INTEGRATING PYTHAGOREAN FUZZY SOFT SETS WITH SWARA AND WASPAS FOR ENHANCED MCDM IN EDUCATION 5.0: A NOVEL APPROACH FOR NEXT GENERATION ENGINEERING SOLUTIONS

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Abstract:

This paper aims to introduce an innovative approach involving lattice implementation within the context of Pythagorean Fuzzy Soft Sets. It demonstrates several properties of this approach. The principal objective of this research is to amalgamate the Stepwise Weight Assessment Ratio Analysis (SWARA) and Weighted Aggregated Sum Product Assessment (WASPAS) methodologies, thus creating a comprehensive Multiple Criteria Decision-Making (MCDM) framework. This integration not only presents a new perspective in mathematical computing but also has intriguing principles. Within this integrated framework, alternative rankings are determined using the WASPAS approach, while the SWARA method is employed to generate criteria weights. The applicability of our proposed strategy is substantiated by addressing an MCDM challenge in the context of Education 5.0. The effectiveness and accessibility of this approach are evaluated by comparing its outcomes with those of previously established techniques.

1 Introduction

Decision making involves selecting a reasonable solution from a wide range of options and it is an essential skill in everyday situations. Better decision-making will alter the procedure for figuring out the restrictions, advantages, and traits of the DM. In an effort to cope with the intended situations, Zaheh[46] introduced the Fuzzy Set(FS) paradigm, which advances several scientific and technological domains by designating the membership grade for each item as real values between 0 and 1. In traditional set theory, a set's elements may either be 0 or 1, whereas in FS, the degree of membership can be any value between 0 and 1. By extending the idea of FS, Atanassov[4] proposed an intuitionistic fuzzy set (IFS) that took membership(MG) and nonmembership grades(NMG) into account. IFS is a potent idea that several scholars have examined since its creation. The prevalent IFS approach, however, has significant drawbacks, including the MG and NMG adopted such that their sum surpasses 1. IFS is unable to get around these restrictions and successfully avoids the aforementioned situations. Yager[45] changed the criteria MG + NMG = 1 to $MG^2 + NMG^2 = 1$, extending the concept of IFS and creating the Pythagorean fuzzy set (PFS).

Experts in a variety of sectors recognize and apply the aforementioned ideas and their accompanying DM techniques[51,52]. These methods, however, are unable to resolve parametrization issues because they lack parametrized values. Molodtsov[24] explored various fundamental operations and their characteristics while also presenting the answer to ambiguity and uncertainty. He also created a soft set (SS). Maji et al.'s extension of the FSS concept[22] and introduction of the IFSS[23] with some basic operations. Several researchers expanded on the SS idea by using the basic FSS definition. By changing the limitation MG + NMG

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= 1 to $MG^2 + NMG^2 = 1$with a few acceptable operations, Peng et al.[34] recommended the theory of IFSS to PFSS.

Yang et al. [44] also looked at an integrated theory of Multi-Fuzzy Soft Set (MFSS). Initiations of many advances are made in the MFSS[72]. Parallel to the development of fuzzy and soft set theories, lattice theory has emerged as a vital mathematical framework. Numerous academics used the context of lattice in FS and created numerous notions[74]. With the introduction of some ordering in the parameter space, the idea of a lattice approach to fuzzy multiset was introduced[73].Lattices, which provide a formal structure for ordering parameters, have found widespread applications in modeling complex decision-making processes[71]. Numerous investigations and findings are detailed in the lattice extensions of fuzzy sets[53]. Also, the theory of lattice was implemented on hyperfuzzy and provided an illustrative application to show their applicability[54].

One of the most crucial elements of the nation's infrastructure development is education. For improved benefits and outcomes, the education sector must evolve in concert with the modernization of general business in order to accommodate industrial and technical expansion. To create an engaging and dynamic learning environment, education must accept the usage of contemporary industrial and technology revolutions including artificial intelligence, robots, machine learning, and data analytics.

Education 5.0 is a revolutionary paradigm change in education that is ready to tackle the problems of our quickly changing global environment as well as the changing demands of students. It's a reaction to the fast-paced, technologically-driven modern world in which big data, augmented reality, artificial intelligence, and the Internet of Things are all prevalent. Education 5.0, the fifth generation of educational approaches, is evidently a logical development from its predecessors. The foundations of Education 1.0: traditional, teacher-centered; Education 2.0: interactive and learner-focused; Education 3.0: technology-enhanced; and Education 4.0: personalized and collaborative are all built upon this approach. Education 5.0 is a major shift from these earlier approaches in that it prioritizes individualized, technologically assisted, and goal-oriented learning experiences. With the extraordinary range of digital resources and instructional technology available to them in this day and age, students may now customize their educational paths to fit their individual requirements, hobbies, and professional goals. The foundation of Education 5.0 is the idea that learning should not be restricted to traditional classroom settings or set curricula, but rather should be dynamic, flexible, and available at any time and place. The main objective of this approach is to give students the knowledge, abilities, and flexibility they need to prosper in a world that is changing all the time. The purpose of this prologue is to clarify the importance and necessity of Education 5.0, which opens the door to a more flexible and learner-centered educational environment that better meets the needs of the twenty-first century.

There are several studies being conducted on the aspect of Education 5.0. As indicated by M. N. Rahim[36] in his discussion of the need to reform education 5.0, the goal of this study is to lead the global educational transformation in Afghanistan in order to prepare higher education institutions for the post-COVID-19 pandemic. An examination of educators' attitudes regarding the adoption of Education 5.0 was the focus of a study by D. R. Muzira and B. M. Bondai[31] that was carried out at a public university in Zimbabwe. The investigation by M. Togo and C.P. Gandidzanwa[41] outlined the contribution of Education 5.0 to expediting the realization of the SDGs as well as the difficulties the University of Zimbabwe faced. W. Muchabaiwa et al.[30] investigated the effects of the Education 5.0 framework on the prospects for female academics to further their careers in Zimbabwean universities. Implementation of Education 5.0 in Developed and Developing Countries was compared by A.M. Alharbi[3]. Through a thorough literature study, this paper examines the implementation of Education 5.0 in both industrialized and developing nations. Figure 1 depicts the development of an educational system from 1.0 to 5.0 from the ninth to the twenty-first century with their respective educational framework styles. Since both academic institutions and technology have been working together to change education. In this research study, a mathematical model has been presented for selecting a suitable educational organization for the transformation of Education 5.0 to achieve sustainability goals by examining the characteristics of the institution.

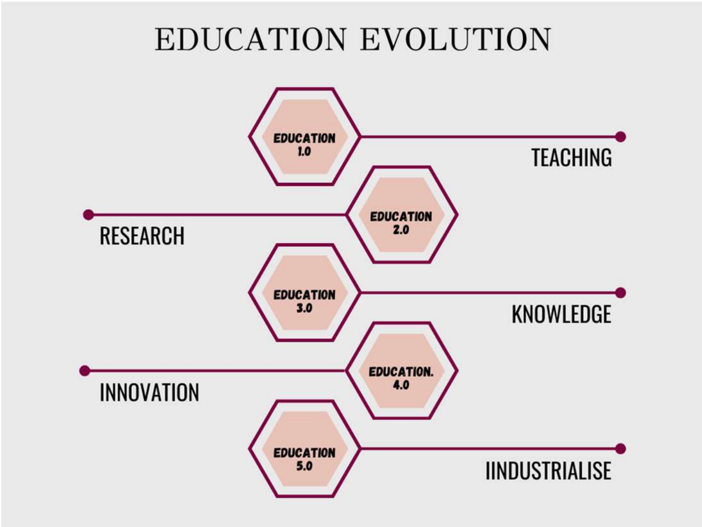

Figure 1: Evolution of Education 5.0

1.1 Literature Review

PFS has wide range of applications in various fields of study. The PFS extension was used to solve a variety of transport problems [51]. Additionally, a number of studies addressed the aggregation operators for IFS [56] and PFS [57]. Spherical fuzzy sets with triplet grade values were introduced as an extension of PFS [58]. Additionally, fuzzy extension was used in algebraic ideas such as bi-ideals [65] and lie subalgebra [64]. S.A. Edalatpanah introduced the basic functions of PFS [66]. Stock portfolio optimisation challenges were solved using Pythagorean fuzzy numbers [6]. Additionally, Adak and Kumar [5] introduced the spherical distance measurement for the PFS framework with MCDM application. The discussion by Subha and Dhanalakshmi [67] focused on the similarity measurements for interval valued PFS.

