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Wireless Sensor Network Design in Grid-Free Structure

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Abstract: Sensors are small devices with limited battery, sensing, data processing, and communication capabilities. When many of them work together to monitor remote or hostile environments, they form a wireless sensor network (WSN). Designing WSNs requires prioritizing long network lifetime and energy efficiency. There are four main WSN design issues that affect the equal distribution of energy loads among sensors to extend network lifetime: sensor locations, sensor activity schedules, sink places/routes and data routes. In addition, instead of grid-based placement of sensors and sinks, a grid-free structure that allows them to be positioned more flexibly according to environmental factors or specific application requirements will allow minimizing energy consumption and increasing network resilience. As a contribution to the extensive literature presented in this framework, a new conic model is proposed that targets mentioned WSN Design Problems without a grid structure. The solution of the model is obtained using a commercial solver. The performance of the conic model is demonstrated by comparing its results against the grid-based design and a random placement strategy. It is shown that the conic model provides more flexibility in application and the resulting network will have a longer lifetime compared to the networks found by grid based model and random placement strategy.

Keywords: conic model; grid-free WSNs; network lifetime maximization; optimization; wireless sensor network

1 INTRODUCTION

Wireless Sensor Networks (WSNs) consist of small electronic devices known as sensors. These sensors have limited battery life and are equipped with capabilities for sensing, data processing, and communication. Sensors monitor the environment within their sensing range and transmit the data they collect to nodes called sinks, either directly or through other sensors. Data transmission can only occur when another sensor or sink is within the sensors' communication range [1]. They are deployed in a specific area of interest, referred to as the sensor field. When many sensors collaborate to achieve a common goal, such as monitoring remote or challenging environments, they form a WSN. An illustration of WSNs structure is given in Fig. 1 [2]. Working together collectively, sensors can sense and monitor various physical events such as temperature, humidity, light, object movements, pressure, noise levels, and animal presence within their range [3].

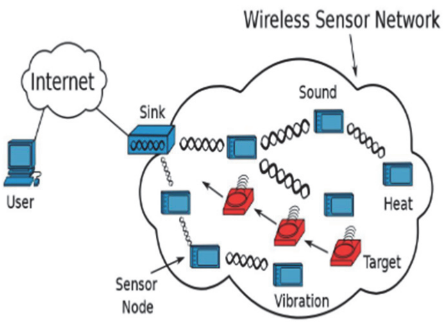

Figure 1 An example WSN

The collaborative nature of WSNs enables a distributed sensing and monitoring system, even in remote or hard-to-reach areas, opening the door to numerous applications [4, 5]. For instance, in military contexts, the rapid deployment, self-organizing capabilities, and fault tolerance of WSNs make them a highly promising technology for command, control, communication, intelligence, surveillance, and targeting systems. In the healthcare sector, sensor nodes can be utilized for patient monitoring and providing assistance to individuals with disabilities. Additionally, WSNs have a broad spectrum of real-world applications in areas such as homeland security, healthcare, environmental monitoring, agriculture, logistics, inventory management, product quality control, disaster area surveillance, and the design of smart homes or offices [6].

One of the most significant limitations of WSNs is the limited energy capacity of the sensors. Given that a typical WSN application involves a large number of sensors, replacing their batteries is generally not a feasible solution [7]. Consequently, the design and operation of a WSN must be meticulously planned to ensure a sufficiently long network lifetime. To minimize the overall energy consumption of sensor activities or to maximize network lifetime in WSN design, researchers have concentrated on four primary design challenges: the Coverage Problem (CP) related to sensor placement, the Activity Scheduling Problem (ASP) concerning the scheduling of sensor activities, the Sink Routing Problem (SRP) or the Sink Placement Problem (SPP) for determining sink locations or routes, and the Data Routing Problem (DRP) for optimizing data flow routes [1].

Determining sensor locations to satisfy the coverage requirements of the sensor field is referred to as CP [8]. The energy consumed during transmission is influenced by the distance between the transmitter and the receiver sensors, meaning that sensor placement directly impacts energy usage. An efficient sensor placement strategy should minimize energy consumption and prolong the network's lifetime [9]. ASP aims to maximize the network lifetime by determining the working schedules of the sensors. An ideal activity schedule distributes the energy load among the sensors in a balanced manner [10]. Sensors with low energy levels are put into standby mode to save energy. In the meantime, some sensors in standby mode are activated. In addition, the number and locations of active sensors at any time in the network should be such that they meet the coverage requirements of the sensor field. Moreover, active sensors should form a connected network so that every sensor can transmit their data directly or indirectly to the sinks via other sensors [10]. Determining the sink locations is critical to minimizing energy consumption and maximizing network lifetime. These locations have a decisive effect on data transmission paths and therefore on energy consumption. In WSNs, it is widely known that sensors that communicate directly with the sinks (relay sensors) consume their energy faster than other sensors. As a result, while many sensors in the network can still operate at full capacity, the connection with some sinks may be lost due to the energy depletion of these relay sensors. This situation is expressed in the literature with terms such as "crowded center effect" [11], "energy hole problem" [12, 13] and "sinking neighborhood problem" [14]. Innovative methods are needed to eliminate these energy inefficiencies that shorten the network lifetime. Controlled mobility of the sinks and changing the relay sensors by sink mobility stand out as an approach to solve this problem. The SPP is called the SRP when the sinks are mobile [1]. DRP is related to determining the data paths. Since energy consumption in WSNs is largely realized during data transfer, data paths should be selected carefully. Data transfer is a process that involves determining the data flow paths from the sensor to the sink in an energy efficient manner and is an important issue in WSN design. Mathematical programming models developed to determine the optimal data paths usually 0include additional constraints related to routing and include linear or nonlinear network flow models aimed at maximizing the network lifetime or minimizing the total routing energy [9].

Grid-based distribution strategies are widely used in the design of WSNs [15]. These strategies are preferred to provide significant improvements in terms of coverage and connectivity [16, 17]. Grid-based distributions allow sensor nodes to be placed in a regular lattice structure, effectively limiting the search space of sensor locations [15]. Grid design is particularly useful in cases where the cost of sensor nodes is high and the performance of these nodes is significantly affected by their locations [18]. For example, grid distribution strategies are widely used in applications such as aircraft health monitoring, pollution and CO_2 flow monitoring, forest fire sensing, and giant sequoia monitoring [18, 19]. In addition, grid-based design allows WSNs to create a more organized and predictable network structure in areas such as environmental monitoring, smart cities, and industrial applications. In this structure, the careful placement of sensor nodes ensures that the network covers the entire area, while power consumption during data transmission is also efficiently optimized. Especially in applications where continuous monitoring of large areas is required or the number of nodes is very high, grid-based arrangements offer an ideal solution to improve network performance and ensure energy efficiency [20]. In real-world applications, grid-based deployment can create various challenges, such as placement uncertainty and communication irregularity [21, 22]. These challenges can lead to serious connectivity issues, especially in critical applications that require immediate response to natural disasters. Therefore, the connectivity between the deployed sensor nodes and the sink is extremely important to ensure timely transmission of measured data [15]. In WSN designs that do not have a grid structure, the placement of nodes is done more flexibly according to environmental conditions or specific application requirements, rather than a grid-based layout. In this approach, nodes are positioned in accordance with the needs of the application and environmental factors, without being bound to a specific layout [20].

