FIZIKA 9 (1977) Supplement 3, 21–56
(Proceedings of the Int. Symp. on Nuclear Collisions and Their Microscopic Description, Bled, September 1977)

Continuum Shell Model Calculations: Status and Prospects *

R. J. Philpott
Department of Physics,
The Florida State University,
Tallahassee, Florida 32306

In this talk, I would like to review briefly the present status of the continuum shell model, regarded specifically as a framework for carrying out dynamical calculations of various nuclear properties and processes. So many calculations based on so many diverse reaction formalisms have appeared over the last dozen years or so that it will not be possible even to mention them all. However, I will describe a few "typical" calculations with a view towards illustrating the capabilities of the model. These calculations include applications of the model to the description of giant resonance phenomena and resonance phenomena observed in nucleon elastic and inelastic scattering. A recent advance in the theoretical formulation of the continuum shell model has resulted in the ability to treat target recoil effects. Because of its importance in calculations involving light target nuclei, I will discuss this development in some detail. Finally, I will briefly consider the prospects for future calculations within the continuum shell model framework. By comparison with standard nuclear structure calculations, presently available continuum shell model calculations are still quite primitive. There is much scope for

* Supported in part by the National Science Foundation under Grant Nos. NSF-GU-2612, NSF-PHY-74-02673, and NSF-GJ-367.

advancement through the development of valid approximation techniques together with a more automated calculational approach.

The continuum shell model ideology was defined in 1966 by Bloch[1)] following earlier related work by Fano[2)] and others. The original concept, however, can be traced back to an article published by Dirac[3)] in 1927. Many of the earlier treatments focus on the "structural content" of the model, namely the implied mathematical structure of the S matrix and the ability of the model to provide a formal description of resonance phenomena. These and many other aspects of the model have been extensively reviewed in the now classic book by Mahaux and Weidenmüller[4)]. A later, somewhat less comprehensive but very valuable review has been compiled by Barrett et al.[5)]. The present author is pleased to acknowledge his indebtedness to these works.

Early motivation for the development of the continuum shell model was found in the need to account dynamically for resonance phenomena observed in nucleon elastic and inelastic scattering and in photo-nuclear reactions, including the well known giant resonance and analogue resonance phenomena. Some aspects of these phenomena could already be described by standard structure calculations, but since the coupling to the breakup channels was missing, it was impossible to obtain the detailed angular and energy behavior of the cross sections. Later developments include application of the model to muonuclear absorption[6], electron inelastic scattering[7], and direct (p,p') reactions[8] etc. processes in which continuum states of the target nucleus can be excited.

The continuum shell model represents a natural extension of the conventional nuclear shell model to include single nucleon channels. The model contains two basic ingredients: a (non-relativistic) microscopic Hamiltonian which contains only nucleon coordinates and a (truncated) function space within which the Hamiltonian is to be diagonalized. Implicit in the definition of the function space is a set of single particle orbitals which may include both bound and unbound single particle states. The function space is then defined to include multi-particle states in which all nucleons are in bound orbitals as well as multi-particle states in which all but one of the nucleons are in bound orbitals. These states are illustrated schematically in fig. 1. It will be convenient to assume that the Hamiltonian has been pre-diagonalized both with respect to the target states $\Phi_{\alpha A}$ and with respect to the multi-particle states involving only bound orbitals, $\Phi_{\beta B}$. These requirements can be achieved by standard shell model diagonalizations. Then the states on the left of fig. 1 represent possible entrance and exit channels and the states on the right represent so-called "bound states embedded in the continuum"(BSEC). Even in the absence of coupling to the breakup channels, many of the BSEC may lie at energies greater than the lowest breakup threshold. Resonances can arise as single particle resonances within the entrance or exit channels, as a consequence of channel-channel coupling, or by virtue of coupling between the nucleon continua and the BSEC which lie close to the energy region of interest.

Some of the formal and practical limitations of the continuum shell model are already evident from the above description. Firstly, the model does not admit either cluster channels (eg. α-decay channels) or many-body breakup channels. Although some attempts have been made to include these features[9], the methods have not yet been developed sufficiently to be of importance to the present discussion. Secondly, the model suffers from the same limitations as the conventional shell model with regard to the types of basis state and the types of two-body interaction which can conveniently be included. Thus, certain types of clustering phenomena[10] are not likely to be well represented in the model. Nor can interactions which contain hard cores be employed directly. Thirdly, the model is expected to contain errors of unknown magnitude arising from an incorrect treatment of the center of mass motion unless special steps are taken to remove them. In practice, limitations such as the above have not prevented widespread and productive application of the conventional shell model ideology to problems in nuclear physics, and the same is expected to be true of the continuum shell model.

By far the most popular version of the model is that based on the so called "1p - 1h continuum approximation". This model has been widely used[4-7,11,12] to study giant multipole resonance excitations in closed shell or quasi-closed shell nuclei, such as ^{4}He, ^{16}O, ^{40}Ca, ^{90}Zr, ^{208}Pb. The 1p - 1h continuum model can be regarded as a natural extension into the continuum of earlier bound state models based on the 1p - 1h Tamm-Dancoff approximation (TDA) or random phase approximation (RPA). As in the bound state counterpart, it is possible to extend the model to include states of higher particle-hole excitation[4,5,13-15]. In the simplest extension, additional states of 2p - 2h or 3p - 3h character might be employed as BSEC in order to increase the predicted structure in the resonance region.

Let us use the giant dipole exitation in ^{16}O as an example, in order to illustrate what is involved in these calculations and what results might typically be obtained. Some of the channels into which the excited states of ^{16}O can decay are shown in fig. 2. The particular channels which are retained in a 1p - 1h continuum calculation for the dipole states of ^{16}O are those built on the ground ($J^{\pi} = \frac{1}{2}^{-}$) and third excited ($J^{\pi} = 3/2^{-}$) states in ^{15}O and ^{15}N. From angular momentum considerations, one sees that there are just 10 fully labelled channels in the model.