One of the key issues throughout the MCDM process is the significance weights of the criterion[59]. Subjective and objective weights for criteria are the two forms that are covered in the literatures. The information in decision matrices provides the objective weights, whereas the knowledge provided by the DEs is used to estimate the subjective weights[13]. Various writers have devised various methods for assessing the weights of objective criteria[29]. Using the SWARA methodology, Kersuliene et al.[16] introduced a novel effective technique for calculating subjective criterion weights.

In the framework of the FS, Karabasevic et al.[18] presented an object for people analysis based on the Additive Ratio Assessment (ARAS) and SWARA methodologies. The SWARA[62] and WASPAS techniques and their applicability in various fuzzy contexts were thoroughly reviewed by Mardani et al. in their study[27]. For the evaluation of the challenge of choosing renewable energy techniques, Maghsoodi et al.[28] proposed a combined technique of the SWARA and MULTIMOORA approaches. To address MCDM issues, Ghorabaee et al.[17] developed a fuzzy hybrid approach using the SWARA, the CRITIC method, and the Evaluation Based on Distance from Average Solution (EDAS) methodologies. Chusi et.al[61] implemented SWARA method in interval-valued spherical fuzzy set and applied it in carbon credit market. The hybrid concept of SWARA-MARCOS approach was applied in the selection of smart logistic enterprise in triangular fuzzy framework[63].

One of the most recent MCDM techniques developed by Zavadskas et al. is named WASPAS[48]. It incorporates both the Weighted Product Model (WPM) and the Weighted Sum Model (WSM)[49]. The WSM and WMP model outputs are combined to compute the joint generalized criteria value, which is used to rank the options. The WASPAS approach has been shown to be highly efficient and successful in the decision-making process. Its key benefits are computational simplicity, consistency of results, and great resistance to alternate rank reversal. Using the WASPAS with quantitative strategic planning matrix (QSPM) methodologies, Lashgari et al.[19] determined the optimum healthcare outsourcing strategy. The WASPAS approach was utilized by Chakraborty et al.[8] to resolve selection issues involving five unconventional machining process parameters. Madic et al.[26] performed an economic assessment of several machining processes using the WASPAS technique to select the most suitable machining procedures. Reddy et al.'s[43] multi-response optimization using WASPAS and a well-known multi-objective optimization based on the ratio analysis method. The use of WASPAS was demonstrated by Chakraborty et al.[7] by resolving selection issues that arise in the manufacturing sector. Despite all of its benefits, the WASPAS approach cannot manage information that is imprecise, hazy, or ambiguous. Thus Turskis et al.[41] implemented the WASPAS approach using FS, whereas Stanuikis and Karabasevic[39] implemented the WASPAS method based on IFS.

The SWARA technique is used in the suggested approach to find the subjective weights of the assessment criteria, and the WASPAS method is used in the PF context to determine the preference order of the alternatives. The imprecision of experts' judgments is considered in the criterion weights examined by this method, which makes them easier to understand. A case study of Education 5.0 in a PF context is used to demonstrate the computational process of the suggested technique, demonstrating its relevance and effectiveness. This research additionally performs sensitivity analysis to demonstrate the robustness of the developed framework.

1.2. Contribution

The study made the following contributions:

- **Introduction of the proposed Structure:** This study offers a new method for organizing and arranging parameters in decision-making processes with the Lattice Ordered Pythagorean Fuzzy Soft Sets (LOPFSS) framework. A crucial necessity in complicated choice scenarios is met by LOPFSS, which offers a framework for handling ambiguity and vagueness.
- **Integration of WASPAS and SWARA:**The integration of the SWARA and WASPAS methodology within the LOPFSS framework is an important aspect of this study. By integrating the advantages of different approaches, this integration improves the decision-making process.
- **Development of LOPFS-SWARA-WASPAS Model:**The study describes the development of the LOPFS-SWARA-WASPAS model. This model efficiently addresses ambiguity and uncertainty in decision-maker judgments by offering a comprehensive solution for ranking alternatives and calculating criterion weights.
- **Application to Educational Structure Evaluation:**Using an assessment of a qualified educational structure, especially in light of Education 5.0, the study illustrates the usefulness of the integrated framework. This application demonstrates the LOPFS-SWARA-WASPAS model's flexibility and applicability in actual decision-making situations.

1.3. Research Gap and Novelty

Despite the growing body of research on Pythagorean fuzzy soft sets and their applications, there remains a significant gap in the literature concerning the implementation of lattice structures within this framework. To date, no studies have explored the integration of lattice theory with Pythagorean fuzzy soft sets, leaving their potential unexplored. Furthermore, while methods such as SWARA and WASPAS are well-regarded in Multiple Criteria Decision-Making (MCDM), their combined application within the context of Pythagorean fuzzy soft sets has not been introduced. Even beyond this specific set type, the hybrid integration of SWARA and WASPAS remains absent in the existing body of work across other types of fuzzy sets. To address these gaps, this research introduces a novel approach that incorporates lattice-ordered Pythagorean fuzzy soft sets and merges SWARA and WASPAS methodologies. This innovative framework not only fills the identified research gaps but also establishes a new direction for advancements in decision-making models, highlighting the novelty and significance of the study.

1.3. Motivation

Decision-making processes have grown more complex, characterized by a profusion of alternatives and a growing incidence of ambiguity and vagueness, in an era marked by quickly shifting technological, societal, and economic landscapes. In order to achieve the best results in a variety of areas, such as business, public policy, and education, it is imperative that decision-makers address these difficulties.

The interaction of the LOPFSS framework with the WASPAS methodology and the SWARA method represents a promising solution to these challenges. LOPFSS presents a new paradigm for parameter orders, providing an organized and methodical way to deal with uncertainty. The combination of LOPFSS, SWARA, and WASPAS results in the LOPFS-SWARA-WASPAS model, a strong and complete framework for decision-making that can precisely compute criterion weights and rank alternatives. This research is also driven by the suggested framework's applicability to the field of education, particularly in light of Education 5.0, which prioritizes outcome-focused, technology-driven, and individualized learning methods. Developing a qualified educational structure that aligns with the objectives of Education 5.0 requires the ability to navigate ambiguity and reluctance effectively.

The goal of this research work aims to fill the current gap[66] in decision support systems and give decision-makers a reliable, proven, and strong tool to help them make decisions in a world that is getting more and more complicated. This study intends to enhance decision-making processes and promote breakthroughs in various fields by providing a workable answer to the problems of ambiguity and hesitation. Ultimately, this will lead to better outcomes and the improvement of society.

1.5. Organization of this study

The research is organized as follows: Chapter 2 provides the preliminaries, introducing foundational concepts, terminologies, and existing frameworks related to Pythagorean fuzzy soft sets, lattice theory. Chapter 3 develops the theoretical framework for lattice-ordered Pythagorean fuzzy soft sets, presenting key theorems, propositions, and properties to establish their mathematical foundation. Chapter 4 proposes the integration of the SWARA and WASPAS methodologies tailored for lattice-ordered Pythagorean fuzzy soft sets, detailing the method and showcasing its practical utility through a case study on Education 5.0, including numerical applications. Chapter 5 analyzes the results, providing comparative and superiority analyses to highlight the advantages and effectiveness of the proposed approach against existing methods. Finally, Chapter 6 concludes the study by summarizing the findings, discussing implications, and outlining limitations and potential future research directions, such as extending applications or refining the methodology further.

2 Preliminaries

Throughout this section, Θ is a universal set, Z is a set of parameters.

Definition 1:[46] The FS on Θ is designated as

$$F = \{(p, \mu(p)) : p \in \Theta\}$$

where μ is a membership function from Θ to [0,1].

Definition 2:[22] Let Z be the collection of parameters. The fuzzy soft set (U, Z) is a mapping defined by U : Z → P(Θ) , where P(Θ) is set of all fuzzy subsets in Θ.

Definition 3:[5] The fuzzy soft set (U, Z) on Θ is said to be lattice ordered fuzzy soft set on Θ if

$$\sigma_1 \le \sigma_2 \Rightarrow U(\sigma_1) \subseteq U(\sigma_2), \forall \sigma_1, \sigma_2 \in Z$$

Definition 4:[4] The IFS on Θ is designated as

$$I = \{p, \prec \mu_I(p), \nu_I(p) \succ : p \in \Theta\}$$

where $\mu_I(p), \nu_I(p)$ are MG and NMG which belongs to [0,1] subject to the condition $0 \le \mu_I(p) + \nu_I(p) \le 1$.