In this work, we study a model that integrates CP, ASP, SRP and DRP WSN design issues in a grid-free structure. There are two main factors that affect the grid-free choice approach: First, the conditions of the area where WSNs will be installed may not always be known in advance. Therefore, the design model should decide on the positioning of sensors or sinks. The fact that the points to be positioned are not determined in advance leads to the formation of a grid-free structure. Namely, sensors or sinks can be positioned at any point in the area. In this way, it is evaluated that the network will have a longer lifetime. Secondly, the negative effect of assuming a grid layout for the sensor places on the quality of the WSN, has never been explored before. In this work we analyze to what extent grid layout assumption reduces the final WSN lifetime.

The structure of the paper is organized as follows: we give a brief review of the relevant literature in Section 2. In Section 3, we first present a model developed assuming a grid structure and then extend it to a model that works in a grid-free environment. Both models integrate the CP, ASP, SRP and DRP. We give the numerical results of the two models in Section 4. Finally, in Section 5, we discuss the results and summarize the contribution of the paper to the field.

2 RELATED WORKS

In the literature on grid-based design, critical elements such as different topology models, node placement strategies, and energy consumption have been deeply addressed. Studies focusing on metrics such as energy management at the node level and link quality at the network level evaluate the advantages provided by the grid structure from a comprehensive perspective. These review studies provide various approaches for WSN topologies, while also providing significant contributions to improve energy efficiency, link robustness, and overall network performance [20].

Turjman et al. in [15] state that grid-based deployment increases coverage and connectivity quality by ensuring that sensor nodes are evenly distributed in the monitored area. In addition, the same authors in [18] mention that placing nodes on a regular grid would facilitate deployment planning by narrowing the search space for sensor locations. Using various grid structures in 3D space makes it possible to obtain more precise estimates of the spatial characteristics of the collected data. In addition, they argue that in some applications, the cost of sensor nodes could be high and in cases where their performance depends on their location, grid-based deployments would be useful in increasing cost-effectiveness. Verma et al. [23] develop a proposed energy intelligent routing protocol to reduce energy consumption and enhance network lifetime in WSNs. This protocol involves the first receiver to proactively create a network structure and pave a path to transmit queries. Each sensor node is selected with a cost function and network node failures are taken into account. In addition, expected mobility zones are created in the sensor area to manage the multiplicity and movements of receivers, thus ensuring continuous data transmission.

Alternatively, Han et al. [24] propose a tracking scheme based on two-layer grid model to track continuous object motion in resource-constrained wireless sensor networks. A new mechanism is presented to eliminate the boundary distortion caused by uneven node distribution, and a flow regulation system is designed to reduce the data load. Simulation results show that their approach provides high tracking accuracy and lower communication load without increasing energy consumption. In addition, Li and Ko [25] present grid-based routing algorithms for large-density WSNs. The algorithms address continuous geodetic problems via a geographic model to calculate minimum routing costs. Routing strategies are developed using the fast march method for location-only costs and the finite element method for traffic-proportional costs. Simulation results show that the finite element method provides lower-cost routing paths and is more effective in energy-load balancing. The study presents a new approach for developing routing algorithms within a systematic framework and suggests potential research directions for the applicability of existing techniques in WSNs. Anupong et al. [26] propose a fault-tolerant network system to cope with network congestion and communication errors. This architecture provides efficiency in energy usage and packet distribution, reducing network latency, delivery loss and response times. The proposed method exhibits higher reliability and performance than existing methods.

Turjman et al. [15] state that although grid-based design provides many benefits for WSNs, it also has some disadvantages in practice such as connectivity issues and failures. They propose techniques such as node redundancy and node mobility to address these issues. On the other hand, Kodz et al. [27] note that failure to precisely place sensor nodes at targeted grid locations could lead to coverage and connectivity issues. They suggest that this placement uncertainty could be due to unexpected factors such as timing errors, deviations in distance estimation, weather conditions such as rain and wind, and wildlife disturbance of nodes. Hashim and Stavrou [28] emphasize the problem of communication irregularity in grid-based distribution. They state that natural or man-made obstacles such as trees, mountains, walls, cliffs in the field, as well as extreme weather conditions encountered in outdoor applications, can affect the transmitted signals and cause irregularities in the communication distance. Tolle et al. [29] note that in some applications, sensors must be positioned at different heights, which adds additional challenges to the deployment process. For example, in experiments to monitor giant sequoia trees in California, sensors had to be placed at different heights on the trunks of the trees, covering a wide vertical range. Wei et al. [30] address the placement of sensors and various battery types in a 3D area in Electrochemical Energy Storage (EES) systems. It offers a mixed integer nonlinear programming (MINLP) model for efficient and secure sensor placement. The system is shown to respond efficiently to real-time changes and is compatible with different geometric structures and sensor costs, confirming the effectiveness of the model.

Keskin et al. [10] present a mathematical optimization model that simultaneously addresses four main problems in WSN design assuming a grid structure. This formulation provides an optimal solution to the sensor location, activity planning, mobile sink and data routing problems. Later, the same authors in [8] present a mathematical programming model with event-based grid structure that integrates these problems. The presented approach significantly reduces the number of binary variables by using variable-length periods compared to traditional time-slot based models. Due to the complexity of the problem, the authors propose a nested heuristic. The proposed method gives promising results, especially when compared to commercial solvers such as Gurobi for larger sensor networks.

In this study, a WSN design without a grid structure is developed in response to these difficulties in grid-based design. Hence, we also present a brief literature of the grid-free WSN design studies below.

