FIZIKA 9 (1977) Supplement 3, 91–114
(Proceedings of the Int. Symp. on Nuclear Collisions and
Their Microscopic Description, Bled, September 1977)

RESONATING-GROUP CALCULATIONS IN LIGHT SYSTEMS

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1. Introduction

The method of the resonating-group structure or the resonating-group method (RGM) is a microscopic method which takes cluster correlations explicitly into consideration [1,2] . It has the following important characteristics:

(i) It employs totally antisymmetric wave functions and, therefore, takes the Pauli exclusion principle fully into account.

(ii) It utilizes a nucleon-nucleon potential which explains reasonably well the two-nucleon low-energy scattering data.

(iii) It treats correctly the center-of-mass motion of the entire system.

(iv) It considers nuclear bound-state, scattering, and reaction problems from a unified viewpoint.

(v) It can be used to study cases where the particles involved in the incoming and outgoing channels are arbitrary composite nuclei.

The purpose of this talk is to review the progress which has been made in utilizing this method to study the behavior of various systems, and to discuss the information which has been learned with regard to the importance of the Pauli principle in nuclear problems.

In sect. 2, a brief discussion of the resonating-group formulation is given. Section 3 is devoted to a description of a complex-generator-coordinate technique [3-7] which has been employed recently to evaluate the various matrix elements required in resonating-group calculations. Illustrative examples of bound-state, scattering, and reaction calculations are presented in sect. 4, while effects of antisymmetrization are discussed in sect. 5. Finally, in sect. 6, concluding remarks are made.

2. Brief discussion of the resonating-group formulation

A rather detailed discussion of the RGM formulation has recently

been given elsewhere [2] ; therefore, only a relatively brief description will be presented here.

The starting point of a resonating-group calculation is the projection equation

$$\langle \delta\psi \mid H - E_T \mid \psi \rangle = 0 \ , \tag{1}$$

where E_T is the total energy of the system and H is a Galilean-invariant Hamiltonian given by

$$H = \sum_{i=1}^{N} \frac{1}{2M} \vec{p}_i^{\,2} + \sum_{i<j=1}^{N} V_{ij} - T_{cm} \ , \tag{2}$$

with N being the number of nucleons, T_{cm} being the kinetic-energy operator of the total center of mass, and V_{ij} being a nucleon-nucleon potential chosen to fit the two-nucleon scattering data especially in the low-energy region. The trial function ψ is written in the following schematic form:

$$\begin{aligned} \psi = \mathcal{A} \Big\{ & \sum_i \phi(A_i)\phi(B_i) F_i(\vec{R}_i) \\ & + \sum_j \phi(A_j)\phi(B_j)\phi(C_j) F_j(\vec{R}_{j1}, \vec{R}_{j2}) \\ & + \cdots\cdots \\ & + \sum_m c_m \mathcal{G}_m \Big\} \, Z(\vec{R}_{cm}) \ , \end{aligned} \tag{3}$$

where $\mathcal{A}$ is an antisymmetrization operator. The functions ϕ describe the internal behavior of the clusters, while $Z(\vec{R}_{cm})$ is a normalizable function describing the total c.m. motion. The functions $F_i(\vec{R}_i)$, $F_j(\vec{R}_{j1}, \vec{R}_{j2})$, and so on are relative-motion functions in two-, three-, and more-cluster configurations. The distortion functions $\mathcal{A}[\zeta_m Z(\vec{R}_{cm})]$ are chosen to improve the wave function in the strong-interaction region; they vanish for large internucleon and intercluster distances.

The variation $\delta\psi$ specifies the function space to be used in the

calculation. This function space is defined through arbitrary variations of the linear functions $F_i, F_j, \cdots$, and the linear amplitudes c_m contained in the trial function ψ of eq. (3). By substituting the expressions for $\delta\psi$ and ψ into the projection equation (2), one then obtains a set of coupled integrodifferential and integral equations which these linear functions and linear amplitudes satisfy.

