projections, such as Gauss-Krüger and UTM with linear scale of 0.9999, that is not the case.



a) b)

Figure 10 Comparative analysis of the greatest (a) and average (b) absolute linear deformation within the frame and within the state boundaries

The greatest linear deformation within the frame for Gauss-Krüger projection (linear scale 0.9999) is 6.7 dm/km, while within the state boundaries it is 2.8 dm/km. Figure 10a clearly shows the importance of analysis within the state boundaries, not just within the wider frame. If values obtained within the frame would be the only ones taken into consideration the conclusions could be completely different. More precisely, this would lead to the conclusion that the Gauss-Kruger and UTM projections with a linear scale of 0.9999, which gave almost the highest deformation value in this analysis, are the most unsuitable, but in the area that is actually important (analyzed), they give the best results.

Further, considering values within the analyzed frame stereographic projection with linear scale of 0.9999 has the highest values of absolute linear deformations. However, within the state boundaries this is not the case. Especially if average values of absolute linear deformations are taken into account, since these are by far smallest and are equal to 1.0 (the same case as with Gauss-Krüger projection with linear scale of 0.9999).

UTM and Gauss-Krüger projection yielded same results in analyzed area for each value of linear scale. Highest value of linear deformation for UTM projection with linear scale of 0.9996 is 4 dm/km, which is worse result than Gauss-Krüger projection with 0.9999 linear scale. However, there are some other benefits of this projection that justify its acceptance. For instance, the entire territory of Serbia lies within one zone of projection (34T), while in the case of Gauss-Krüger projection most of the territory is within zone 7, but there is a part that lies within zone 6, and even one small part that should be within zone 8, but is considered to be in zone 7 (theoretically there are 3 zones, but in practice there are 2). Also, UTM projection is standardized and used globally which makes it easy to use in international projects. Further, distribution of deformations is more uniform and the values are acceptable for many geodetic and mapping applications. Regular grid (90° angle between paralles and meridians in projection) is another advantage of cylindrical projections. On the other hand, in case of normal cylindrical projection, large deformations in areas closer to poles is a drawback that often requires exclusion of these areas form the mapping zones.

The highest values of average linear deformations are obtained for variant I and II of Kavrayskiy projection and Lambert variant V, while the lowest average value within analyzed frame (1.9 dm/km) is obtained for stereographic (linear scale 0.9996) and Gauss-Krüger projection (linear scale 0.9999).

6 CONCLUSION

The analysis of optimal conformal map projection for the territory of the Republic of Serbia included eight variants of projections. For all conic projections obtained values were the same within both the analyzed frame and the state boundaries. Gauss-Krüger and UTM projections yielded identical results for all examined criteria. The lowest values of linear deformation within the state boundaries of the Republic of Serbia were obtained using cylindrical Gauss-Krüger/UTM projection with linear scale 0.9999. However, if entire frame is taken into consideration, different conclusion can be made. Then stereographic projection is the only one yielding higher values of linear deformation than Gauss-Krüger projection.

Based on the values of linear deformations, transverse cylindrical projection with 0.9999 linear scale along the central meridian is the optimal projection for the territory of Serbia. However, benefits of UTM projection with 0.9996 linear scale (e.g. entire territory in one zone, more even distribution of deformations usage in international projects, etc.) and still acceptable value of linear deformation contributed to its adoption as the official projection.

Also, the results of this study show that better conclusions can be made when analyzed area is the exact area of interest and not wider, approximate area bounded by two parallels and two meridians. Therefore, in future analyses only area within the state boundaries should be taken into account. By analyzing linear scales cartographic projects with the smallest linear deformations can be realized, as well as topographic maps and various areas of interest can be visually interpreted.

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