Among the studies addressing the design of grid-free WSNs, studies discussing how different placement and routing strategies can be implemented stand out. Such studies have examined how irregularly placed sensors can be used to increase energy efficiency and coverage in network connectivity. For instance, Ojeda et al. [20] discuss modeling methods to increase energy savings in node placement and data transmission. Pantazis et al. [31] describe how multilayer protocols can be optimized to increase resilience in networks where sensor nodes are placed in flexible structures. The aim of these studies is to provide flexible and energy-efficient communication according to the location of the nodes and to increase usability in applications such as environmental monitoring [32]. Abidi [33] discussed automatic sensor placement in an environment where there is no prior knowledge about the object. The volume occupied by the objects is tracked by creating and updating an occupation grid. Sensor placement is decided freely based on images. Chepuri and Leus [34] address the problem of finding the minimum number of sensors that achieve the lowest estimation error by performing sensor placement in a grid-free structure. Unlike the traditional grid structure, it considers the points where sensors will be placed as a continuous area. Poudel and Cowlagi [35] focus on the problem of optimum sensor placement to make the necessary measurements in an unknown environment that changes spatially and temporally. It addresses this problem together with the path planning problem to reduce uncertainty. They conducted simulation studies with a new criterion and algorithm they developed and showed that the conjugate approach they presented is efficient. Dong [36] developed different alternatives to the traditional grid-structured sensor placement and made sensor placement. The developed sensor placement model was applied to lighting and shading control systems in the building. According to the model, sensor placement was made by considering the demands of different locations and optimum energy management was provided. Chowdhury [37] offers a grid-free placement approach for sensors deviating from the grid-structured area. Sensors are placed at random or optimized positions. The approach was evaluated in terms of signal identification ability of antenna arrays and performance in the presence of mixed strong/weak signals. It has been reported that although grid-free sensor placement has advantages in terms of interference suppression and invisible signal detection, it makes signal identifiability difficult.

In studies on how to manage the continuity of wireless network connection and connection problems in grid-free structures, issues such as the placement of nodes according to environmental factors and the dynamic provision of communication features are emphasized. For instance, Keskin et al. [38] present two mathematical models in its study to maximize the lifetime of a WSN by considering the travel time of a mobile sink in a grid-free structure. The models aim to create realistic scenarios by considering the sink making multiple trips and the data accumulated during the trip. In addition, effective heuristic methods are proposed to solve these models and their performance is stated to be successful. It is shown that considering sink travel times can significantly increase network lifetime, especially in large networks and situations requiring a large number of trips. Çıbuk and Cengiz [39] also investigate the energy consumption of WSN based monitoring and management systems depending on the topological structure in grid-free renewable energy sourced lighting systems. In these lighting systems, it is aimed to maximize the WSN life and provide minimum energy consumption with energy storing accumulator-batteries.

It has been observed that grid-free sensor placement is preferred in modern and variable environments and advanced optimization methods are used. In studies, linear models have been replaced by nonlinear or mixed structure models. In addition, it has been revealed that the developed methods are generally advantageous in terms of coverage and accuracy, but disadvantageous in terms of computational cost and complexity.

In our study, we do not limit the points to be positioned and use the entire area. In this way, the model we develop is supposed to provide longer network lifetimes. Our study is qualitatively similar to other studies in the literature that examine sensor placement in a grid-free structure. On the other hand, its quantitative performance is demonstrated by comparing it against a random placement strategy.

3 MATHEMATICAL MODELS

In this section, we first present a mathematical model assuming a grid structure first and then extend it to a model that works for the grid-free structure. We also include a random placement strategy to demonstrate the superiority of our grid-free model in real life.

3.1 The Model with Grid Structure

The network given in Fig. 2 depicts an example of a sensor network belonging to a grid-model with a two-period lifetime. In this network, there are two sinks with 15 visit points and 30 candidate sensor locations. The figure graphically presents the locations of the sensors and sinks, while the data paths in both periods are indicated by arrows. Some of the sensors remain in active mode in both periods, while some are active in one period and switch to standby mode in the other period. Coverage points are taken as the centroid points of the grids which are also the sink visit locations. Each coverage point of the field must be covered by at least two sensors in each time period. Assume that there is a distance of 60 meters between sensors and a sensing range of each sensor is taken equal to 75 meters. Note that the given sensor network meets this coverage requirement and each point is seen to be in the sensing range of at least two sensors in both time periods. In addition, some sensors transmit the collected data directly to the sinks, while some transmit the data to neighboring sensors, which act as relays. In this way, a solution is created in which both the active/sleep modes of the sensors and the data transmission paths are optimized [10].

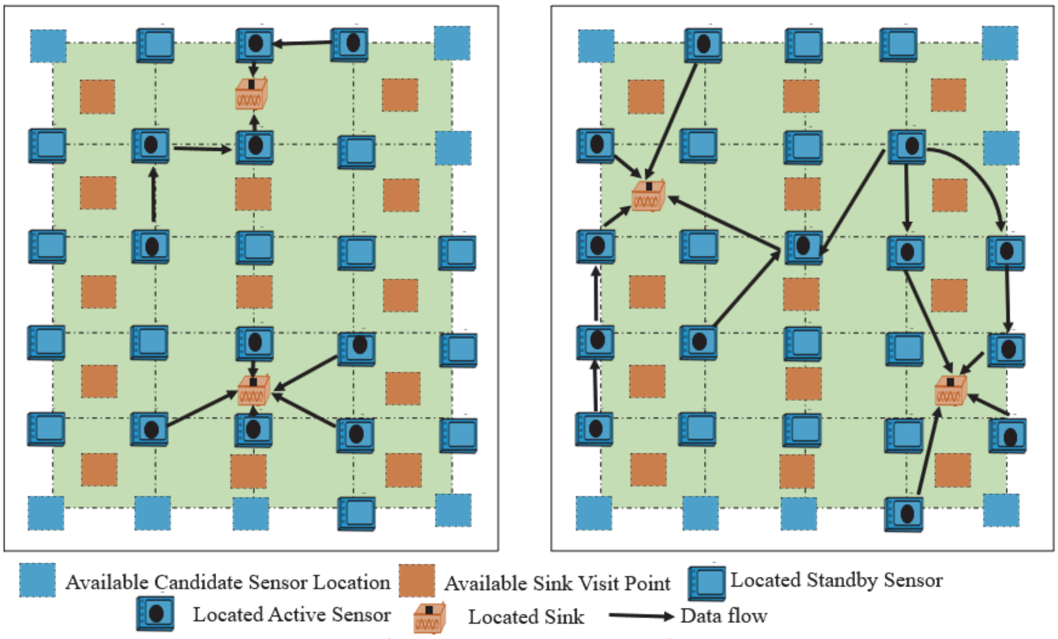

Figure 2 Sensor network implemented in the Grid model

The set of locations to be covered is represented by K. The set of candidate sensor locations and the set of sink visit points, (candidate sink locations), are denoted as S and N, respectively. The data collected by sensor i is transmitted to a sink located within the communication range of sensor *i*. If there is no nearby sink within the communication range of sensor *i*, it transmits its data to the sink via relay sensors. The set of neighbor sensor locations within the communication range of sensor *i* is represented by S_i, and the set of neighbor sink locations is represented by N_i. K_i represents the set of locations that can be covered by a sensor located at point *i*. The set of time periods is denoted by *T*.