In order to develop our ideas further, let us first consider the corresponding, rather well understood[16] conventional shell model (TDA) calculation. For the moment, we will assume that the model parameters, i.e. those that determine the Hamiltonian and the single particle orbitals, are already known. In the conventional calculation, the 10 channels are represented by 10 distinct 1p - 1h states, obtained by promoting a nucleon from the p shell into the s-d shell. Since the radial functions are all assumed to be known, the only unknowns are the discrete eigenvalues of the Hamiltonian matrix and the expansion amplitudes of the eigenfunctions expressed as linear superpositions of the 10 given 1p - 1h basis states:

$$\Psi_{SM(1p\,-\,1h)} = \sum_{ph} A_{ph} \, \phi_{ph} \, (\vec{r}_p, \vec{r}_h) \tag{1}$$

These eigenvalues and amplitudes are determined by diagonalization of the Hamiltonian within the space spanned by the basis states. As a result of the diagonalization, one or two states are shifted up into the giant resonance region. Moreover, these states are found to have captured almost all of the available dipole strength, a point well demonstrated by the schematic model of Fallieros and of Brown[16).

The continuum shell model calculation differs from the conventional shell model calculation in that, although the target states and hence the hole orbitals are supposed to be known, the radial functions describing the particle motion are not. Each eigenstate of the Hamiltonian is now expressed as a linear superposition of the 10 channel states:

$$\Psi_{CSM(1p\,-\,1h)} = \sum_{ph} \mathcal{A} \, [r_p^{-1} \, u_p(r_p) \, \phi_{ph}(\hat{r}_p, \vec{r}_h)] \tag{2}$$

The radial functions corresponding to a given energy in the continuum are now to be obtained by solving a set of coupled integro-differential equations. Thus, 10 continua of eigenstates are obtained, rather than 10 discrete eigenstates. Eigenstates which describe specific reactions are now formed by taking linear superpositions of the 10 independent solutions in order to satisfy the appropriate boundary conditions. Cross sections for photo absorption or emission are calculated as usual by treating the interaction with the radiation field in perturbation theory. In practice, the original integro-differential equations of the 1p - 1h continuum model are often simplified by adopting a residual interaction of zero range and neglecting certain other non-local terms generated by the antisymmetrization of the wavefunction[11].

As an example of the kind of result which is obtained, we show in fig. 3 the calculated versus experimental integrated cross sections for the (γ,n_0) and (γ,n_3) reactions on ^{16}O. This figure was taken from the pioneering work of Buck and Hill[11]. These authors attempted to represent the effects of decay channels not explicitly included in the model by means of a phenomenological absorptive potential. The calculations certainly yield resonance effects not unlike those seen in the data, but much of the observed structure is missing. Other kinds of data, such as angular distributions and proton to neutron branching ratios are also not reproduced in detail by these calculations[4,5].

Of course, this is to be expected because of the many simplifying assumptions upon which the 1p - 1h model is based. Higher particle-hole excitations are expected to give rise to additional structure both by coupling with the 1p - 1h states in the resonance region and by introducing correlations into the ^{16}O ground state. Without ground state correlations, the photo-absorption process excites only states of 1p - 1h character. But when the target ground state includes many-body correlations, states of higher particle-hole excitation can be excited directly, thus enriching the resonance structure and possibly also shifting the centroid of resonance strength.

Although the calculations can become quite complicated there have been a few attempts[4,5,13-15] to include additional structure states in the model. Some work carried out by Wang and Shakin[13] has produced a very nice (although somewhat artificial) agreement with the (γ,n_0) data. Combinations of low-lying negative parity 1p - 1h eigenstates derived from a structure model were used to construct a set of correlated 3p - 3h states. These states were then added to the 1p - 1h continuum model as BSEC in order to provide a means of describing the intermediate structure. Rather than solving the coupled channels equations directly, these authors employed a projection-operator formalism. By this means, the single particle bound states and narrow resonances could be formally separated off from the single particle continua. A number of approximations were made in these calculations, including neglect of residual channel-channel coupling, neglect of Pauli blocking during construction of the 3p - 3h states and neglect of certain contributions to the matrix elements of the Hamiltonian among the 3p - 3h states. Nevertheless, after a few relatively minor adjustments were made, the results shown in fig. 4 were obtained. The full line in the figure includes energy averaging with an energy resolution of 200 keV. In later work by the same authors[17)], angular distributions and polarizations have also been reproduced with remarkable accuracy. Results such as these illustrate the level of detail which may be described within the 1p - 1h model and its straightforward extensions.

Another area in which the continuum shell model offers great promise, although few calculations have appeared as yet, is the study of low energy nucleon scattering states. At low enough energies, it is often possible to include all (or almost all) open channels explicitly. Moreover, conventional shell model descriptions of the BSEC appear to be more reliable at lower energies. The ^{15}N + n system has been treated quite extensively[4,5,18,19)] within the 1p - 1h approximation. Takeuchi and Moldauer[20)] have reported a calculation for ^{17}O + n scattering. A calculation for ^{89}Y + n scattering has been done by Ramavataram et al.[21)].

A prototype calculation for neutron scattering from ^{14}C was done by the present author[22)]. The ^{15}C nucleus was treated in terms of 3 active neutrons outside an inert spherical ^{12}C

closed-shell core. For the construction of the BSEC and ^{14}C eigenstates, upon which the channels were built, the active nucleons were restricted to the $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ orbits. The experimental spectrum for ^{15}C, see fig. 5, shows two bound states whose spins are known, and several states of unknown spin in the continuum. In the region of excitation energy extending from 1.22 MeV up to 7.31 MeV there is only one open channel. In these calculations, the nuclear Hamiltonian contained a single-body potential of Woods-Saxon form, representing the interaction of the valency neutrons with the ^{12}C core and a two-body term representing the interactions of the valency neutrons with each other:

$$H = \sum_i (T_i + U_i) + \tfrac{1}{2}\sum_{ij} v_{ij} \tag{3}$$

The parameters of the single body potential were fitted to observed single particle energies in ^{13}C, and the parameters of the two-body interaction were those used in a previous shell model study of ^{14}N and ^{14}C carried out by True[23]. The continuum calculation was done within the framework of the dynamical R-matrix method[24] of Lane and Robson[25], which facilitated the direct application of standard nuclear structure (i.e. shell model) techniques to the problem. We note that the residual interaction has a finite range and that all terms resulting from the antisymmetrization of the wavefuntion are retained.