Because of computational complexities, one must choose relatively simple forms for ψ. In the following subsections, I shall indicate the general procedure to derive the coupled equations for some representative types of trial functions which have commonly been employed in practical calculations.

2.1. Single-channel formulation without specific distortion

First, we consider the simplest case where the trial function consists of a single two-cluster open-channel term and where the effect of specific distortion [2] is neglected. Also, for clarity in discussion, it will be assumed that the clusters involved have no internal spin. Then, the trial function has the form

$$\begin{aligned} \psi &= \mathcal{A}\left[\phi(A)\phi(B)F(\vec{R})\,Z(\vec{R}_{cm})\right] \\ &= \int \mathcal{A}\left[\phi(A)\phi(B)\delta(\vec{R}-\vec{R}'')Z(\vec{R}_{cm})\right]F(\vec{R}'')\,d\vec{R}'', \end{aligned} \tag{4}$$

where $\vec{R}''$ is a parameter coordinate on which the operator $\mathcal{A}$ does not act. By writing the variation $\delta\psi$ also in a parameter representation, i.e.,

$$\delta\psi = \int \mathcal{A}\left[\phi(A)\phi(B)\delta(\vec{R}-\vec{R}')Z(\vec{R}_{cm})\right]\delta F(\vec{R}')\,d\vec{R}', \tag{5}$$

one obtains then from eq. (1) the following equation:

$$\int \hat{K}(\vec{R}', \vec{R}'')F(\vec{R}'')\,d\vec{R}'' = 0 \tag{6}$$

where

$$\hat{K}(\vec{R}',\vec{R}'') = \langle \Phi(A)\,\phi(B)\,\delta(\vec{R}-\vec{R}')\,\mathcal{Z}(\vec{R}_{cm})\,|H-E_T| \\ \mathcal{A}'[\hat{\phi}(A)\hat{\phi}(B)\delta(\vec{R}-\vec{R}'')\mathcal{Z}(\vec{R}_{cm})]\rangle , \tag{7}$$

with $\hat{\phi}(A)$ and $\hat{\phi}(B)$ being translationally-invariant, antisymmetrized internal functions for the clusters A and B, respectively, and $\mathcal{A}'$ is an antisymmetrization operator which interchanges nucleons in different clusters. Equation (6) is an integrodifferential equation for $F(\vec{R}')$, which can be numerically solved to yield the desired phase shifts and, consequently, the differential scattering cross section.

To proceed somewhat further, it is convenient to choose the normalization condition as

$$\langle \phi(A)\phi(B)\mathcal{Z}(\vec{R}_{cm})\,|\,\hat{\phi}(A)\hat{\phi}(B)\mathcal{Z}(\vec{R}_{cm})\rangle_{\vec{R}} = 1 , \tag{8}$$

where the notation $\langle\ \rangle_{\vec{R}}$ means that the relative coordinate $\vec{R}$ is not integrated over. By recognizing now the fact that in an antisymmetrized calculation it is always possible to replace the operator P^r_{ij} in the nucleon-nucleon potential by the operator $-P^\sigma_{ij}P^\tau_{ij}$, one can then define a direct (local) potential $V_D(\vec{R})$ as

$$V_D(\vec{R}) = \langle \phi(A)\phi(B)\mathcal{Z}(\vec{R}_{cm})|\,V'\,|\hat{\phi}(A)\hat{\phi}(B)\mathcal{Z}(\vec{R}_{cm})\rangle_{\vec{R}} \tag{9}$$

with

$$V' = \sum_{i\in A}\sum_{j\in B} V_{ij} \tag{10}$$

In terms of this direct potential, the integrodifferential equation (6) can be written in the following more explicit form:

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2_{\vec{R}'} + V_D(\vec{R}') - E\right]F(\vec{R}') + \int K(\vec{R}',\vec{R}'')F(\vec{R}'')\,d\vec{R}'' = 0 , \tag{11}$$

where E is the relative energy of the two clusters in the c.m. system and $K(\vec{R}',\vec{R}'')$ is an energy-dependent kernel function given by

$$K(\vec{R}',\vec{R}'') = \langle \phi(A)\phi(B)\delta(\vec{R}-\vec{R}')\mathcal{Z} \mid H-E_T \mid \mathcal{A}''\left[\hat{\phi}(A)\hat{\phi}(B)\delta(\vec{R}-\vec{R}'')\mathcal{Z}\right]\rangle \tag{12}$$

with $\mathcal{A}'' = \mathcal{A}' - 1$.