The explanation of the parameters used in the model is as follows: *e* is the initial sensor battery energy, *h* is the amount of data produced per unit time by each sensor. c^s, c^r and c^t indicate the amount of energy a sensor spends to sense and process data, receive data, and transmit data, respectively. d_k is the coverage requirement of coverage point *k*. Namely, the number of active sensors that can sense coverage point *k* must be at least d_k at any time during the network lifetime. f_i is the cost of placing the sensor at point *i*. *B* indicates the available budget allocated for the sensor placement and the total number of sinks to be located at each period is denoted by *P*.

The decision variables used in the models are as such; w_t represents the length of period *t*, x_{ijt} and y_{ilt} represent the amount of data transferred from sensor *i* to sensor *j* and from sensor *i* to sink *l* at period *t*, respectively. p_i and z_{lt} are the binary location variables. They represent the decisions to place sensors and sinks, respectively. If the sensor is placed at location *i* in the sensor area, p_i = 1, otherwise p_i = 0. Similarly, if the sink is placed at location *l* in period *t*, z_{lt} = 1, otherwise z_{lt} = 0. Finally, q_{it} = 1 indicates whether or not sensor *i* is active in period *t*.

A summary of the definitions of the cluster, parameter and decision variables used in the model is given in Tab. 1.

The mathematical optimization model for WSN design assuming a grid-based structure is given below.

Grid Model:

$$\max \sum_{t\in T} w_t \quad (1)$$

Subject to: $$\sum_{j:i\in S_j} x_{jit} + h w_t q_{it} = \sum_{i:l\in N_i} y_{ilt} + \sum_{i:j\in S_i} x_{ijt}.$$

$$i\in S,\ t\in T \quad (2)$$

$$\sum_{t\in T}\left(c^s w_t q_{it} + c^r \sum_{j:i\in S_j} x_{jit} + c^t \sum_{i:l\in N_i} y_{ilt} + c^t \sum_{i:j\in S_i} x_{ijt} \right) \le e$$

$$i\in S \quad (3)$$

$$\sum_{i:l\in N_i} y_{ilt} \le M z_{lt},\ l\in N,\ t\in T \quad (4)$$

$$\sum_{\in N} z_{lt} = P,\ t\in T \quad (5)$$

$$\sum_{i:k\in K_i} q_{it} \ge d_k,\ k\in K,\ t\in T \quad (6)$$

$$\sum_{i\in S} f_i p_i \le B \quad (7)$$

$$q_{it} \le p_i\ \ i\in S,\ t\in T \quad (8)$$

$$\sum_{i:j\in S_i} x_{ijt} \le M q_{it},\ i\in S,\ t\in T \quad (9)$$

$$\sum_{j:i\in S_j} x_{jit} \le M q_{it},\ j\in S,\ t\in T \quad (10)$$

$$w_t, x_{ijt}, y_{ilt} \ge 0$$

$$i,j\in S,\ l\in N,\ t\in T \quad (11)$$

$$p_i, q_{it}, z_{lt} \in \{0, 1\},\ \ i\in S,\ l\in N,\ t\in T \quad (12)$$

Table 1 Sets, parameters and decision variables

Sets	Definition
S	Set of candidate sensor locations
S_i	Set of sensor locations neighboring sensor
N	Set of candidate sink locations
N_i	Set of sink locations neighboring sensor *i*
T	Set of time periods
K	Set of locations to be covered
K_i	Set of locations covered by sensor *i*
Parameters	Definition
h	Amount of data generated by a sensor per unit time
c^s	Amount of energy of unit sensing and coordination
c^r	Amount of energy of unit data reception
c^t	Amount of energy of unit data transmission
e	Battery energy of sensor
M	A very large number
P	Total number of sink
d_k	Coverage requirement of location *k*
f_i	Cost of placing the sensor at point *i*
B	Sensor placement budget
Decision variables	Definition
w_t	Length of period t
x_{ijt}	Amount of data flow from sensor*i* to sensor *j* in period *t*
y_{ilt}	Amount of data flow from sensor *i* to sink *l* in period *t*
p_i	Binary variable indicating whether sensor *i* is located or not
q_{it}	Binary variable indicating whether sensor *i*is active in period *t* or not
z_{lt}	Binary variable indicating whether sink *l* is located in period *t* or not

The network lifetime is defined as the total of period lengths in the objective Eq. (1) and it is maximized. Eq. (2) represents the data flow balance equation for each sensor in each period, ensuring that the data generated by the sensor or received from neighboring sensors equals the total data transmitted by the sensor to its neighboring sensors and sinks. Eq. (3) is the energy constraint written for each sensor. Accordingly, the sensing energy amount of sensor *i* in each period, the amount of energy spent while receiving data from neighboring sensors, the amount of energy spent while sending data to the sinks and neighboring sensors are limited by the initial energy of the sensor. Eq. (4) prevents data transmission to location *l* if there is no sink located at that location in period t. Eq. (5) ensures that the total number of sinks located for each t in T is equal to P. In other words, it ensures that all sinks are located in each period. Eq. (6) ensures that the coverage requirements of all nodes are met by the number of active sensors throughout the network lifetime. This ensures that not only sensors are placed, but also that a subset of them is kept active to meet all coverage requirements for each period. Eq. (7) is the budget constraint, ensuring that the total amount of money to be spent on sensor deployment does not exceed the available budget B. Eq. (8) states that in order for a sensor to be active, it must first be placed. Eq. (9) and Eq. (10) state that no data can be transmitted to or transmitted from a sensor that is not active. Eq. (11) states that continuous variables must be positive and Eq. (12) states that binary variables must have a value of either 0 or 1.

In Eq. (2) and Eq. (3), $w_t q_{it}$, which is used as the active time length of sensor i, makes the model nonlinear because it is the product of binary variable q_{it} and continuous variable w_t. Therefore, this product is expressed by the term b_{it} and 4 more linear constraints are added to the model to linearize this term, as shown below.

$$b_{it} \le w_t \;\; i \in S,\; t \in T \tag{13}$$

$$b_{it} \le M q_{it} \;\; i \in S,\; t \in T \tag{14}$$

$$b_{it} \ge w_t - M\left(1 - q_{it}\right) \;\; i \in S,\; t \in T \tag{15}$$

$$b_{it} \ge 0 \;\; i \in S,\; t \in T \tag{16}$$

Eq. (13), Eq. (14), Eq. (15) and Eq. (16) force the continuous variable b_{it} to be equal to the term $w_t q_{it}$. According to Eq. (13) and Eq. (15), if sensor i is in active mode in period t, then (q_{it} = 1), then b_{it} = w_t. Similarly, according to constraints Eq. (14) and Eq. (16), if sensor i is in standby mode in period t, then (q_{it} = 0), then b_{it} = 0.

Four main problems in WSN design are addressed simultaneously and the sensor and sink placement area is considered as grid-based. Note that this model is similar to the mathematical model given in [10].