Initial results for ^{15}C showed that the binding relative to

^{14}C was predicted too large by 2.5 MeV. Rather than attempting to remove this discrepancy by modifying the effective Hamiltonian, a correction was achieved by arbitrarily adjusting the channel threshold energy. When this adjustment was made, the energies of both bound states were well reproduced within the model. In addition, an intriguing spectrum of resonances was predicted in the continuum, see fig. 6. The predicted $d_{3/2}$ single particle resonance is visible as a broad hump at about 2 MeV: the other resonances have a more complicated many-body character. We note that fig. 6 also shows a neutron inelastic cross section which was obtained by including the excited $J^{\pi} = 0^{+}$ states of ^{14}C in the model. Apart from the arbitrary shift in the channel energy, this was a completely parameter-free calculation. Unfortunately, there is no data of which I am aware against which these results may be directly compared. Nor is it expected that the comparison would be particularly good at this stage because of the truncations involved and the general lack of experience with models of this type. However, the calculation is indicative of the kind of prediction which is rapidly becoming feasible for rather general p-shell and certain sd-shell nuclei, for example. Perhaps by mentioning these particular results again, I will encourage an experimental group to take a look at the neutron cross section of ^{14}C.

A calculation quite similar to the above was performed by George and Philpott[26] for the low energy states of the $^{16}O + n$ and $^{16}O + p$ systems. In this investigation , there were 5 active nucleons outside an inert spherical ^{12}C closed-shell core. The shell model diagonalizations which determine the eigenstates of ^{16}O and the BSEC in ^{17}O were taken from the work of Zuker[27)] and Zuker, Buck, and McGrory[28)]. Thus, the ^{16}O ground state contains np - nh correlations where n = 0,2,and 4, and the ^{17}O BSEC include states of 5p - 4h character. Many-body structure coefficients needed in the continuum shell model calculations were obtained from the Oak Ridge-Rochester shell model code[29)]. Fortunately there exists a lot of data for the A = 17 system so that many interesting comparisons could be made. In fig. 7, we show the comparison between the measured and predicted phase shifts for neutron scattering from ^{16}O in the region 0 to 4 MeV. The agreement is striking. In fact there is almost complete correspondence between the two sets of phase shifts below about 3 MeV. The spectrum of bound states and resonances as seen in the total neutron elastic scattering cross section is shown in fig. 8. Generally speaking, the positions of the resonances have been quite well reproduced but the calculated widths tend to be too narrow. The cross section which would be obtained from a single channel calculation without BSEC is given by the dashed line. It exhibits only the $d_{3/2}$ single particle resonance at 1 MeV.

Calculations similar in spirit to the above (but formulated quite independently[14)]) were also carried out by Birkholz and Heil[30)] for the rather complicated case of neutron scattering from ^{11}B. Neutron channels were built on 6 different negative parity states of the target nucleus. The target states were taken from an intermediate coupling p-shell calculation of Kurath[31]. All neutron waves up to $\ell = 3$ were combined with the ground state of the target nucleus and even parity neutron waves up to $\ell = 2$ were combined with the excited states. The resulting large dimensional coupled channels equations were solved by an effective technique involving separation of the energy and radial dependence of the single particle continuum functions which was developed by Birkholz[14]. In order to avoid difficulties with antisymmetrization, the 1s and 1p orbits were projected out of the continua. Also, no BSEC were included. As a consequence of these restrictions, a few positive parity resonances based on the $(1p)^8$ configuration are missing from the calculated spectrum. In fig. 9, we show the Legendre expansion coefficients B_L of the neutron elastic differential cross section in the energy region below 2 MeV both before and after addition of fitted resonance terms. The extra resonance terms were arbitrarily included in order to represent omitted positive parity resonances at 0.43 and 1.77 MeV. Considering the delicacy of such comparisons, the agreement with experiment is extraordinarily good.

It is evident from the foregoing that most of the detailed dynamical calculations which have been carried out within the continuum shell model framework have been for rather light nuclear systems. The reasons for this are obvious. The light nuclei have relatively few important degrees of freedom and the shell model descriptions of these nucle- are relatively well developed. However, as one proceeds to study the lighter nuclear systems, errors introduced by the incorrect treatment of target recoil, which are inherent in the continuum shell model as originally formulated, are expected to become increasingly important. This point was brought home strongly to the present author when he tried to reproduce the results of a resonating group calculation[32] for neutron scattering from ^{4}He using a computer code designed to solve the corresponding continuum shell model problem[33]: The p-wave phase shifts were up to 30° different and showed a different dependence on energy. Significant differences can also be found in the ^{16}O + n system and even in the ^{40}Ca + n system. Indeed, uncertainty about the recoil errors has been a major impediment preventing the more widespread development of continuum shell models for light nuclei.

Recently, a solution has been found to the recoil problem[34]. The key to the problem can be seen in fig. 10, which shows the coordinates employed in a continuum shell model description of a nucleon at $\vec{r}_0$ scattering from a target nucleus, whose center of mass is located at $\vec{R}_c$. The shell model states are constructed from products of single particle orbitals expressed in terms of coordinates such as $\vec{r}_1$ measured from a single fixed point. The product nature of these states and the independence of the particle coordinates greatly facilitates the application of the Pauli principle and the computation of matrix elements. But it also introduces into the wavefunction an extra unwanted coordinate $\vec{R}$, which describes the location of the center of mass of the system as a whole. In general, the dependence on $\vec{R}$ cannot be factored out of the total wavefunction: so the latter contians a fundamental error[35].

These ideas can be made more explicit by assuming that the single particle orbitals are all given by oscillator eigenfunctions and that the target nucleus is specified by a non-spurious[35] shell model eigenstate. The coordinate $\vec{R}_c$ is then in a 0s state which can be factored from the target state. If the extra nucleon is in an oscillator eigenstate specified by $n_0\ell_0$, we may employ a Talmi-Moshinsky transformation[36] (with unequal masses) to recast the wavefunction in terms of the coordinates $\vec{R}$ and $\vec{r}$. In general, one obtains a superposition of states, which we write schematically as follows:

$$\psi_{0s}(\vec{R}_c)\psi_{n_0\ell_0}(\vec{r}_0) = \sum_{n\ell NL} \psi_{n\ell}(\vec{r})\psi_{NL}(\vec{R}) < n\ell NL|0sn_0\ell_0 > \quad (4)$$

It is clear that the functional dependence on $\vec{R}$ cannot be factored from this expression even approximately.