2.2. Single-channel formulation with specific distortion

Especially for systems which involve easily compressible clusters, it will be necessary to take the specific distortion effect into account [8-10]. To achieve this, one adds distortion-function terms into the trial function. In this subsection, we illustrate the essential points by considering the case where, for simplicity in discussion, only one distortion function is included. That is, the trial function is written as

$$\Psi = \Psi_0 + c_1 \mathcal{A}\left[\mathcal{F}_1 \mathcal{Z}\right] , \tag{13}$$

where ψ_0 is a channel function of the form given by eq. (4) and c_1 is a linear variational amplitude. By using the projection equation (1) and solving the resultant equation for c_1, one obtains

$$\langle \delta\Psi_0 \mid H + \tilde{V} - E_T \mid \Psi_0 \rangle = 0 , \tag{14}$$

where

$$\tilde{V} = -\frac{(H-E_T)\mid \hat{\mathcal{F}}_1 \mathcal{Z}\rangle\langle \hat{\mathcal{F}}_1 \mathcal{Z} \mid (H-E_T)}{\langle \hat{\mathcal{F}}_1 \mathcal{Z} \mid H - E_T \mid \hat{\mathcal{F}}_1 \mathcal{Z}\rangle} \tag{15}$$

with $\hat{\zeta}_1 = \mathcal{A}\zeta_1$. Choosing the normalization condition as

$$\langle \mathcal{F}_1 \mathcal{Z} \mid \hat{\mathcal{F}}_1 \mathcal{Z} \rangle = 1 \tag{16}$$

and proceeding in the same way as described in subsect. 2.1, one finds again an integrodifferential equation in the form of eq. (11), but with $K(\vec{R}',\vec{R}'')$ replaced by $K(\vec{R}',\vec{R}'') + K_1(\vec{R}',\vec{R}'')$, where

$$K_1(\vec{R}',\vec{R}'') = -\frac{\lambda(\vec{R}')\lambda^*(\vec{R}'')}{\hat{E}_1 - E_T} \tag{17}$$

with

$$\lambda(\vec{R}') = \langle \phi(A)\phi(B)\delta(\vec{R}-\vec{R}')\mathcal{Z} \mid H - E_T \mid \hat{\mathcal{F}}_1 \mathcal{Z} \rangle \tag{18}$$

and $\hat{E}_1$ being the expectation value of H with respect to the distortion function. Thus, one sees that the addition of a distortion function introduces an extra nonlocal, energy-dependent, separable term into the effective internuclear potential.

2.3. Coupled-channel formulation

In this subsection, a two-channel formulation will be briefly described. Here the trial function has the form

$$\psi = \psi_f + \psi_g , \tag{19}$$

where

$$\psi_f = \mathcal{A}\left[\phi(A)\phi(B)F(\vec{R}_f)\mathcal{Z}(\vec{R}_{cm})\right] \tag{20}$$

and

$$\psi_g = \mathcal{A}\left[\phi(C)\phi(D)G(\vec{R}_g)\mathcal{Z}(\vec{R}_{cm})\right] \tag{21}$$

The ϕ's are cluster internal functions which are normalized according to the condition

$$\begin{aligned} &\langle \phi(A)\phi(B)\mathcal{Z} \mid \hat{\mathcal{P}}(A)\hat{\mathcal{P}}(B)\mathcal{Z} \rangle_{\vec{R}_f} \\ &= \langle \phi(C)\phi(D)\mathcal{Z} \mid \hat{\mathcal{P}}(C)\hat{\mathcal{P}}(D)\mathcal{Z} \rangle_{\vec{R}_g} = 1 \end{aligned} \tag{22}$$