3.2 The Model with Grid-Free Approach

The land area conditions where the model will be applied may be unknown and hence, we may not assume specific discretized sensor or sink placement locations. In such an environment, sensors/sinks can be placed on to any location in the sensor field making the approach as grid-free. This grid-free approach will make the network structure to use the available battery energy in the most efficient way while transmitting data from sensor to sensor or from sensor to sink. In addition, WSN design problems are still addressed and the network lifetime is maximized, consequently.

The explanation of the additional parameters used in the grid-free model is as follows. $\left(a_k^3, b_k^3\right)$ represents the coordinates of the location k to be observed. The communication range (CR) and sensing range (SR) of each sensor are considered as parameters in the model.

5 of the 8 decision variables used in the model are continuous variables, the remaining 3 are binary variables. Continuous variables are as such; $\left(a_i^1, b_i^1\right)$ represents the coordinates of the sensor i location, $\left(a_{lt}^2, b_{lt}^2\right)$ represents the coordinates of the location where sink l is placed at time t. In addition, d_{ij}^1 represents the distance between sensor i and sensor j locations, d_{ilt}^2 represents the distance between sensor i and sink l in period t, and d_{ik}^3 represents the distance between sensor i and coverage point k. On the other hand, β_{ij} indicates whether sensor j is in the communication range of sensor i or not, ϑ_{ilt} indicates whether sink l is in the communication range of sensor i in period t or not, and α_{ik} indicates whether location k is in the sensing range of sensor i or not.

A summary of new parameters and decision variables used in the grid-free model is given in Tab. 2.

Table 2 Parameters and decision variables of grid-free model

Parameters	Definition
$\left(a_k^3, b_k^3\right)$	Coordinates of the location k to be observed
CR	Communication range
SR	Sensing range
Decision variables	Definition
$\left(a_i^1, b_i^1\right)$	Coordinates of sensor i
$\left(a_{lt}^2, b_{lt}^2\right)$	Coordinates of sink l at period t
d_{ij}^1	Distance between sensor i and sensor j
d_{ilt}^2	Distance between sensor i and sink l at period t
d_{ik}^3	Distance between sensor$i$ and coverage point k
β_{ij}	Binary variable indicating whether sensor i is in the communication range of sensor j or not
ϑ_{ilt}	Binary variable indicating whether sensor i is in the communication range of sink l in period t or not
α_{ik}	Binary variable indicating whether sensor i is in the sensing range of point k or not

Now, we give the formulation of the grid-free model below.

Grid-free model: $\max \sum_{t \in T} w_t$.

Subject to: Eq. (2), Eq. (3), Eq. (7) to Eq. (16).

$$\left(d_{ij}^1\right)^2 = \left(a_i^1 - a_j^1\right)^2 + \left(b_i^1 - b_j^1\right)^2 \qquad i, j \in S \tag{17}$$

$$d_{ij}^1 \ge CR - M\beta_{ij} \qquad i, j \in S \tag{18}$$

$$d_{ij}^1 \le CR + M\left(1 - \beta_{ij}\right) \qquad i, j \in S \tag{19}$$

$$\left(d_{ilt}^2\right)^2 = \left(a_i^1 - a_{lt}^2\right)^2 + \left(b_i^1 - b_{lt}^2\right)^2 \qquad i \in S,\; l \in N,\; t \in T \tag{20}$$

$$d_{ilt}^2 \ge CR - M\vartheta_{ilt} \qquad i \in S,\; l \in N,\; t \in T \tag{21}$$

$$d_{ilt}^2 \leq CR + M\left(1-\vartheta_{ilt}\right) \quad i \in S, l \in N, t \in T \quad (22)$$

$$\left(d_{ik}^3\right)^2 = \left(a_i^1 - a_k^3\right)^2 + \left(b_i^1 - b_k^3\right)^2 \quad i \in S, k \in K \quad (23)$$

$$d_{ik}^3 \geq SR + M\alpha_{ik} \quad i \in S, k \in K \quad (24)$$

$$d_{ik}^3 \leq SR + M\left(1-\alpha_{ik}\right) \quad i \in S, k \in K \quad (25)$$

$$\sum_{t \in T} x_{ijt} \leq M\beta_{ij} \quad i, j \in S \quad (26)$$

$$y_{ilt} \leq M\vartheta_{ilt} \quad i \in S, l \in N, t \in T \quad (27)$$

$$\sum_{i \in S} \alpha_{ik} q_{it} \geq d_k \quad k \in K, t \in T \quad (28)$$

$$d_{ij}^1,\ d_{ilt}^2,\ d_{ik}^3,\ a_i^1, b_i^1, a_{lt}^2, b_{lt}^2 \geq 0$$
$$i, j \in S, k \in K, l \in N, t \in T \quad (29)$$

$$\beta_{ij}, \alpha_{ik}, \vartheta_{ilt} \in \{0, 1\} \quad i \in S, l \in N, t \in T \quad (30)$$

Eq. (17) provides the equation that gives the distance between sensor i and sensor j. Eq. (18) and Eq. (19) ensure that the distance between sensor i and sensor j is equal to or less than the communication range if sensor j is in the communication range of sensor i or vice versa. Eq. (20) provides the equation that gives the distance between sensor i and sink l in period t. Eq. (21) and Eq. (22) ensure that the distance between sensor i and the sink l in period t is equal to or less than the communication range if sensor i is in the communication range of the sink l in period t or vice versa. Eq. (23) provides the equation that gives the distance between sensor i and the coverage point k. Eq. (24) and Eq. (25) ensure that the distance between sensor i and coverage point k is less than or equal to the sensing range if coverage point k is in the sensing range of sensor i or vice versa. Eq. (26) prevents data output from sensor i to sensor j if sensor j is not in the communication range of sensor i. Eq. (27) prevents data output from sensor i to sink l in period t if sink l is not in the communication range of the sensor i in period t. Eq. (28) ensures that the coverage requirements of all locations are met by the number of active sensors in the sensing range during the network lifetime. Eq. (29) states that continuous variables must be positive. Eq. (30) states that binary variables must take the value 0 or 1.

In Eq. (28), the product $\alpha_{ik} q_{it}$ is expressed with the term δ_{ikt} and 4 linear constraints are added to the model instead of the nonlinear product as shown below:

$$\delta_{ikt} \leq \alpha_{ik} \quad i \in S, k \in K, t \in T \quad (31)$$

$$\delta_{ikt} \leq q_{it} \quad i \in S, k \in K, t \in T \quad (32)$$

$$\delta_{ikt} \geq \alpha_{ik} + q_{it} - 1 \quad i \in S, k \in K, t \in T \quad (33)$$

$$\delta_{ikt} \in \{0,1\} \quad i \in S, k \in K, t \in T \quad (34)$$

Eq. (31), Eq. (32), Eq. (33) and Eq. (34) force the binary variable δ_{ikt} to be equal to the term $\alpha_{ik} q_{it}$. According to constraints Eq. (31) and Eq. (33), if sensor i is in active mode in period t, then $\left(q_{it} = 1\right)$, then $\delta_{ikt} = \alpha_{ik}$. Similarly, according to constraints Eq. (32) and Eq. (34), if sensor i is in standby mode in period t, then $\left(q_{it} = 0\right)$, then $\delta_{ikt} = 0$.