The solution to the recc.. problem is to employ wavefunctions expressed entirely in terms of internal coordinates, so that the overall center of mass coordinate $\vec{R}$ never appears in the formulation. Such "intrinsic" wavefunctions are normally very hard to deal with directly, because the partial separability associated with independent particle coordinates has been lost. However, we do not actually need to know the intrinsic wavefunctions themselves, but only the <u>matrix elements</u> of certain operators between the intrinsic wavefunctions.

Equation (4) may be used to develop a relationship between the desired intrinsic matrix elements and the corresponding, but much easier to calculate, shell model matrix elements. We introduce the notations:

$$|\alpha A n_0 \ell_0 \rangle_{SM} \equiv \mathcal{A}[\Phi_{\alpha A}(\xi_c, \vec{R}_c)\ \psi_{n_0 \ell_0}(\vec{r}_0)] \tag{5}$$

$$|\alpha A n \ell \rangle_{int} \equiv \mathcal{A}[\Psi_{\alpha A}(\xi_c)\ \psi_{n\ell}(\vec{r})] \tag{6}$$

where $\Psi_{\alpha A}$ is the intrinsic target state, related to $\Phi_{\alpha A}$ through $\Phi_{\alpha A}(\xi_c, \vec{R}_c) = \Psi_{\alpha A}(\xi_c)\ \psi_{0s}(\vec{R})$. We then obtain, schematically,

$$_{SM}\langle \alpha A n_0 \ell_0 |\hat{O}| \alpha' A' n_0' \ell_0' \rangle_{SM} = \sum_{n\ell n'\ell' NL} \langle n\ell NL | 0s n_0 \ell_0 \rangle \langle n'\ell' NL | 0s n_0' \ell_0' \rangle$$

$$_{int}\langle \alpha A n \ell |\hat{O}| \alpha' A' n' \ell' \rangle_{int} \tag{7}$$

where $\hat{O}$ is any operator which acts only on internal coordinates. Equation (7) is the relation we seek. Although it may, perhaps, not be obvious at first glance, this equation can readily be inverted[34] to express the desired intrinsic matrix elements directly in terms of basic shell model matrix elements. Similar relations can be developed for matrix elements containing BSEC. The whole procedure can also be generalized[34] to include properly the spin and isospin coordinates. The additional complexity introduced by the above transformation is in general expected to be much less than the complexity typically encountered in the construction of the many-body shell model matrix elements themselves.

Since the above ideas are expressed with respect to oscillator basis functions, it is necessary to employ a reaction theory which is capable of generating the S matrix from these functions. The Lane-Robson R-matrix method mentioned earlier is such a reaction formalism. The formalism considered by Birkholz[14] and others, in which continuum eigenstates are represented in terms of oscillator functions also appears to be suitable.

We have verified the above treatment of the recoil problem by repeating within the continuum shell model framework various resonating group calculations[32,37,38] which have been carried out for nucleon scattering from closed shell nuclei. In these calculations, the target is treated as an inert LS-closed core. The nucleon-nucleon interaction includes both central and spin-orbit terms with finite-range Gaussian form factors. There is no explicit one-body interaction. Some calculated phase shifts for the ^{4}He + p system are shown in fig. 11. The full lines (exact) give the CSM results obtained when recoil was treated correctly. They agree perfectly with the corresponding resonating group results[32]. The dashed lines (Approx. 1) give the CSM results obtained when recoil was ignored, except that the relative wavefunction for the incident nucleon was calculated as usual with a reduced mass. The other lines are irrelevant to the present discussion. A corresponding calculation for the polarization at 13.94 MeV is shown in fig. 12. Clearly, the recoil correction is of vital importance for such a light target nucleus.

Similar calculations have also been carried out[39] for $^{16}O + p$ and $^{40}Ca + p$ at somewhat higher energies in order to check the trend of the recoil correction as the target mass is increased. It is noteworthy that the continuum shell model approach permits these calculations to be done exactly and relatively straightforwardly. There is no need to approximate the Coulomb exchange matrix elements and a spin-orbit two-body interaction of finite range can still be employed. (Indeed, it would even be quite feasible to introduce non-local or velocity dependent two-body interactions.) We did however find that the Lane-Robson R- matrix method, based on oscillator functions alone, displayed an unacceptably large sensitivity to the particular choice of matching radius.

This sensitivity has been remarked on earlier[40]. In fig. 13, we show the dependence on the matching radius of the $d_{3/2}$ and $s_{1/2}$ phase shifts at low energies, obtained from a potential model for $^{16}O + n$. Note that the scale on the left is greatly expanded, being graduated in 1/10's of a degree. One sees that good convergence is obtained, even with oscillator functions alone: but only if the matching radius is very carefully chosen. For many purposes, small errors in the phase shifts such as those shown here, can be tolerated. However, if the physical observable of interest, such as a cross section or a polarization, is formed as a result of rather complete cancellation betweeen several partial waves, it may become very tedious to obtain all the phase shifts to the desired accuracy.

A way to avoid this sensitivity to the matching radius has also been proposed[39]. It involves the use of energy dependent radial functions in each channel. If this new approach is to be useful for continuum shell model calculations, it is important to show that the method can generate an accurate S matrix even though the only available many-body matrix elements are those evaluated between oscillator eigenstates. We believe that this has been accomplished[39], although I will not attempt to develop the details here. Let it suffice to say that we may still use the Lane-Robson R-matrix formalism. The energy dependent functions are then simply treated as additional basis states. Alternatively, we may now also employ the Kohn-Hulthen variational method[41], or the "least squares" method of Schmid and Schwager[42]. The converged results are identical in each case. Typically one needs about 10 or 12 oscillator functions to obtain the s-wave phase shifts to an accuracy of about $1/100^{\circ}$. Smaller numbers of oscillator functions are required for the higher ℓ's.