Definition 5: [45] The PFS on Θ is designated as

$$P = \{p, \prec \mu_p(p), \nu_P(p) \succ : p \in \Theta\}$$

where $\mu_P(p), \nu_P(p)$ are MG and NMG which belongs to [0,1] depending on the circumstance $0 \le \mu_P^2(p) + \nu_P^2(p) \le 1$.

Definition 6: [32] Let $L \subseteq Z$ The Pythagorean fuzzy soft set (R, L) is a mapping defined by R : L → $PFS(\Theta)$ where PFS is the collection of Pythagorean fuzzy set over Θ.

Note: Hereafter, the set (R, L) is simply denoted as (R_L, Z).

3 Lattice ordered Pythagorean Fuzzy Soft set

In this section, the concept of lattice ordered Pythagorean fuzzy soft set, anti-lattice ordered Pythagorean fuzzy soft set, and some of its basic operations are defined.

Definition 7: Let $L \subseteq Z$. A pythagorean fuzzy soft set (R_L, Z) is called lattice ordered pythagorean fuzzy soft set, if for the mapping, $R_L: L \to PFS(\Theta)$, for all $\sigma_1, \sigma_2 \in L$,

$$\sigma_1 \le \sigma_2 \Longrightarrow R_L(\sigma_1) \subseteq R_L(\sigma_2)$$

i.e., $\mu_{R_L(\sigma_1)}(p) \le \mu_{R_L(\sigma_2)}(p)$ and $\nu_{R_L(\sigma_1)}(p) \le \nu_{R_L(\sigma_2)}(p), \forall p \in \Theta$.

In further discussion, LOPFSS(Θ) denotes the set of all lattice ordered Pythagorean fuzzy soft sets of Θ.

Example 1. Let X = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set of designs for necklaces, E = {e_0 (Bronze), e_1 (Silver), e_2 (Gold), e_3 (White Gold), e_4 (Platinum), e_5 (Diamond)} be the set of parameters to represent manufacturing materials and A ⊆ E.

Let A = {e_0 (Bronze), e_1 (Silver), e_2 (Gold), e_3 (White Gold), e_4 (Platinum)}. Then the lattice ordered Pythagorean fuzzy soft sets are given in Table 1.

R_L	x_1	x_2	x_3
e_0	(0.1,0.5)	(0.1,0.6)	(0.15,0.8)
e_1	(0.3,0.4)	(0.1,0.2)	(0.15,0.5)
e_2	(0.2,0.5)	(0.4,0.4)	(0.15,0.6)
e_3	(0.1,0.4)	(0.2,0.5)	(0.3,0.6)
e_4	(0.6,0.4)	(0.8,0.1)	(0.6,0.4)

Table 1: Example of LOPFSS

Clearly, $R_A(e_0) \subseteq R_A(e_1) \subseteq R_A(e_4)$,
$R_A(e_0) \subseteq R_A(e_2) \subseteq R_A(e_4)$,
$R_A(e_0) \subseteq R_A(e_3) \subseteq R_A(e_4)$ Hence, it is a LOPFSS.

Definition 8. Let $(R_L, Z), (R_M, Z)$ be two LOPFSS over Θ. Then (R_L, Z) is said to be pythagorean fuzzy soft subset of (R_M, Z) if $L \subseteq M \Rightarrow R_L(\sigma) \subseteq R_M(\sigma), \forall \sigma \in L$.
We represent it as $(R_L, Z) \subseteq (R_M, Z)$.

Definition 9. Let $(R_L, Z), (R_M, Z)$ be two LOPFSS over Ω. Then (R_L, Z) and (R_M, Z) is said to be equal if $(R_L, Z) \subseteq (R_M, Z)$ and $(R_M, Z) \subseteq (R_L, Z)$.

Definition 10. Let (R_L, Z) be a LOPFSS over Θ. Then the complement of (R_L, Z) is defined by $R_L^c = (R_L^c, \neg L)$ where $R_L^c: \neg L \to PFS(\Omega)$ is a mapping defined by $R_L^c(\sigma) = \big(R_L(\neg\sigma)\big)^c \ \forall \neg\sigma \in L$.

Definition 11. Let $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. Then restricted union of (R_L, Z) and (R_M, Z) is defined by $(R_L, Z) \cup_R (R_M, Z) = (R_U, Z)$, where $U = L \cap M$.
$\forall \sigma \in U, p \in \Theta$

$R_U(\sigma) = R_L(\sigma) \cup R_M(\sigma)$
i.e., $\mu_{R_U}(\sigma)p = max\{\mu_{R_L(\sigma)}(p), \mu_{R_M(\sigma)}(p)\}, \ \nu_{R_U(\sigma)}(p) = min\{\nu_{R_L(\sigma)}(p), \nu_{R_M(\sigma)}(p)\}$.

Definition 12. Let $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. Then restricted intersection of (R_L, Z) and (R_M, Z) is defined by $(R_L, Z) \cap_R (R_M, Z) = (R_T, Z)$, where $T = L \cap M$.
$\forall \sigma \in T, p \in \Omega$
$R_T(\sigma) = R_L(\sigma) \cup R_M(\sigma)$
i.e., $\mu_{R_U}(\sigma)(p) = min\{\mu_{R_L(\sigma)}(p), \mu_{R_M(\sigma)}(p)\}, \ \nu_{R_U(\sigma)}(p) = max\{\nu_{R_L(\sigma)}(p), \nu_{R_M(\sigma)}(p)\}$

Definition 13. Let $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. Then extended union of (R_L, Z) and (R_M, Z) is defined by $(R_L, Z) \cup_E (R_M, Z) = (R_L, U)$, where $U = L \cup M$.

$$R_U(\sigma) = \begin{Bmatrix} R_M(\sigma) & \text{if } \sigma \in M - L \\ R_L(\sigma) & \text{if } \sigma \in L - M \\ max\{R_L(\sigma), R_M(\sigma)\} & \text{if } \sigma \in M \cap L. \end{Bmatrix}$$

Definition 14. Let $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. Then extended intersection of (R_L, Z) and (R_M, Z) is defined by $(R_L, Z) \cap_E (R_M, Z) = (R_T, Z)$, where $T = L \cup M$.

$$R_T(\sigma) = \begin{Bmatrix} R_M(\sigma) & \text{if} \sigma \in M - L \\ R_L(\sigma) & \text{if} \sigma \in L - M \\ min\{R_L(\sigma), R_M(\sigma)\} & \text{if} \sigma \in M \cap L. \end{Bmatrix}$$

Theorem 15. *Let* $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. *Then* $(R_L, Z) \cup_R (R_M, Z) \in LOPFSS(\Theta)$.

Proof. Given that $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$.
By the definition, we have
$R_L(e) \cup R_M(e) = R_N(e)$, where $N = L \cap M$, if $L \cap M = \emptyset$, then the result is true.
Now, for $L \cap M \neq \emptyset$.
Since, $L, M \subseteq Z$, so L and M inherit the partial order from Θ, therefore for any $l_1 \leq_L l_2$, we have $R_L(l_1) \subseteq R_M(l_2), \forall l_1, l_2 \in L$, also for any $m_1 \leq_M m_2$, we have $R_M(m_1) \subseteq R_M(m_2), \forall m_1, m_2 \in M$.
Now, for any $n_1, n_2 \in N$ and $n_1 \leq_N n_2$.
$\Rightarrow n_1, n_2 \in L \cap M$
$\Rightarrow n_1, n_2 \in L$ and $n_1, n_2 \in M$.
$\Rightarrow R_L(n_1) \subseteq R_L(n_2)$ and $R_M(n_1) \subseteq R_M(n_2)$, whenever $n_1 \leq_L n_2, n_1 \leq_M n_2$
$\Rightarrow \mu_{R_L(n_1)}(p) \leq \mu_{R_L(n_2)}(p), \mu_{R_M(n_1)}(p) \leq \mu_{R_M(n_2)}(p)$ and $\nu_{R_L(n_1)}(p) \leq \nu_{R_L(n_2)}(p), \nu_{R_M(n_1)}(p) \leq \nu_{R_M(n_2)}(p)$
$\Rightarrow max\{\mu_{R_L(n_1)}(p), \mu_{R_L(n_2)}(p)\} \leq max\{\mu_{R_M(n_1)}(p), \mu_{R_M(n_2)}(p)\}$ and $min\{\nu_{R_L(n_1)}(p), \nu_{R_L(n_2)}(p)\} \leq min\{\nu_{R_M(n_1)}(p), \nu_{R_M(n_2)}(p)\}$
$\Rightarrow \mu_{R_N(n_1)}(p) \leq \mu_{R_N(n_2)}(p)$ and $\nu_{R_N(n_1)}(p) \leq \nu_{R_N(n_2)}(p)$
$\Rightarrow R_N(n_1) \subseteq R_N(n_2), \forall n_1 \leq_N n_2$.
$\therefore (R_L, Z) \cup_R (R_M, Z) \in LOPFSS(\Theta)$.