The model we present is the first mathematical optimization model that simultaneously addresses four main problems in grid-free WSN design.

In addition, this model is a conical model that includes linear and nonlinear constraints. Eq. (17), Eq. (20) and Eq. (23) are written as quadratic constraints of variables.

In conical optimization, decision variables form a convex cone set which helps to model problems in various fields. There are 5 different mathematical conical optimization models; Linear Programs, Second Order Conical Programs, Semi-Definite Programs, Copositive and Completely Positive Programs, and Mixed Integer Conical Optimization Programs.

The grid-free formulation given above is of the type Copositive and Completely Positive Programming and Mixed Integer Conical Optimization. Copositive and Completely Positive Programming is used for complex problems involving binary variables such as Quadratic problems. Constitutive cone and completely positive cone are used for formulation. In Mixed Integer Conic Optimization, some variables must take integer values. In other words, in addition to conic constraints there are integrality restrictions on the variables. It should be noted that the grid free model of this study is the first conical model used for design of WSNs.

3.3 The Random Placement Strategy

A random placement strategy is produced to demonstrate the superiority of the grid free placement model with a realistic WSN scenario. For this purpose, the coordinates of sensor i, $\left(a_i^1,\ b_i^1\right)$ and the coordinates of the sink l at period $t\left(a_{lt}^2, b_{lt}^2\right)$ are randomly generated. In this way, the sensors and sinks are randomly distributed and located. The sensor and sink coordinates are then fixated as parameters in the grid free model and the lifetime of the resulting network is considered as the main performance metric. We provide a comparative analysis of the lifetimes of the networks created by random placement, networks found by the grid model and the networks produced by the grid free model in the numerical results section.

4 NUMERICAL RESULTS

In this section, we first explain how the parameters used in the formulations are selected. Then, the created test examples are solved with Gurobi mixed-integer linear/non-linear program solver [40] and the superiority of the grid-free approach is revealed.

4.1 Creating Test Problems

The creation of parameters and test problems to be used in the formulation is as follows: The grid structure was used to implement the first model. The number of sensors in the generated test instances is 4, 6, 9, 12, 15, 18 and 20. Four different problem sets are created for each instance with different number of sensors in which the coverage requirements of the locations are randomly generated. Candidate sensor locations are determined as intersection locations of the grid structure. The dimensions of the grid are adjusted to ensure that the area is as close to a square as possible. For example, the grid sizes for candidate sensor locations 12 and 20 are selected as 3 × 4 and 4 × 5, respectively. The distance between neighboring candidate sensor locations is assumed to be 60 meters and sinks are determined as the center points of the sensor grids. This grid structure created for candidate sensor locations will be called the "sensor grid". The candidate sink locations will also have a similar grid structure and therefore sensor and sink grids will exhibit an integrated structure. Fig. 3 shows a grid structure with a 3 × 3 square candidate sensor area for 9 candidate sensor locations.

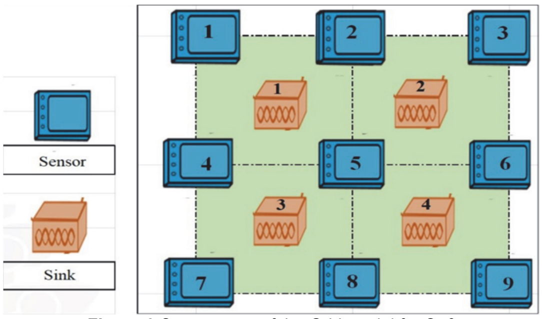

Figure 3 Sensor area of the Grid model for S=9

The second model is assumed to be applied in areas that do not have a grid structure. The locations of candidate sensors and sinks are not determined in advance, they can be placed anywhere. The locations where the sensors and sinks are placed in the model are considered as decision variables. An example is given in Fig. 4.

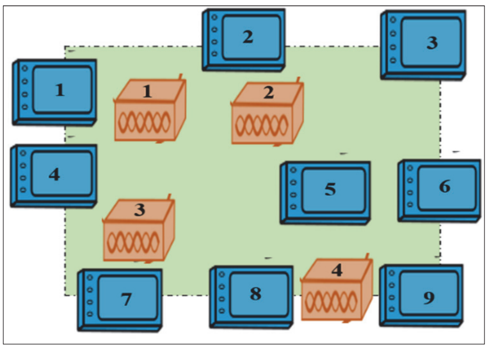

Figure 4 Sensor area of the Grid-free model for *S* = 9

The test parameters used in the study are given in Tab. 3. The amount of data produced by a sensor per unit time is 4096 bits/hour for both models [10]. The unit sensing and processing energy amount c^s and the unit data reception energy amount c^r of an active sensor are 50×10^{-6} joules per bit per hour. We assume that the battery energy of the sensor is 42 624 joules. We take the cost of placing sensor f_i at point i equal for each point i and consider the sensor placement budget B as 70% of the number of candidate sensor locations. On the other hand, we make the coverage locations coincide with the candidate sink locations. For each coverage location, coverage demands (d_k) are randomly assigned as 1, 2 or 3. The sensor placement budget is selected as the 70% of the required budget if all sensors are located. The CR and SR, are the same in both models. Assuming that there is an imaginary grid designed sensor area with S candidate sensor locations, we design the grid dimensions to have an area of $n_1 \times n_2$ (For example, for $S = 20$, $n_1 = 4$, $n_2 = 5$ is selected). Accordingly, the coordinates of coverage point of the grid-free model are determined as the same coverage points of the grid model. The determination details are given by Eq. (35) to Eq. (38) as:

$$C_1 = mod\left(k, n_2 - 2\right) \tag{35}$$

$$C_2 = \frac{k - C_1}{n_1 - 1} \tag{36}$$

$$a_k^3 = 30 + 60 \times C_1 \tag{37}$$

$$b_k^3 = 30 + 60 \times C_2 \tag{38}$$

where C_1 and C_2 are auxiliary variables used to determine the horizontal and vertical coordinates of the sensor area. n_2 represents the vertical dimension of the sensor area. C_1 gives the remainder of dividing k by (n_2 − 2) and determines which horizontal row k falls in the sensor area. n_1 represents the horizontal dimension of the sensor area. C_2 is calculated by subtracting k from C_1 and dividing by (n_1 − 1) and determines the vertical position of k in the sensor area.