Figure 14 shows the results of some continuum shell model calculations[39] for proton scattering on ^{16}O, treated as an LS-closed core. In these calculations, the flux lost to omitted channels is described by means of a phenomenological absorptive potential. The pecked lines (S.R. L-S) were obtained using the two-body interaction developed for this system by Tang and co-workers[37]. They also very closely reproduce the resonating group results[37]. The full lines (F.R. L-S) show the effect of using a finite range two-body spin orbit interaction. The dashed lines show what happens when the recoil corrections are omitted from the latter calculation. Without the recoil corrections, the calculated differential cross section tends to be enhanced in the forward direction and is greatly reduced at back angles. In terms of reaction mechanisms[43], the recoil corrections are needed here in order to obtain the heavy particle pickup amplitude which is dominant at back angles. Without the recoil corrections, the model polarization is very poorly reproduced at back angles. (The improvement in agreement with the data is of no significance.)

Results[39] for proton scattering on ^{40}Ca shown in fig. 15 display similar features, although the changes introduced by the recoil corrections are now much less noticeable. The main reason for showing these results is to point out a small discrepancy between them and those derived by resonating group techniques. The polarization obtained from our calculations remains positive at backward angles whereas the polarization obtained from the resonating group calculations goes negative over the angular region 170° - 180°. Since both calculations appear to have converged[44], we believe that this discrepancy must be attributed to small differences in the way in which the phenomenological absorptive potential is introduced into the calculations or differences in the manner in which the Coulomb exchange matrix elements are treated.

I hope that the above discussion has succeeded in transmitting some idea of the scope of present-day continuum shell model calculations. Following early investigations of giant resonance and analog resonance phenomena, the model is expected to become increasingly important as a tool in the study of neutron and proton elastic and inelastic scattering and charge exchange reactions, both at low energies where resonance phenomena dominate the cross-sections and at higher energies where the reaction mechanism is increasingly direct. Some of the advantages offered by the model are:

(1) A fully microscopic description.

(2) Equations are solved to all orders of the interaction among the retained channels.

(3) All exchange amplitudes are retained.

(4) Great flexibility with regard to internal structures (BSEC) and correlations in the wavefunctions of target and residual nuclei.

(5) Great flexibility with regard to possible residual interactions. (One may even use forms such as the Sussex interaction[45], for which only the relative matrix elements are known.)

(6) Target recoil may be treated correctly.

Paralleling the development of the continuum shell model there has arisen a large number of different reaction formalisms designed to facilitate the numerical implementation of the model. The different approaches include[4,5] the coupled channels method[46], the Feshbach[47] projection operator method[48], the separation method of Birkholz[14] and others, the Lane-Robson[25] R-matrix method, the eigenchannel method[5] and many others. It is generally expected, and indeed has often been shown in special cases, that any of these approaches is capable of yielding a correct converged solution when properly implemented. The question as to whether any particular formalism will ultimately lead to a significant reduction in numerical effort appears to be as yet unanswered.

Despite the many attractive features of the continuum shell model (or perhaps because of them) any non-trivial calculation is likely to be a rather heroic undertaking. It is hoped that in the near future some of the technical difficulties associated with the introduction of increasingly complex internal states will be alleviated by more efficient use of structure codes developed for conventional shell model applications. In this way, one can hope to take increasing advantage of the inherent flexibility of the model.

On the other hand, there is a pressing need to develop valid and powerful approximations. Many authors[4,19,49] have proposed ways in which the model equations might be simplified. But considerable care is needed when approximations are made. Small changes in the model can have quite large effects in the calculated results, as was seen above in connection with the target recoil problem, see for example fig. 14.

For the lighter nuclear systems, $A \lesssim 20$, one should presumably employ a method based directly on oscillator expansion states. By this means, one retains adequate generality for the wavefunctions and is also able to handle general interactions and treat the recoil problem. We remark that the use of effective single particle potentials of Woods-Saxon form is highly questionable for these light systems. Such a potential is correctly expressed in terms of the coordinate $\vec{r}$ of fig. 10. The matrix elements of such a potential are not easy to evaluate either with respect to the shell model functions or with respect to the (antisymmetrized) intrinsic functions. Nor does such a potential generate the expected odd-even differences[50)] obtained from a more realistic two-body interaction.

For the heavier nuclear systems, A $\gtrsim$ 40, there is less need to implement the recoil corrections (although one must still take care to eliminate spurious states from the structure problem) and less need to employ a residual interaction of finite range. Presumably one can take advantage of these simplifications to implement many of the suggested approximations[4,19,49)]. Ideally, of course, one should always tailor the model (and hence the numerical effort which will be required) to the type of information which one is attempting to obtain. Although this may require considerable physical insight, it is a problem with which physicists are familiar!

My feeling for the future of the continuum shell model is one of considerable optimism. Many pioneering investigations have already demonstrated the general usefulness of the model. The necessary calculational techniques exist and have in most cases been adequately tested. I therefore expect that in the future many more investigators will be encouraged to take advantage of the unique opportunities which this model has to offer.

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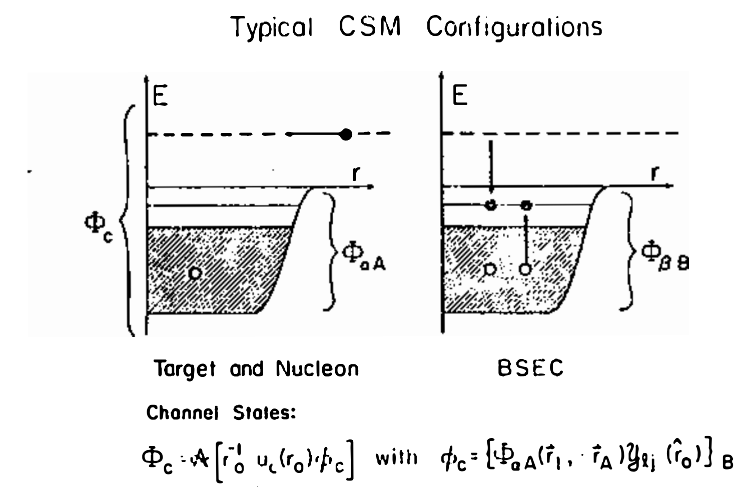

Fig. 1 The two types of configuration retained in the continuum shell model as considered in this article. States $\phi_{\alpha A}$ and $\phi_{\beta B}$ may have arbitrary complexity.