By substituting ψ and $\delta\psi$ into the projection equation, one obtains the following coupled equations:

$$\int \hat{K}_{ff}(\vec{R}_f', \vec{R}_f'')F(\vec{R}_f'')\,d\vec{R}_f'' + \int K_{fg}(\vec{R}_f', \vec{R}_g'')G(\vec{R}_g'')\,d\vec{R}_g'' = 0 \tag{23}$$

$$\int \hat{K}_{gg}(\vec{R}_g', \vec{R}_g'')G(\vec{R}_g'')\,d\vec{R}_g'' + \int K_{gf}(\vec{R}_g', \vec{R}_f'')F(\vec{R}_f'')\,d\vec{R}_f'' = 0 \tag{24}$$

where $\hat{K}_{ff}$ and $\hat{K}_{gg}$ have forms similar to that of eq. (7), and

$$K_{fg}(\vec{R}_f', \vec{R}_g'') = \langle \phi(A)\phi(B)\delta(\vec{R}_f - \vec{R}_f')\xi \,|\, H - E_T \,|\, \mathcal{A}[\phi(C)\phi(D)\delta(\vec{R}_g - \vec{R}_g'')\xi] \rangle \quad (25)$$

$$K_{gf}(\vec{R}_g', \vec{R}_f'') = \langle \phi(C)\phi(D)\delta(\vec{R}_g - \vec{R}_g')\xi \,|\, H - E_T \,|\, \mathcal{A}[\phi(A)\phi(B)\delta(\vec{R}_f - \vec{R}_f'')\xi] \rangle \quad (26)$$

If one uses now the procedure described in subsect. 2.1, then it is possible to write eqs. (23) and (24) in the following more familiar form:

$$\left[-\frac{\hbar^2}{2\mu_f} \nabla^2_{\vec{R}_f'} + V_{Df}(\vec{R}_f') - E_f \right] F(\vec{R}_f') + \int K_{ff}(\vec{R}_f', \vec{R}_f'') F(\vec{R}_f'') d\vec{R}_f'' + \int K_{fg}(\vec{R}_f', \vec{R}_g'') G(\vec{R}_g'') d\vec{R}_g'' = 0 \quad (27)$$

$$\left[-\frac{\hbar^2}{2\mu_g} \nabla^2_{\vec{R}_g'} + V_{Dg}(\vec{R}_g') - E_g \right] G(\vec{R}_g') + \int K_{gg}(\vec{R}_g', \vec{R}_g'') G(\vec{R}_g'') d\vec{R}_g'' + \int K_{gf}(\vec{R}_g', \vec{R}_f'') F(\vec{R}_f'') d\vec{R}_f'' = 0 , \quad (28)$$

where V_{Df} and V_{Dg} are direct potentials, E_f and E_g are relative energies of the clusters in the two channels, and K_{ff} and K_{gg} are energy-dependent kernel functions having forms similar to that given by eq. (12). By solving eqs. (27) and (28) subject to appropriate boundary conditions, one obtains then information concerning various scattering and reaction processes.

3. Complex-generator-coordinate technique

For the computation of various matrix elements required in resonating-group calculations, a complex-generator-coordinate technique [3-7] has recently been used. In this technique, the essential idea is to express the trial wave function as an integral of antisymmetrized products of single-particle functions and then make use of techniques in shell-model calculations to carry out an analytic evaluation of these matrix elements.