Example 2. Let $\xi = \{x_1, x_2, x_3\}$ be the set of houses, E = $\{e_1$ (large houses), e_2 (very large houses), e_3 (huge houses), e_4 (very huge houses)$\}$ be the set of parameters and A, B $\subseteq$ E, A = $\{e_1$ (large houses), e_2 (very large houses), e_3 (huge houses)$\}$, B = $\{e_2$ (very large houses), e_3 (huge houses), e_4 (very huge houses)$\}$.

(R_A, E)	x_1	x_2	x_3
e_1	(0.1,0.5)	(0.1,0.6)	(0.15,0.8)
e_2	(0.3,0.4)	(0.1,0.2)	(0.15,0.5)
e_3	(0.6,0.4)	(0.8,0.1)	(0.6,0.4)

Table 2: LOPFSS (R_A, E)

(R_B, E)	x_1	x_2	x_3
e_2	(0.15,0.9)	(0.2,0.7)	(0.3,0.7)
e_3	(0.2,0.7)	(0.3,0.6)	(0.4,0.6)
e_4	(0.3,0.5)	(0.6,0.4)	(0.55,0.4)

Table 3: LOPFSS (R_B, E)

Therefore $(R_A, Z), (R_B, E) \in LOPFSS(\Theta)$. Then the restricted union of $(R_A, E), (R_B, E)$ is given in the table 4.

(R_C, E)	x_1	x_2	x_3
e_2	(0.3,0.4)	(0.2,0.2)	(0.3,0.5)
e_3	(0.6,0.4)	(0.8,0.1)	(0.6,0.4)

Table 4: LOPFSS (R_C, E)

$\therefore (R_A, \Theta) \cup_R (R_B, \Theta) \in LOPFSS(\Theta)$.

Theorem 16. *Let (R_L, Z), $(R_M, Z) \in LOPFSS(\Theta)$. Then $(R_L, Z) \cap_R (R_M, Z) \in LOPFSS(\xi)$.*

Proof. The proof follows from the definition and theorem. □

Proposition 17. Let $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$. Then $(R_L, Z) \cup_E (R_M, Z) \in LOPFSS(\Theta)$ if one of them is a lattice ordered pythagorean fuzzy soft subset of other.

Proof. Given that $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$.
Then $(R_L, Z) \cup_E (R_M, Z) = (R_N, \Theta)$, $R_L(e) \cup R_M(e) = R_N(e)$, where $N = L \cup M$, $\forall e \in N, p \in \xi$
By definition, Suppose, $(R_L, Z) \subseteq (R_M, Z)$.
Then $L \subseteq M$ implies $\mu_{R_L(e)}(x) \leq \mu_{R_M(e)}(x)$ and $\nu_{R_L(e)}(x) \geq \nu_{R_M(e)}(x)$, $\forall p \in \xi$.
Since $L, M \subseteq \Theta$, so L and M inherit the partial order from Θ, therefore for any $l_1 \leq_L l_2$, we have $R_L(l_1) \subseteq R_L(l_2), \forall l_1, l_2 \in L$, also for any $m_1 \leq_M m_2$, we have $R_M(m_1) \subseteq R_M(m_2), \forall m_1, m_2 \in M$.
Therefore, for any $n_1, n_2 \in N$ and $n_1 \leq_N n_2$, implies $n_1, n_2 \in L \cap M$ implies $n_1, n_2 \in L \cap M$ or $n_1, n_2 \in L$ and $n_1, n_2 \notin M$, because $L \subseteq M$
Let $n_1, n_2 \in L \cap M$.
$\Rightarrow n_1, n_2 \in L$ and $n_1, n_2 \in M$.
$\Rightarrow R_L(n_1) \subseteq R_L(n_2)$ and $R_M(n_1) \subseteq R_M(n_2)$, whenever $n_1 \leq_L n_2, n_1 \leq_M n_2$
$\Rightarrow \mu_{R_L(n_1)}(p) \leq \mu_{R_L(n_2)}(p), \mu_{R_M(n_1)}(p) \leq \mu_{R_M(n_2)}(p)$ and $\nu_{R_L(n_1)}(p) \leq \nu_{R_L(n_2)}(p), \nu_{R_M(n_1)}(p) \leq \nu_{R_M(n_2)}(p)$
$\Rightarrow max\{\mu_{R_L(n_1)}(p), \mu_{R_L(n_2)}(p)\} \leq max\{\mu_{R_M(n_1)}(p), \mu_{R_M(n_2)}(p)\}$ and $min\{\nu_{R_L(n_1)}(p), \nu_{R_L(n_2)}(p)\} \leq min\{\nu_{R_M(n_1)}(p), \nu_{R_M(n_2)}(p)\}$

$\Rightarrow \mu_{R_N(n_1)}(x) \leq \mu_{R_N(n_2)}(x)$ and $\nu_{R_N(n_1)}(x) \leq \nu_{R_N(n_2)}(x)$
$\Rightarrow R_N(n_1) \subseteq R_N(n_2), \forall n_1 \leq_N n_2$.
$\therefore (R_L, Z) \cup_E (R_M, Z) \in LOPFSS(\Theta)$, if $n_1, n_2 \in L \cap M$.
Now, suppose for any $n_1, n_2 \in L$ and $n_1, n_2 \notin M$ and $n_1 \leq_L n_2$,
$\Rightarrow R_L(n_1) \subseteq R_L(n_2)$, whenever $n_1 \leq_L n_2$, implies this is also a $LOPFSS$.
Hence, $(R_L, Z) \cup_E (R_M, Z) \in LOPFSS(\Theta)$, for both cases.
$\Rightarrow (R_L, Z) \cup_E (R_M, Z) \in LOPFSS(\Theta)$, if one of them is a $LOPFS$ subset of other.

Proposition 18. *Let $(R_L, Z), \in LOPFSS(\Theta)$. Then*

1. $(R_L, Z) \cap_R (R_L, Z) = (R_L, Z)$
1. $(R_L, Z) \cup_R (R_L, Z) = (R_L, Z)$
2. $(R_L, Z) \cap_R (R_\emptyset, E) = (R_\emptyset, E)$
3. $(R_L, Z) \cap_R (R_\emptyset, E) = (R_L, Z)$

Proof. The proof is obvious.

Proposition 19. Let $(\mathrm{R_L}, \mathrm{Z}), \in$ LOPFSS(Θ). Then the complement of $(\mathrm{R_L}, \mathrm{Z})$ is an anti lattice ordered pythagorean fuzzy soft set over Θ.

Proof. Given that $(R_L, Z) \in LOPFSS(\Theta)$.
For any $\sigma_1 \leq_L \sigma_2$, we have $R_L(\sigma_1) \subseteq R_L(\sigma_2)$

$$\Rightarrow \mu_{R_L(\sigma_1)}(p) \leq \mu_{R_L(\sigma_2)}(p), \nu_{R_L(\sigma_1)}(p) \leq \nu_{R_L(\sigma_2)}(p)$$
$$\Rightarrow \mu_{R_L(\sigma_1)}(p) \leq \mu_{R_L(\sigma_2)}(p), \nu_{R_L(\sigma_1)}(p) \leq \nu_{R_L(\sigma_2)}(p)$$

$\Rightarrow R_L(\sigma_1) \subseteq R_L(\sigma_2)$, whenever $\sigma_1 \leq_L \sigma_2$.
$(R_L, Z)^c$ is an anti lattice ordered pythagorean fuzzy soft set over Θ.

Proposition 20. *Let* $(R_L, Z), \in LOPFSS(\Theta)$. *Then* $((R_L, Z)^c)^c = (R_L, Z)$.