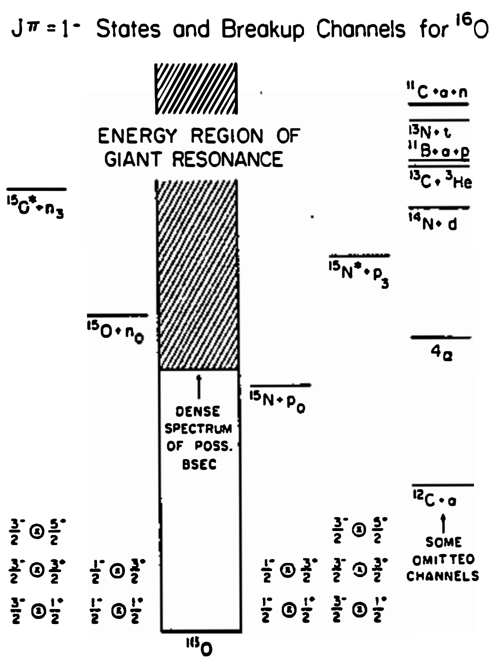

Fig. 2. Schematic representation of the states and breakup channels in ^{16}O encountered in a typical continuum shell model calculation of photonuclear cross-sections in the giant dipole resonance region. Target and nucleon spins and parties (in the jj-coupling scheme) are listed below each retained channel. The nucleon spins and parities are those requred for the formation of $J^{\pi} = 1^-$ states in ^{16}O.

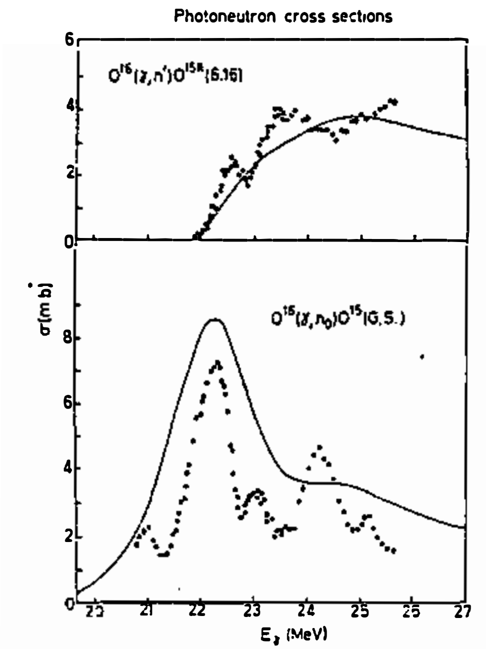

Fig. 3.
Photoneutron cross-sections from the 1p - 1h continuum shell model calculations of Buck and Hill [11].

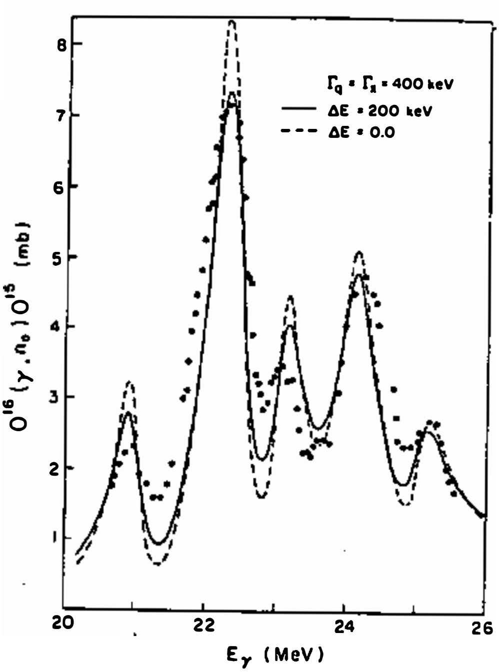

Fig. 4.
Photoneutron cross-sections from the calculations of Wang and Shakin [13]. The 1p-1h continuum model is augmented by the addition of some 3p - 3h states.

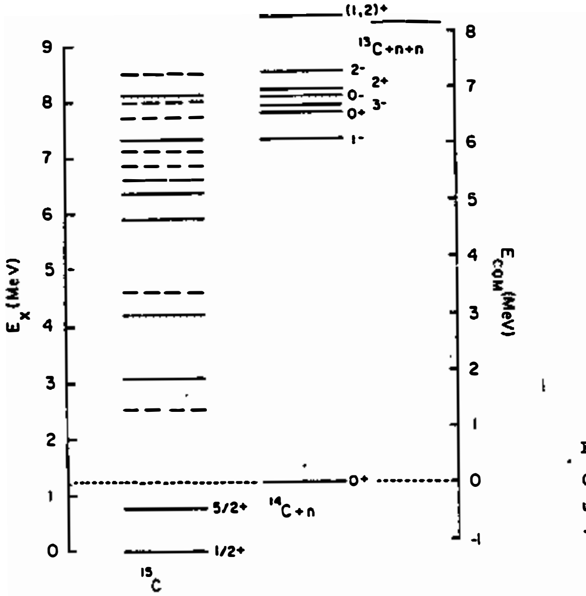

Fig. 5.
Observed states and break-up channels in the low-energy spectrum of ^{15}C.

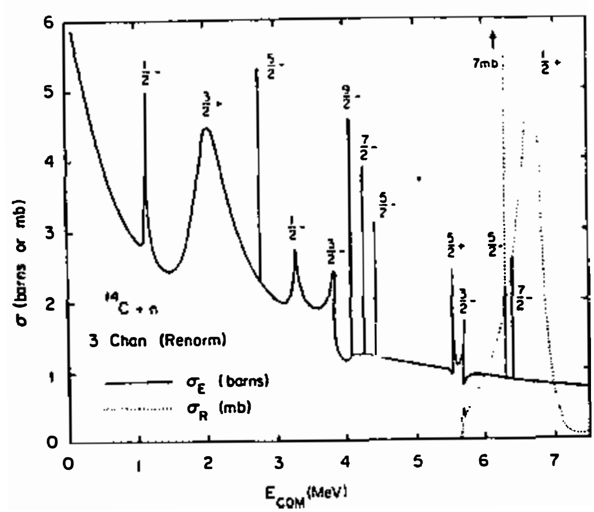

Fig. 6 Calculated[22] total elastic and reaction cross sections for the ^{14}C + n system.