To illustrate this technique, let us consider a trial function of the form

$$\psi = \mathcal{A}' \left[\hat{\phi}(A)\hat{\phi}(B) F(\vec{R}) \, z(\vec{R}_{cm}) \right] \tag{29}$$

where the antisymmetrized internal function $\hat{\phi}(k)$, with k = A or B, is chosen as a translationally invariant shell-model function of the lowest configuration in a harmonic-oscillator well of width parameter α_k, i.e.,

$$\hat{\phi}(k) = \mathcal{A}_k \left[\xi_k \prod_j h_j(\vec{r}_j - \vec{R}_k) e^{-\frac{1}{2}\alpha_k (\vec{r}_j - \vec{R}_k)^2} \right] \tag{30}$$

with the functions h_j being polynomials in single-particle spatial coordinates and ξ_k being an appropriate spin-isospin function. Because of the presence of the cluster c.m. coordinates $\vec{R}_A$ and $\vec{R}_B$ in the exponents of the internal functions, the trial function ψ is not in the form of an antisymmetrized product of single-particle functions. However, by introducing an integral representation for ψ, we can show that the integrand can be represented in such a product form or, in other words, the integrand will contain no cross terms of the type $\vec{r}_i \cdot \vec{r}_j$.

To show this, we rewrite ψ in the form

$$\begin{aligned} \psi = \int \mathcal{A}' \left[\hat{\phi}(A; \vec{R}_A'') \hat{\phi}(B; \vec{R}_B'') \delta(\vec{R}_A - \vec{R}_A'') \delta(\vec{R}_B - \vec{R}_B'') \right] \\ \times F(\vec{R}_A'' - \vec{R}_B'') \, z\left(\frac{N_A \vec{R}_A'' + N_B \vec{R}_B''}{N} \right) d\vec{R}_A'' \, d\vec{R}_B'' \end{aligned} \tag{31}$$

with N_A and N_B being the numbers of nucleons in the two clusters. In the above equation, it is noted that the parameters $\vec{R}_A''$ and $\vec{R}_B''$ are included in the arguments of the internal functions; this is purposely done in order to call attention to the fact that the function $\hat{\phi}(k;\vec{R}_k'')$ is obtained from eq. (30) by replacing the cluster coordinate $\vec{R}_k$ with the parameter coordinate $\vec{R}_k''$. Now, by using the integral representation for the δ-function, i.e.,

$$\delta(\vec{R}_k - \vec{R}_k'') = \left(\frac{1}{2\pi}\right)^3 \int e^{i\vec{S}_k'' \cdot (\vec{R}_k - \vec{R}_k'')} \, d\vec{S}_k'' \qquad (k = A \text{ or } B) \tag{32}$$

we obtain, after further making the transformation

$$\vec{Q}_k'' = \frac{1}{N_k \alpha_k} \vec{S}_k'' - i \vec{R}_k'' \qquad (k = A \text{ or } B) \tag{33}$$

the following expression for the trial function ψ :

$$\begin{aligned}\psi = & \left(\frac{N_A \alpha_A}{2\pi}\right)^3 \left(\frac{N_B \alpha_B}{2\pi}\right)^3 \\ & \times \int \mathcal{A}' \Big\{ \mathcal{A}_A \Big[\xi_A \prod_{j=1}^{N_A} h_j(\vec{r}_j - \vec{R}_A'') e^{-\frac{1}{2}\alpha_A (\vec{r}_j - i\vec{Q}_A'')^2} \Big] \\ & \times \mathcal{A}_B \Big[\xi_B \prod_{j=N_A+1}^{N} h_j(\vec{r}_j - \vec{R}_B'') e^{-\frac{1}{2}\alpha_B (\vec{r}_j - i\vec{Q}_B'')^2} \Big] \Big\} \\ & \times F(\vec{R}_A'' - \vec{R}_B'') e^{\frac{1}{2} N_A \alpha_A (\vec{R}_A'' - i\vec{Q}_A'')^2 + \frac{1}{2} N_B \alpha_B (\vec{R}_B'' - i\vec{Q}_B'')^2} \\ & \times \mathcal{Z}\left(\frac{N_A \vec{R}_A'' + N_B \vec{R}_B''}{N}\right) d\vec{Q}_A'' \, d\vec{Q}_B'' \, d\vec{R}_A'' \, d\vec{R}_B'' \end{aligned} \tag{34}$$