Proof. $(R_L, Z), \in LOPFSS(\Theta)$.
Then the complement of (R_L, Z) is $\mu_{R_L^c(\sigma_1)}(p) = \nu_{R_L(\sigma_2)}(p), \nu_{R_L^c(\sigma_1)}(p) \leq \mu_{R_L(\sigma_2)}(p)$
Now, the complement of $(R_L, Z)^c$ is $\mu_{(R_L^c)^c(\sigma_1)}(p) = \nu_{R_L^c(\sigma_2)}(p) = \mu_{R_L(\sigma_1)}(p), \nu_{(R_L^c)^c(\sigma_1)}(p) \leq \mu_{R_L^c(\sigma_2)}(p) = \nu_{R_L(\sigma_1)}(p)$
$\Rightarrow ((R_L, Z)^c)^c = (R_L, Z)$.

Proposition 21. *Let* $(R_L, Z), (R_M, Z) \in LOPFSS(\Theta)$, *then* $(R_L, Z) \cap_R (R_M, Z)$ *is the largest* $LOPFSS$ *over* Θ, *which is contained in* (R_L, Z) *and* (R_M, Z).

Proof. Let $(R_N, \Theta) = (R_L, Z) \cap_R (R_M, Z)$, where $N = L \cap M \neq \emptyset$, and
for any $n \in N$, we have
$\mu_{R_N(n)}(p) = min\{\mu_{R_L(n)}(p), \mu_{R_M(n)}(p)$ and
$\nu_{R_N(n)}(p) = max\{\nu_{R_L(n)}(p), \nu_{R_M(n)}(p)$,
then $\mu_{R_N(n)}(p) \leq \mu_{R_L(n)}(p)$ and $\mu_{R_N(n)}(p) \leq \mu_{R_L(n)}(p)$
and also, $\nu_{R_N(n)}(p) \leq \nu_{R_L(n)}(p)$ and $\nu_{R_N(n)}(p) \leq \nu_{R_L(n)}(p)$
$\Rightarrow R_N(n) \subseteq R_L(n)$ and $R_N(n) \subseteq R_M(n)$

$\Rightarrow R_N(n) \subseteq R_L(n) \cap_R R_M(n)$
$\Rightarrow R_N(n) \subseteq (R_L \cap_R R_M)(n)$
Thus, $(R_N, Z) \subseteq (R_L, Z) \cap_R (R_M, Z)$.
Suppose (R_O, Z) is another $LOPFSS(\Theta)$ contained in both (R_L, Z) and (R_M, Z).
Then $R_O(n) \subseteq R_L(n) \cap_R R_M(n)$ and $R_O(n) \subseteq (R_L \cap_R R_M)(n)$
$\Rightarrow R_O(n) \subseteq (R_N, Z), \forall n \in O$, where $O = L \cap N \neq \emptyset$,
it implies $(R_O, Z) \subseteq (R_N, Z)$.
$\Rightarrow (R_N, Z)$ is the largest $LOPFSS$ over Θ, that contained in (R_L, Z) and (R_M, Z).

4 Lattice Ordered Pythagorean Fuzzy Soft Number

For the sake of simplicity, the pair $\beta = P(\mu_\beta, \nu_\beta)$ is said to be Lattice Ordered Pythagorean fuzzy soft number ($LOPFSN$).

Definition 22. Consider any two $LOPFSNs$, $\beta_1 = P(\mu_{\beta_1}, \nu_{\beta_1}), \beta_2 = P(\mu_{\beta_2}, \nu_{\beta_2})$, then the operational laws can be defined as follow:

1. $\beta_1 \oplus \beta_2 = P\left(\sqrt{\mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1}^2 . \mu_{\beta_2}^2}, \nu_{\beta_1}^2 . \nu_{\beta_2}^2\right)$
2. $\beta_1 \otimes \beta_2 = P\left(\mu_{\beta_1}^2 . \mu_{\beta_2}^2, \sqrt{\nu_{\beta_1}^2 + \nu_{\beta_2}^2 - \nu_{\beta_1}^2 . \nu_{\beta_2}^2}\right)$
3. $\lambda\beta_1 = P\left(\sqrt{1 - \left(1 - \mu_{\beta_1}^2\right)^{\lambda}}, \left(\nu_{\beta_1}\right)^{\lambda}\right), \lambda \geq 0$
4. $\beta_1^{\lambda} = P\left(\left(\mu_{\beta_1}\right)^{\lambda}, \sqrt{1 - \left(1 - \nu_{\beta_1}^2\right)^{\lambda}}\right), \lambda \geq 0$

4 A hybrid SWARA-WASPAS algorithm for LOPFSS

Step 1: Construct the LOPFS decision matrix for each decision expert M_i.

$$M_i = \begin{bmatrix} \eta_{11}^{(i)} & \eta_{12}^{(i)} & \cdots & \eta_{1n}^{(i)} \\ \eta_{21}^{(i)} & \eta_{22}^{(i)} & \cdots & \eta_{2n}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{m1}^{(i)} & \eta_{m2}^{(i)} & \cdots & \eta_{mn}^{(i)} \end{bmatrix} \tag{1}$$

where $\eta_{11}^{(i)}$ is a LOPFSN.

Step 2: Aggregate the matrix M_i using the following PFWA operator.

$$PFWA(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n) = w_1\alpha_1 \oplus w_2\alpha_2 \oplus w_3\alpha_3 \oplus \ldots \oplus w_n\alpha_n$$

$$= \sqrt{1 - \prod_{i=1}^{n}(1 - \mu_{\alpha_i}^2)^{w_i}}, \prod_{i=1}^{n}(\vartheta_{\alpha_i})^{w_i} \tag{2}$$

where $w_i \in [0,1] and \sum_{i=1}^{n} w_i = 1$. Then the weighted aggregrated matrix is

$$M = \begin{bmatrix} \oplus w_i\eta_{11}^{(i)} & \oplus w_i\eta_{12}^{(i)} & \cdots & \oplus w_i\eta_{1n}^{(i)} \\ \oplus w_i\eta_{21}^{(i)} & \oplus w_i\eta_{22}^{(i)} & \cdots & \oplus w_i\eta_{2n}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ \oplus w_i\eta_{m1}^{(i)} & \oplus w_i\eta_{m2}^{(i)} & \cdots & \oplus w_i\eta_{mn}^{(i)} \end{bmatrix} \tag{3}$$

Step 3: Normalize the weighted aggregated matrix as follows:

$$M = [(\eta_{ij})] = \begin{Bmatrix} \eta_{ij} & for\ \ same\ type \\ \eta_{ij}^c & for\ different\ type \end{Bmatrix} \tag{4}$$

Step 4: Find the weight for each criterion by SWARA-Method.
The weights of the criterion are determined using the SWARA technique, and its steps are as follows.
4.1: Evaluate the normalized score values of each criteria by employing the following formula.

$$\Delta^*(\psi) = \frac{1}{2}(\Delta(\psi) + 1) \tag{5}$$

where $\Delta(\psi)$ is a score function for the criteria ψ.
4.2: Sort the criteria from highest normalized score values.
4.3: Calculate the comparative value c_j for normalized score values by differencing the jth value to (j-1)th

value, which starts from the second position.
4.4: Compute the comparative coefficient k_j as follows:

$$k_j = \begin{Bmatrix} 1 & for\ j = 1 \\ C_j + 1 & for\ j \neq 1 \end{Bmatrix} \tag{6}$$

4.5: Evaluate the recalculated weight d_j as:

$$d_j = \begin{Bmatrix} 1 & for\ j = 1 \\ \frac{k_{j-1}}{k_j} & for\ j \neq 1 \end{Bmatrix} \tag{7}$$

4.6: Calculate the criteria weight w_j as

$$w_j = \frac{d_j}{\sum d_j} \tag{8}$$

Step 5: Calculate weighted sum measures (WSM)

$$Q_i^s = \oplus_{j=1}^{m}(w_j \otimes N_{ij}) \tag{9}$$

Step 6: Calculate weighted product measures (WPM)

$$Q_i^p = \otimes_{j=1}^{m}\left(N_{ij}^{w_{ij}}\right) \tag{10}$$

Step 7: Calculate the measures of WASPAS,

$$Q_i = \lambda Q_i^s \oplus (1-\lambda) Q_i^p \tag{11}$$

Step 8: Sort the alternative based on positive score values of Q_i.
The figure 2 displays a visual representation of the algorithmic process.