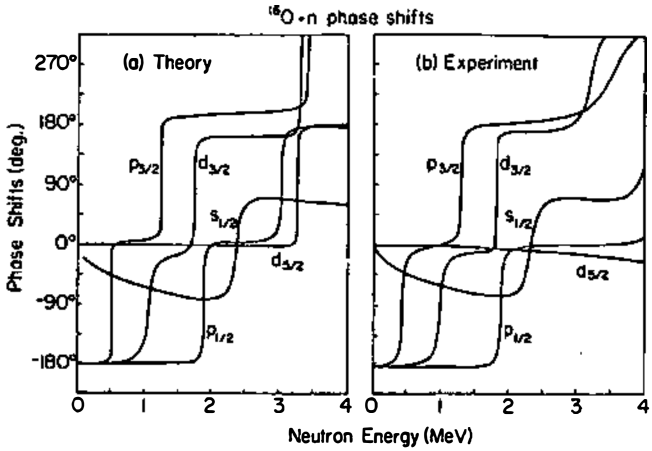

Fig. 7 Comparison between calculated[26)] and experimental neutron phase shifts for the $^{16}O + n$ system.

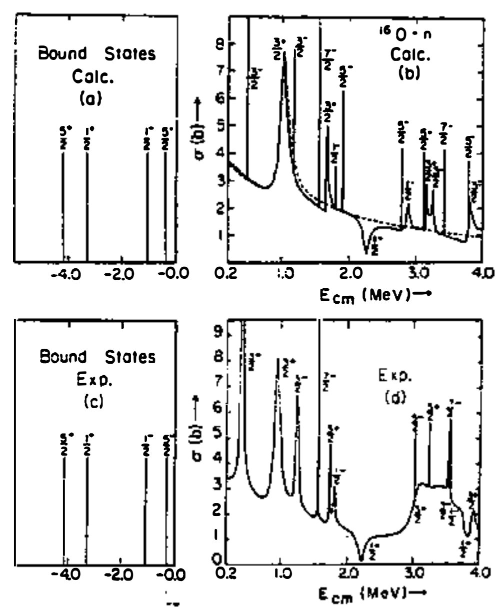

Fig. 8.
Comparison between calculated[26)] and experimental bound state spectrum and neutron total cross-section for the $^{16}O + n$ system.

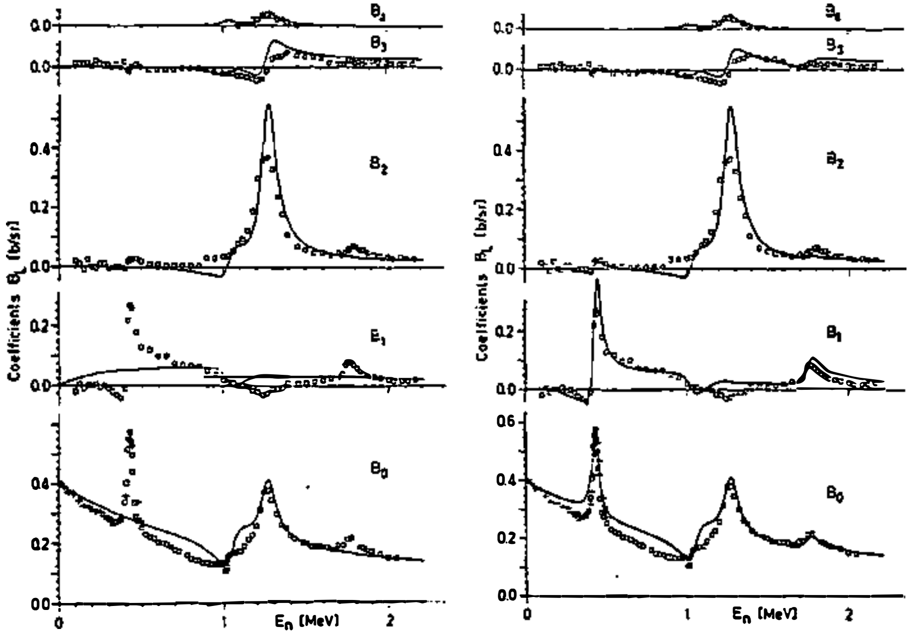

Fig. 9 Legendre expansion coefficients B_L of the elastic differential cross section taken from the calculations of Birkholz and Heil[30] for the $^{11}B + n$ system. On the left are shown results before, and on the right after the addition of fitted resonance terms describing omitted positive parity resonances at 0.43 and 1.77 MeV.

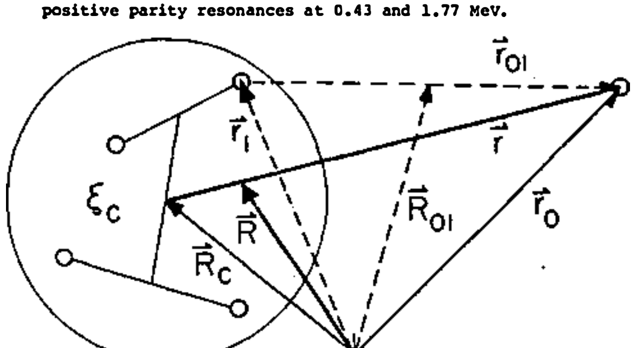

Fig. 10 Schematic illustration of coordinates encountered in continuum shell model calculations.