Table 3 Test parameters

Symbol	The Grid Model Value	The Grid-Free Model Value
h	4.096 bit/hour	
c^s	50×10^{-6} joule	
c^r	50×10^{-6} joule	
c^t	$(50 + 1000) \times 10^{-6}$ joule	
e	42624 joule	
M	$10000 \times \lvert S \rvert$	
P	1 for $\lvert S \rvert = 4$, 2 for other $\lvert S \rvert$ values	
d_c	It is determined randomly within [0, 1, 2, 3]	
B	$0.7 \times \lvert S \rvert$	
f_i	1	
$\left(a_k^3, b_k^3\right)$	-	$\left(30 + 60 \times C_1, 30 + 60 \times C_2\right)$
CR	100	
SR	75	

4.2 Performance of the Grid Free Model

In this section, the network lifetimes obtained by Gurobi from four problem sets for all three approaches are calculated and compared. All models are coded in the Visual Studio environment with C# language, and problem tests are performed on an Intel Core i7-4770 quad-core CASPER computer with 8 GB RAM.

All three approaches are given a three-hour computation time for each test example. If the models reach the optimal solution in less than 3 hours, they immediately report the best solution found and moved to the next instance. The network lifetimes obtained for all models are presented in Tab. 4. The first column of the table shows the problem size, while the second, third and fourth columns contain the network lifetimes (denoted as L_G, L_R and L_{GF}) found for the grid model, random placement strategy and grid-free model respectively. The fifth and sixth columns show the percentage deviation between the network lifetimes calculated with the formula $100\times\frac{\left(L_{GF}-L_G\right)}{L_G}$ and $100\times\frac{\left(L_{GF}-L_R\right)}{L_R}$ respectively. Finally, the seventh, eighth and ninth columns present the computation times used by Gurobi for grid model and grid-free model (denoted as T_G, T_R and T_{GF}), respectively.

Table 4 Network lifetimes found by the grid model, random placement strategy and the grid-free model

Problem set	|S|, |N|	L_G	L_R	L_{GF}	%Deviation (G-GF)	%Deviation (R-GF)	T_G	T_R	T_{GF}
1	(4,1)	200 000,00	200 000,00	200 000,00	0,00	0,00	0,10	0,11	0,16
	(6,2)	37 840,91	37 840,91	37 840,91	0,00	0,00	0,16	0,31	9,67
	(9,4)	9 460,23	9 460,23	18 920,45	100,00	100,00	0,09	0,08	10 800,36
	(12,6)	18 920,45	18 920,45	28 380,68	50,00	50,00	0,11	0,19	10 800,44
	(15,8)	9 460,23	0,00	15 767,05	66,67	NA	0,16	0,05	10 800,26
	(18,10)	18 920,45	NA	NSF	NA	NA	0,32	NA	10 800,24
	(20,12)	9 460,23	NA	NSF	NA	NA	1,08	NA	10 800,05
2	(4,1)	18 920,45	18 920,45	18 920,45	0,00	0,00	0,14	0,21	0,20
	(6,2)	9 460,23	12 613,64	12 613,64	33,33	0,00	0,11	0,11	28,53
	(9,4)	12 613,64	15 767,05	18 920,45	50,00	20,00	0,18	0,07	10 800,04
	(12,6)	12 613,64	9 460,23	18 920,45	50,00	100,00	0,14	0,20	10 800,02
	(15,8)	9 460,23	NA	NSF	NA	NA	0,35	NA	10 800,08
	(18,10)	12 613,64	NA	NSF	NA	NA	4,34	NA	10 800,39
	(20,12)	9 460,23	NA	NSF	NA	NA	0,31	NA	10 800,04
3	(4,1)	18 920,45	18 920,45	18 920,45	0,00	0,00	0,14	0,21	0,13
	(6,2)	37 840,91	37 840,91	37 840,91	0,00	0,00	0,15	0,09	9,29
	(9,4)	14 190,34	9 460,23	28 380,68	100,00	200,00	0,20	0,04	10 800,03
	(12,6)	12 613,64	15 767,05	25 227,27	100,00	60,00	0,22	0,13	10 800,02
	(15,8)	12 613,64	0,00	18 920,45	50,00	NA	0,69	0,05	10 800,09
	(18,10)	12 613,64	NA	NSF	NA	NA	0,79	NA	10 800,05
	(20,12)	12 613,64	NA	NSF	NA	NA	0,55	NA	10 800,07
4	(4,1)	9 460,23	9 460,23	9 460,23	0,00	0,00	0,16	0,13	0,26
	(6,2)	12 613,64	9 460,23	12 613,64	0,00	33,33	0,20	0,07	160,95
	(9,4)	12 613,64	14190,34	15 767,05	25,00	11,11	0,14	0,09	10 800,04
	(12,6)	9 460,23	9460,23	9 460,23	0,00	0,00	0,20	22,31	10 800,05
	(15,8)	12 613,64	NA	NSF	NA	NA	0,53	NA	10 800,43
	(18,10)	9 460,23	NA	NSF	NA	NA	0,24	NA	10 800,07
	(20,12)	12 613,64	NA	NSF	NA	NA	0,50	NA	10 800,06
Ave.	(4,1)	61 825,28	61 825,28	61 825,28	0,00	0,00	0,14	0,17	0,19
	(6,2)	24 438,92	24 438,92	25 227,27	3,23	3,23	0,16	0,15	52,11
	(9,4)	12 219,46	12 219,46	20 497,16	67,74	67,74	0,17	0,07	10 800,04
	(12,6)	13 401,99	13 401,99	20 497,16	52,94	52,94	0,19	5,71	10 800,13
	(15,8)	11 036,93	0,00	17 343, 75	57,14	NA	0,52	0,05	10 800,22
	(18,10)	13 401,99	NA	NSF	NA	NA	1,79	NA	NA
	(20,12)	11 036,93	NA	NSF	NA	NA	0,45	NA	NA

As can be seen in Tab. 4, the grid-free model outperforms both the grid model and the random placement strategy in terms of network lifetime for instances with 15 or less candidate sensor locations. For example, in case (4,1), there is no difference between the models, while in case (6,2), the grid-free model yielded a network lifetime of 25227,27 units compared to an average network lifetime of 24238, 92 units for both the grid model and the random placement strategy. Thus, it yielded 3,23% better lifetime. In case (9,4), the grid-free model outperformed the grid and the random placement strategy by 67,74% with an average network lifetime of 20497,16 units compared to an average network lifetime of 12219,46 units for the grid model and random placement strategy. Similarly, in case (12,6), the grid-free model outperformed the grid and random placement strategy by 52.94%, with an average network lifetime of 20 497, 16 units, whereas the average network lifetime for the grid model and random placement strategy was 13 401, 99 units. Also, in case (15,8), the grid-free model provided a network lifetime of 57,14% longer than the grid model.