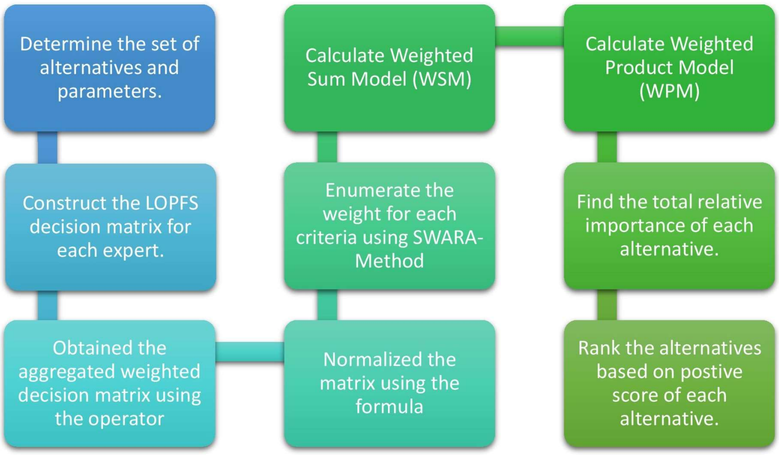

Figure 2: Algorithmic workflow of SWARA-WASPAS

5 A Decision Management on Education 5.0

The goal of Education 5.0 is to empower students to direct their own academic, social, professional, and personal progress. It is a digital transformation that is fueled by the use of cutting-edge technology and places students and teachers at the center of the real-world teaching and learning process. The necessity for education to adapt quickly to the significant changes brought about by globalization, technology, and the changing demands of the 21st century is addressed by Education 5.0, which makes it essential in our ever-changing world. Education 5.0 is crucial for providing students with the information and skills they need to succeed in this day and age when automation and innovation are changing industries and opening up new opportunities. It acknowledges the variety of individual learning styles and enables tailored, technologically assisted learning experiences that are tailored to the particular interests and strengths of every learner. Additionally, Education 5.0 promotes the growth of critical thinking, problem-solving, and adaptability skills—all of which are becoming more and more important in a workforce where complicated problems call for original answers and regular activities are mechanized. Education 5.0 advocates for a research-based educational system in which numerous obstacles and a never-ending list of difficulties are surmounted in the race from Education 1.0 to Education 4.0. Continuous improvement and an open mentality are necessary to remodel and align the current educational system with cutting-edge digital technology. The goal of Education 5.0 is to apply all cutting-edge technologies while preparing for societal change through value creation, human-to-human contact, research skills, and a problem-solving mindset among all stakeholders.

5.1 Scope of Education 5.0

Education 5.0 makes sure that people are not just equipped for the jobs of today, but also have the lifelong learning, inclusion, and development of soft and technical skills necessary to face the challenges of the future. It makes education more relevant and accessible by coordinating academic programs with business demands and promoting international cooperation. Moreover, Education 5.0 gives students the means to confront urgent problems like social inequality and climate change and instills in them a feeling of responsible citizenship in the face of global difficulties. In the end, Education 5.0 is the cornerstone of a society that is more knowledgeable, flexible, and involved, molding people who can prosper in a changing environment and make beneficial contributions to both their local communities and the earth.

Education 5.0 encourages the use of technologies and methods to enhance students' individualized learning with a more humane approach. Nowadays, individualized instruction improves student success. Professional abilities are more crucial to comprehending social demands once pupils have graduated from school and entered the working world. Here, the transition from education 4.0 to 5.0 and its implementation face the following major challenges:

1. Does the organization have the tools necessary to reach education 5.0?
2. What can be expected and what areas require improvement?

This study offers an investigation of educational institutions that are excellent for education 5.0 in an effort to identify the solutions to these research questions.

The emergence of this new paradigm in education depends on the choice of educational institutions that can accommodate Education 5.0. Selecting a flexible institution is crucial if learners are to get a comprehensive, progressive education that equips them with the opportunities and challenges of the twenty-first century. These schools are better suited to support students in thriving in a world that is changing quickly by encouraging the development of critical thinking, problem-solving, and adaptability—all essential elements of Education 5.0. Leading educational establishments in the context of Education 5.0 are actively advancing the field rather than merely following the newest trends. These establishments carry out state-of-the-art research that propels the creation of instructional materials and approaches that meet the needs of a world that is changing quickly.

Some of the key criteria for Education 5.0 are discussed below:

- **Pedagogical Supervision:** This involves guiding and mentoring educators to enhance teaching practices and ensure alignment with Education 5.0 goals. It emphasizes continuous professional development, innovative instructional strategies, and fostering a learner-centered approach to education. Effective pedagogical supervision helps educators adapt to evolving educational needs.

- **Transformative Learning Environment:** This criterion focuses on creating an environment that inspires critical thinking, creativity, and personal growth. It integrates collaborative and experiential learning opportunities, empowering students to apply knowledge to real-world challenges. Such environments encourage inclusivity, innovation, and lifelong learning.

- **Ergonomics Assessment:** Ergonomics in education emphasizes designing physical and digital learning spaces that optimize comfort, safety, and productivity. Regular assessments ensure that classrooms, tools, and technologies support student and teacher well-being, minimizing physical strain and maximizing engagement and efficiency.
- **Coherent Curriculum:** A coherent curriculum ensures that all educational components are logically connected and aligned with the principles of Education 5.0. It integrates interdisciplinary knowledge, future-oriented skills, and digital literacy, fostering a seamless progression from foundational concepts to advanced applications.
- **AI-Enhanced Classroom Orchestration:** This criterion leverages artificial intelligence to optimize classroom management and personalize learning experiences. AI tools can analyze student performance, suggest adaptive learning paths, and streamline administrative tasks, enabling educators to focus on interactive and impactful teaching.

5.2 Numerical Example

A government wants to implement the Evolution of Education 5.0 in their country. So, they want to choose the one appropriate educational institution to implement Education 5.0 among the five institutions $\{L_1, L_2, L_3, L_4, L_5\}$. Let us consider a set of decision-makers $\{M_1, M_2, M_3\}$. Some of the essential criteria for Education 5.0 are taken as a set of parameters $\{\psi_1$ (pedagogical supervision), ψ_2 (Transformative learning environment), ψ_3 (Ergonomics assessment), ψ_4 (Coherent curriculum), ψ_5 (AI enhanced classroom orchestration)$\}$. The parameters are shown in figure 3.

5.2.1 Calculation of weight by SWARA-Method:

Step 4: Let ψ_1 = (0.35,0.71), ψ_2 = (0.42,0.51), ψ_3 = (0.67,0.52), ψ_4 = (0;74,0.42), ψ_5 = (0.91,0.21) be the PFN of the five parameters. Then the score function is computed as:
$\Delta(\psi_1)$ = -0.3816; $\Delta(\psi_2)$ = -0.0837; $\Delta(\psi_3)$ = 0.1785; $\Delta(\psi_4)$ = 0.8920; $\Delta(\psi_5)$ = 0.7840;

By employing steps 4.1 to 4.5, we determine the weight for each criteria using SWARA-Method and it is shown in the table 5.

Table 5: Weights of parameters by SWARA-Method

Parameters	Ordered score value	c_j	k_j	d_j	Weight (w_j)
ψ_3	0.892	-	1	1	0.261
ψ_4	0.6856	0.2064	1.2064	0.829	0.216
ψ_5	0.5893	0.0964	1.0964	0.756	0.197
ψ_2	0.4582	0.1311	1.1311	0.668	0.174
ψ_1	0.3092	0.1490	1.1490	0.581	0.152

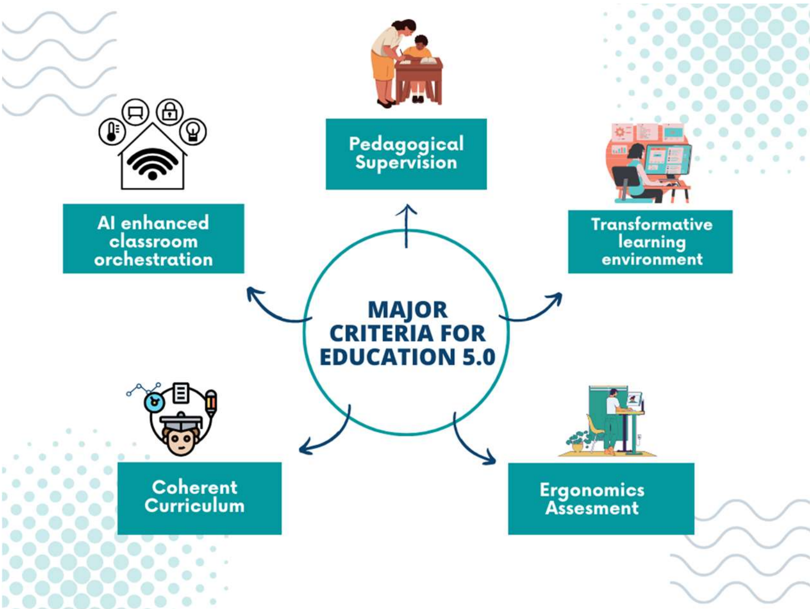

Figure 3: Criteria for Education 5.0

5.2.2 Computational Workflow of SWARA-WASPAS

Step 1: The LOPFS values given by each expert (M_i:$1 = 1,2,3$) are given in the table 6,7,8.