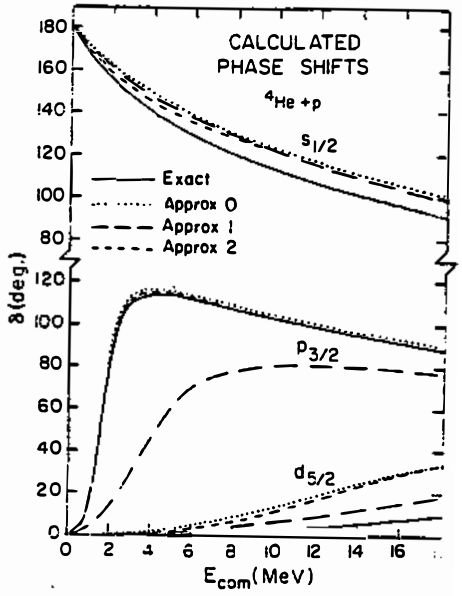

Fig. 11.
Comparison between elastic scattering phase shifts for the ^{4}He + p system calculated[34] with and without corrections for target recoil.

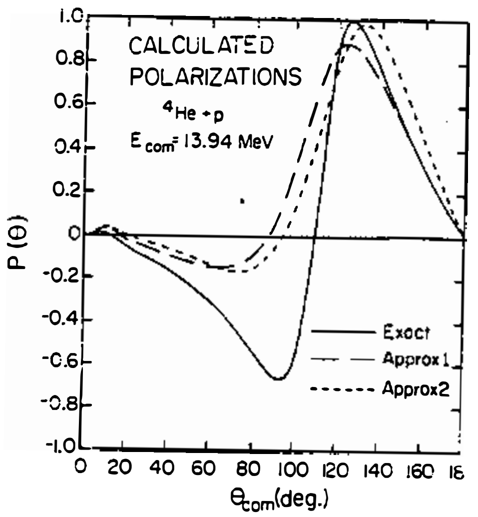

Fig. 12.
Comparison between proton polarizations for the ^{4}He + p system calculated[34] with and without corrections for target recoil.

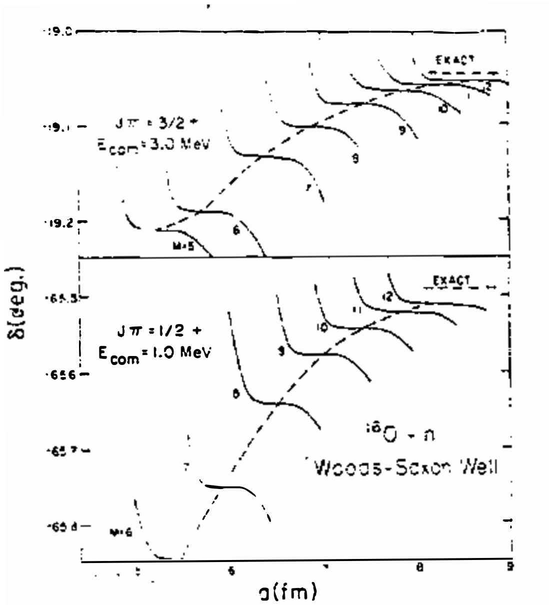

Fig. 13.

Dependence on the matching radius of elastic scattering phase shifts for the ^{16}O + n system calculated[40] from a potential model. The internal wave function was represented as a superposition of M oscillator radial functions.

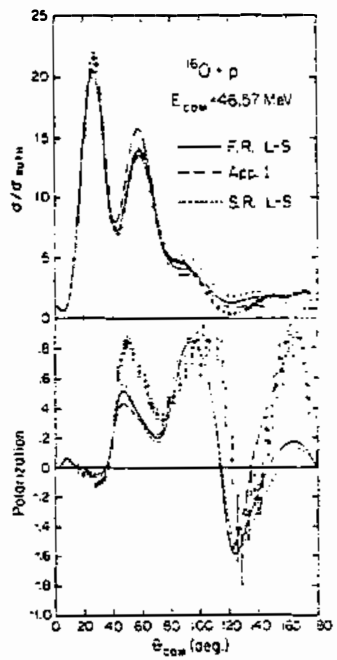

Fig. 14.

Comparison between elastic scattering cross-sections and polarizations for the ^{16}O + p system calculated[39] with and without corrections for target recoil.

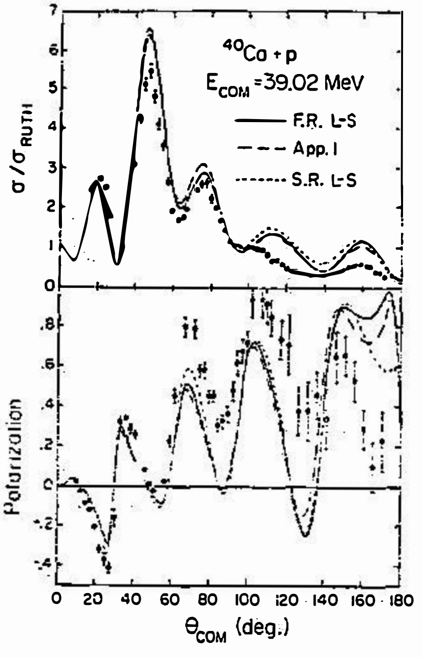

Fig. 15.

Comparison between elastic scattering cross-sections and polarizations for the ^{40}Ca + p system calculated[39] with and without corrections for target recoil.

DISCUSSION

K. Dietrich: One of the problems in the shell-model descriptions of nuclear reactions was the inclusion of configurations with several nucleous in the continuum. Has there been recent progress in resolving this difficulty?

R.J. Philpott: States in which several nucleons are excited above the threshold for particle emission can be handled by including appropriate BSEC. In a calculation for $^{16}O+n$, for example, it would be possible to include configurations in which several nucleons have been excited into the $1d_{3/2}$ oscillator orbit. However, the treatment of multiparticle breakup remains a fundamental unsolved problem. I am not aware of any substantial calculation which attempts to include multiparticle breakup within the continuum shell model framework.

G. Delić: Could you give us an idea of the additional work involved in the separation method of Birkholz and Heil if recoil corrections are included.

R.J. Philpott: I cannot answer this question directly. The main problem here is that the transformation discussed in the talk mixes nucleon orbital angular momentum quantum numbers. Exact calculation of intrinsic matrix elements with small l may require knowledge of shell model matrix elements of considerably higher l. Hopefully it will be possible to find approximate ways to calculate the shell model matrix elements for higher l which are both accurate and fast. However, I have not looked into this problem yet. Once the shell model matrix elements are known, the transformations is not difficult to carry out.