However, "Not Solvable Found" (NSF) occurs in the grid-free model in large-scale problems ((18,10) and (20,12)). This can be explained by the increase in memory and processing load at large problem sizes due to the complex optimization structure of the grid-free model. The random placement strategy generated based on the results of the grid-free model gave a "Not Applicable" (NA) error in cases where the grid-free model could not find a solution. This situation shows that the grid-free model has high memory requirements for large-scale problems and its applicability is limited.In terms of computational time, the random placement strategy is much faster than both the grid model and the grid-free model. For example, in the case of (9,4), the grid model produced results in an average of only 0,17 seconds, while the random placement strategy and the grid-free model required an average of 0,07 and 10 800,04 seconds for the same problem, respectively. The nonlinear conical structure of the grid-free model is the main reason for this long computational time. In addition, the solution times of the grid model and the random placement strategy are significantly faster than the grid-free model for other small and medium-sized problems. However, despite the fast solution times of the grid model and the random placement strategy, their performance in network lifetime optimization is limited.

The results are presented graphically in Fig. 5, Fig. 6 and Fig. 7 depending on the size of the problem (number of candidate sensor locations). Fig. 5 shows that the grid-free model provides higher network lifetime than the grid model and random placement strategy in networks with 6, 9 and 12 candidate sensor locations. However, it is observed that the network lifetimes of all models decrease as the problem size increases. This can be explained by the fact that it becomes more difficult to produce feasible solutions as the problem size increases and it becomes difficult to assist all critical sensors with a limited number of receivers. The percentage deviations shown in Fig. 6 support the network lifetime findings presented in Fig. 5 and confirm the superiority of the grid-free model in small and medium-sized networks.

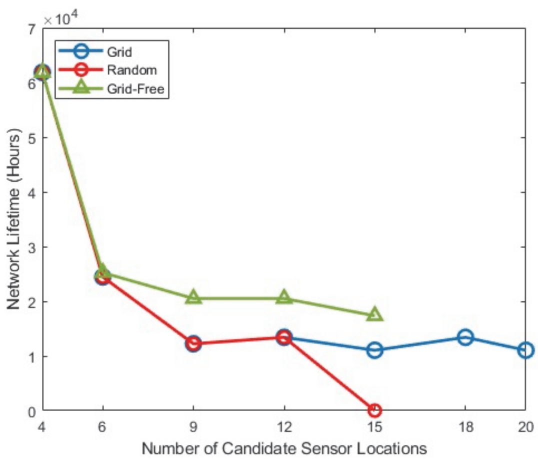

Figure 5 Grid Model and Grid-Free Model: Network lifetime

According to the results given in Fig. 7, the computation times are evaluated depending on the problem size. While the grid model and the random placement strategy exhibit very low and consistent computation times in all problem sizes, the grid-free model shows a low computation time at 4 and 6 candidate sensor locations, while it shows a constant trend at 9 and later candidate sensor locations.

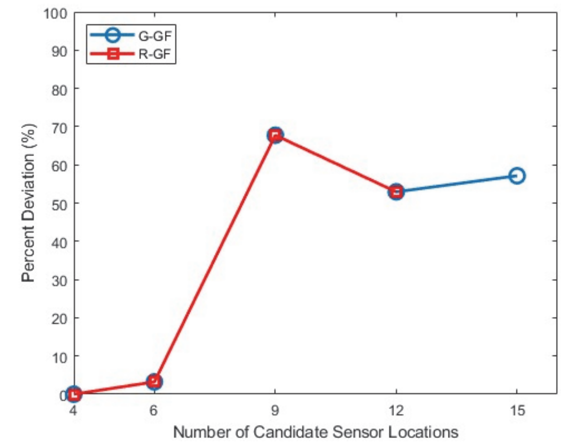

Figure 6 Percent Deviation

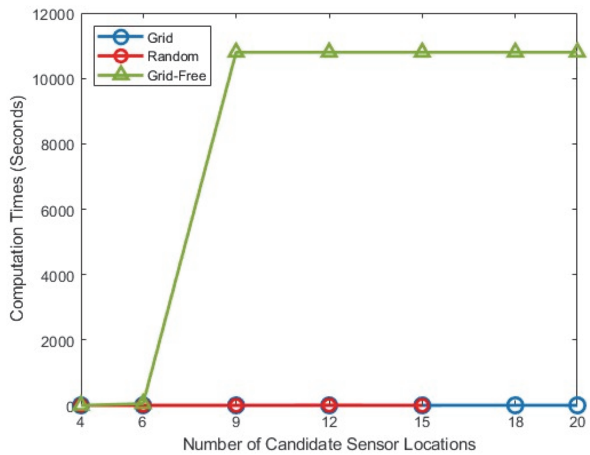

Figure 7 Computation Times

5 DISCUSSION AND CONCLUSIONS

In this study, a mathematical model is proposed to determine the locations and operation plans of sensors, sinks and data paths, including energy, budget and coverage constraints. This model aims to provide a longer network lifetime for a WSN without assuming a network based on the network structure. The proposed grid-free model is presented by comparing it with the mathematical optimization model developed by [10] for grid-based WSN design. In both models, decisions regarding location selection, operation planning and data path determination are made simultaneously. However, the grid-free model does not require a prior determination of candidate sensor and receiver locations by ignoring the network structure in the sensor domain. This provides more flexibility to the design process.

In the study, a random placement strategy was also developed in which sensors were placed randomly in order to demonstrate the performance of the grid-free model in real WSN networks. The test examples show that the grid-free model significantly increases the network lifetime in small and medium-sized wireless sensor networks. However, the relatively long solution times of this model and its inability to produce results for large-scale problems have emerged as an important limitation. Therefore, it is likely that a heuristic method will be developed in the future to reduce the computational time and that computer systems with larger memory will be needed.

It should be noted that we are not using simulation but mixed-integer and conical mixed-integer formulations. As a consequence, the use of custom simulation environments is not possible for our case. Hence, a lot of performance metrics reported by simulation tools are missing in the manuscript. We are able to report the network lifetime performance metric, though since it is optimized in the objective function of our formulations. However, our methodology provides a theoretical analysis for WSN design which is missing in most of the WSN simulation studies. For instance, mathematical optimization tools are able to lead to optimal WSN designs guaranteeing the longest network lifetimes without a loss in coverage of the sensor field.

Grid-based design offers an important approach to optimize coverage and connectivity for WSNs. However, the strengths and weaknesses of grid-based structures should be carefully evaluated to determine the most appropriate design method for each application. In this context, the flexibility advantage offered by grid-free structures with different protocols and algorithms is highlighted and research on alternative approaches is encouraged. This study can be extended by experimentally validating grid free approach in a simulation environment with realistic conditions which we plan to carry out in the future.

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