Table 6: LOPFS decision matrix - Expert 1

expert 1	S1	S2	S3	S4	S5
ψ_1	(0.4,0.7)	(0.6,0.7)	(0.5,0.8)	(0.5,0.7)	(0.2,0.6)
ψ_2	(0.6,0.4)	(0.7,0.6)	(0.6,0.5)	(0.6,0.7)	(0.3,0.5)
ψ_3	(0.7,0.3)	(0.7,0.5)	(0.7,0.4)	(0.7,0.5)	(0.5,0.4)
ψ_4	(0.8,0.3)	(0.8,0.3)	(0.8,0.3)	(0.8,0.3)	(0.6,0.4)
ψ_5	(0.9,0.2)	(0.9,0.2)	(0.8,0.2)	(0.9,0.2)	(0.7,0.3)

Table 7: LOPFS decision matrix - Expert 2

expert 2	S1	S2	S3	S4	S5
ψ_1	(0.5,0.7)	(0.3,0.8)	(0.6,0.7)	(0.4,0.6)	(0.5,0.6)
ψ_2	(0.6,0.6)	(0.6,0.5)	(0.7,0.5)	(0.6,0.6)	(0.5,0.5)
ψ_3	(0.8,0.6)	(0.7,0.3)	(0.7,0.4)	(0.6,0.4)	(0.6,0.4)
ψ_4	(0.8,0.5)	(0.8,0.2)	(0.8,0.3)	(0.7,0.3)	(0.7,0.3)
ψ_5	(0.9,0.3)	(0.9,0.2)	(0.9,0.2)	(0.8,0.3)	(0.8,0.2)

Table 8: LOPFS decision matrix - Expert 3

expert 3	S1	S2	S3	S4	S5
ψ_1	(0.5,0.6)	(0.5,0.6)	(0.5,0.8)	(0.5,0.6)	(0.4,0.6)
ψ_2	(0.6,0.5)	(0.7,0.5)	(0.6,0.4)	(0.5,0.4)	(0.5,0.4)
ψ_3	(0.7,0.2)	(0.7,0.3)	(0.6,0.3)	(0.6,0.2)	(0.6,0.4)
ψ_4	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)	(0.7,0.2)	(0.6,0.3)
ψ_5	(0.9,0.1)	(0.9,0.2)	(0.9,0.1)	(0.9,0.1)	(0.7,0.2)

Step 2: The weighted aggregated LOPFS decision matrix is computed and shown in the table 9. The weight for each expert is taken as (0.3,0.4,0.3). The first element of the table 9 is calculated as:

$$\sqrt{1-\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}}, \prod_{i=1}^{n}(\vartheta_{\alpha_i})^{w_i}$$

$$= (\sqrt{1-(1-0.4^2)^{0.3}(1-0.5^2)^{0.4}(1-0.5^2)^{0.3}}, (0.7)^{0.3}\ (0.7)^{0.4}\ (0.6)^{0.3})$$

$$= (\sqrt{1-(1-0.84)^{0.3}(1-0.75)^{0.4}(1-0.75)^{0.3}}, 0.67)$$

$$= (\sqrt{(1-0.7759)}, 0.67)$$

$$= (0.47, 0.67)$$

Step 3: The criteria are all of the same type. Hence, there is no need for normalization.

Table: 9 Weighted aggregated LOPFS decision matrix

M	S1	S2	S3	S4	S5
ψ_1	(0.47,0.67)	(0.48,0.71)	(0.54,0.76)	(0.46,0.63)	(0.41,0.60)
ψ_2	(0.60,0.50)	(0.67,0.53)	(0.65,0.76)	(0.57,0.56)	(0.45,0.47)
ψ_3	(0.75,0.35)	(0.70,0.35)	(0.67,0.37)	(0.63,0.35)	(0.57,0.40)
ψ_4	(0.80,0.33)	(0.80,0.23)	(0.80,0.27)	(0.74,0.27)	(0.65,0.33)
ψ_5	(0.90,0.19)	(0.90,0.20)	(0.87,0.16)	(0.87,0.19)	(0.75,0.23)

Step 4: The weight for each parameter is computed by the SWARA Method. The weights are (0.152, 0.174, 0.261, 0.216, 0.197)

Step 5: The Weighted Sum Model (WSM) for each alternative are enumerated as

$$Q_1^s = \sqrt{1-(1-0.47^2)^{0.152}(1-0.60^2)^{0.174}(1-0.75^2)^{0.261}(1-0.80^2)^{0.216}(1-0.90^2)^{0.197}},\ 0.67^{0.152}\ 0.50^{0.174}\ 0.35^{0.261}\ 0.35^{0.216}\ 0.19^{0.197})$$

$= (0.5849, 0.3598)$

Similarly,

$Q_2^s = (0.5798, 0.3427)$

$Q_3^s = (0.5519, 0.3705)$

Q_4^s = (0.4966,0.3482)
Q_5^s = (0.3632,0.3764)

Step 6: The Weighted Product Model (WPM) for each alternative are enumerated as

Q_1^p = $(0.47^{0.152}\ 0.60^{0.174}\ 0.75^{0.261}\ 0.80^{0.216}\ 0.90^{0.197},$

$\sqrt{1-(1-0.67^2)^{0.152}(1-0.50^2)^{0.174}(1-0.35^2)^{0.261}(1-0.33^2)^{0.216}(1-0.19^2)^{0.197}})$
= (0.7063,0.1869)
Q_2^p = (0.7094,0.1958)
Q_3^p = (0.7056,0.2889)
Q_4^p = (0.6512,0.1813)
Q_5^p = (0.5650,0.1750)

Step 7: Compute WASPAS measure for each alternative as: Let γ = 0.6

Q_1 =$(\sqrt{1-(1-0.5849^2)^{0.6}(1-0.7063^2)^{0.4}},\ (0.3598)^{0.6}\ (0.1869)^{0.4}$

= (0.6403,0.2768)
Q_2 = (0.6394,0.2740)
Q_3 = (0.6241,0.3354)
Q_4 = (0.5686,0.2682)
Q_5 = (0.4608,0.2771)

Step 8: The score function of each Q_i are computed as:
$\Delta(Q_1)$ = $(0.6403)^2-(0.2768)^2=$ 0.3334; $\Delta(Q_2)$ = 0.3338; $\Delta(Q_3)$ = 0.2270; $\Delta(Q_4)$ = 0.2514; $\Delta(Q_5)$ = 0.1356;
Then the sorting of alternatives is $L_2 > L_1 > L_4 > L_3 > L_5$.

The schematic representation of the results is shown in figure 4. Hence, the educational institution L_2 is the most appropriate for the implementation of Education 5.0.

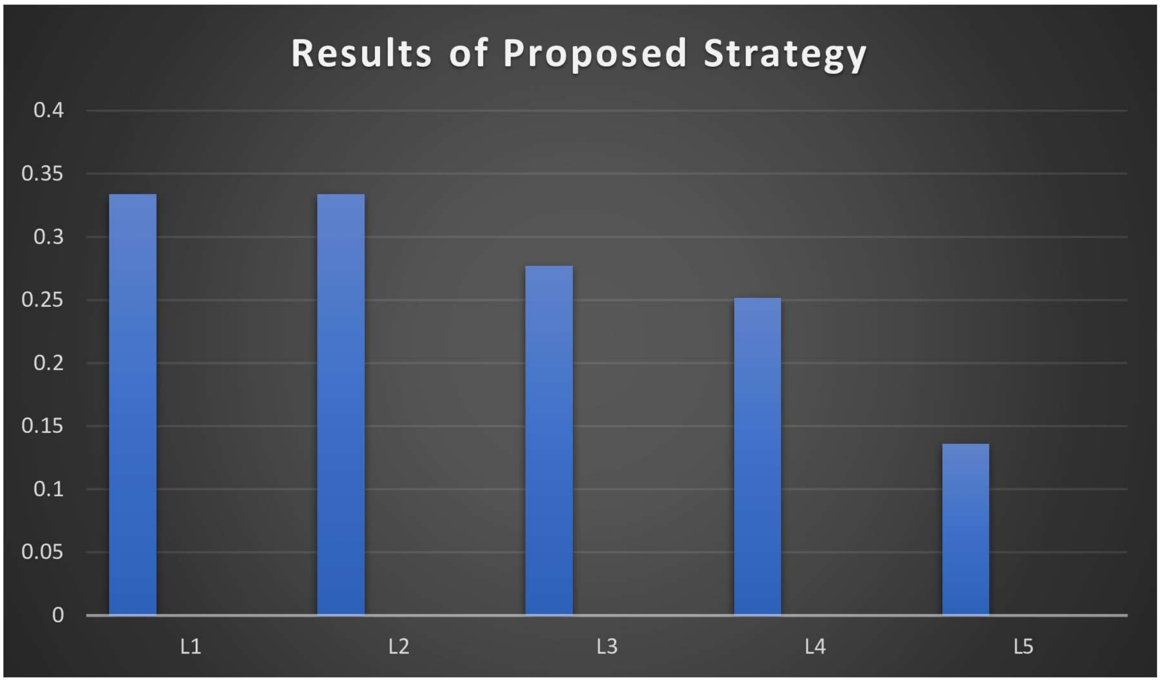

Figure 4: Results of Proposed strategy

6 Analysis of the results

6.1. Comparative Analysis

In this section, we compare the results of our proposed method with those obtained using previously established approaches. The comparison is summarized in Table 10. A notable aspect of our proposed method is the incorporation of operators, which are fundamental in managing the preferences expressed in Pythagorean Fuzzy Soft Sets (PFSS). To evaluate the robustness of our method, we employed a variety of operators commonly used in PFS, such as the Dombi, Hamacher, Einstein aggregation operators, ensuring a comprehensive analysis.

The results indicate a high degree of consistency between our method and the existing approaches. Although minor variations in the ranking of alternatives were observed, the optimal alternative remained consistent across all methods. This consistency highlights the reliability of our approach in handling decision-making problems while maintaining alignment with the outcomes derived from other established methods.

Furthermore, the slight differences in the ranking demonstrate the sensitivity of the ranking process to the choice of operators. This underlines the importance of selecting appropriate operators in PFS-based decision-making methods. Despite these minor changes, the overall robustness of our proposed methodology is evident, as it consistently identifies the optimal alternative, reinforcing its effectiveness and applicability in practical scenarios.

Table 10: Comparative Analysis with Previous Methods

Methods	Ranking
Score function	$L_2 > L_1 > L_3 > L_4 > L_5$
PFS-Weighted Aggregation operator [68]	$L_2 > L_1 > L_3 > L_4 > L_5$
PFS-Weighted Geometric operator [68]	$L_2 > L_1 > L_3 > L_4 > L_5$
PF- Hamacher Aggregation Operator [60]	$L_2 > L_1 > L_3 > L_5 > L_4$
PF- Einstein Aggregation Operator [70]	$L_2 > L_3 > L_1 > L_4 > L_5$
PF-Dombi Aggregation Operator [69]	$L_2 > L_1 > L_4 > L_3 > L_5$
Proposed	$L_2 > L_1 > L_4 > L_3 > L_5$

6.2 Sensitivity Analysis

We perform sensitivity analysis on the current section by adjusting various values for λ to ensure the stability of the new approach. There are three potential cases are present: $\lambda = 0.5$, $\lambda = 0.7$, $\lambda = 0.8$.
Case 1: $\lambda = 0.5$. The WASPAS measures of each alternative along with the score values and their ranking are shown in the table 11.

Table 11: Sensitivity Analysis for $\lambda = 0.5$

Alternatives	WASPAS measure	*Δ*
L_1	(0.7336,0.3868)	0.3886
L_2	(0.7336,0.3782)	0.3951
L_3	(0.7226,0.4321)	0.3354
L_4	(0.6751,0.3767)	0.3139
L_5	(0.5810,0.3930)	0.1831
$L_2 > L_1 > L_3 > L_4 > L_5$		

Case 2: $\lambda = 0.7$ The WASPAS measures of each alternative along with the score values and their ranking are shown in table 12.

Table 12: Sensitivity Analysis for $\lambda = 0.7$

Alternatives	WASPAS measure	Δ
L_1	(0.7437,0.3775)	0.4106
L_2	(0.7425,0.3652)	0.4179
L_3	(0.7289,0.4084)	0.3645
L_4	(0.6839,0.3667)	0.3333
L_5	(0.5867,0.3879)	0.1938
	$L_2 > L_1 > L_3 > L_4 > L_5$	

Case 3: $\lambda = 0.8$. The WASPAS measures of each alternative along with the score values and their ranking are shown in table 13.

Table 13: Sensitivity Analysis for $\lambda = 0.8$

Alternatives	WASPAS measure	Δ
L_1	(0.7485,0.3730)	0.4211
L_2	(0.7468,0.3588)	0.4290
L_3	(0.7320,0.3911)	0.3781
L_4	(0.6882,0.3618)	0.3427
L_5	(0.5896,0.3853)	0.1992
	$L_2 > L_1 > L_3 > L_4 > L_5$	

The outputs are compared and displayed in the figure 6.

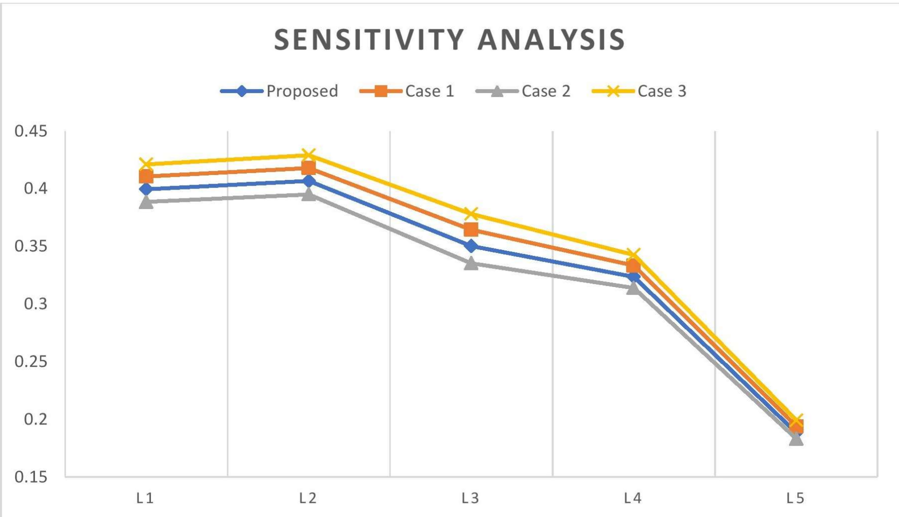

Figure 6: Sensitivity Analysis

6 Conclusion

This study introduces a novel framework that integrates the lattice implementation of Pythagorean Fuzzy Soft Sets with the Stepwise Weight Assessment Ratio Analysis (SWARA) and Weighted Aggregated Sum Product Assessment (WASPAS) methods. The creation of LOPFSS and some of its operations were detailed in this paper. The fundamental characteristics of LOPFSS are explored. By combining these methodologies, a robust Multiple Criteria Decision-Making (MCDM) framework is developed, offering a unique perspective on mathematical computing. The proposed approach effectively determines alternative rankings using the WASPAS method and generates criteria weights through SWARA, ensuring a balanced and comprehensive decision-making process.

The applicability of this framework is validated through an MCDM challenge in the context of Education 5.0, demonstrating its practical relevance and adaptability to real-world scenarios. A comparative analysis with existing techniques underscores its effectiveness and accessibility, highlighting its potential as a valuable tool in the field of MCDM. This integration not only enriches the existing decision-making methodologies but also paves the way for further advancements in the intersection of fuzzy set theory and decision analysis.

Our next research will focus on the development of similarity measures, and entropy measures for LOPFSS, and their applications in the fields of engineering and medicine. Also, the other MCDM approaches like TOPSIS, VIKOR, PROMETHEE, CoCos, and MABAC were implemented in the context of LOPFSS

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