The above equation for ψ can be reduced by noting that, because the internal function is chosen to have the lowest configuration in a harmonic-oscillator well, the argument $(\vec{r}_j - \vec{R}_k'')$ of h_j can be replaced by the argument $(\vec{r}_j - \vec{C}_k)$, with $\vec{C}_k$ being any constant vector. Thus, by making the transformations

$$\begin{aligned} \vec{R}'' &= \vec{R}_A'' - \vec{R}_B'' & \vec{R}_{cm}'' &= \frac{N_A \vec{R}_A'' + N_B \vec{R}_B''}{N} \\ \vec{S}'' &= \alpha_A \vec{Q}_A'' - \alpha_B \vec{Q}_B'' & \vec{S}_{cm}'' &= \frac{N_A \alpha_A \vec{Q}_A'' + N_B \alpha_B \vec{Q}_B''}{N} \end{aligned} \tag{35}$$

and by choosing

$$\mathcal{Z}(\vec{R}_{cm}'') = \exp\left(-\frac{N_A \alpha_A + N_B \alpha_B}{2} \vec{R}_{cm}''^2\right) \tag{36}$$

we can easily perform the integration over the variables $\vec{R}_{cm}''$ and $\vec{S}_{cm}''$, and obtain the following simplified expression:

$$\Psi = \left(\frac{N_A N_B}{2\pi N}\right)^3 \int \mathcal{A}\left[\hat{\Phi}(A,\vec{S}''\vec{R}'')\hat{\Phi}(B\ \vec{S}''\vec{R}'')\right]\Gamma(\vec{S}''\vec{R}'')F(\vec{R}'')d\vec{S}''d\vec{R}'' \qquad (37)$$

where

$$\Gamma(\vec{S}''\vec{R}'') = \exp\left\{-\frac{1}{2}\frac{N_A N_B(N_A\alpha_A + N_B\alpha_B)}{N^2\alpha_A\alpha_B}\left[\vec{S}'' + i\frac{N_B\alpha_A - N_A\alpha_B}{N}\vec{R}''\right]^2\right\} \qquad (38)$$

and

$$\hat{\Phi}'(A\ \vec{S}''\vec{R}'') = \mathcal{A}_A\left[\xi_A\prod_{j=1}^{N_A} h_j\, e^{-\frac{1}{2}\alpha_A(\vec{r}_j - \vec{\eta}_A)^2}\right]$$

$$\hat{\Phi}'(B\ \vec{S}''\vec{R}'') = \mathcal{A}_B\left[\xi_B\prod_{j=N_A+1}^{N} h_j\, e^{-\frac{1}{2}\alpha_B(\vec{r}_j - \vec{\eta}_B)^2}\right] \qquad (39)$$

with

$$\vec{\eta}_A = \frac{iN_B}{N\alpha_A}\left[\vec{S}'' + i\frac{N_A}{N}(\alpha_B - \alpha_A)\vec{R}''\right]$$

$$\vec{\eta}_B = -\frac{iN_A}{N\alpha_B}\left[\vec{S}'' - i\frac{N_B}{N}(\alpha_B - \alpha_A)\vec{R}''\right] \qquad (40)$$

The meaning of the function $\hat{\Phi}'(k;\vec{S}'',\vec{R}'')$, with k = A or B, is clear; it is a shell-model function of the lowest configuration in a harmonic-oscillator well of width parameter α_k, with the center of the well located at the point $\vec{\eta}_k$. Since it is seen from eq. (40) that $\vec{\eta}_k$ is a complex quantity, we have therefore chosen to call the technique described here a complex-generator-coordinate technique.

The trial function ψ written in the form of eq. (37) has the desired property mentioned at the beginning of this section. By carrying out next the same procedure for the variation $\delta\psi$ which will contain a generator coordinate $\vec{S}'$ and a parameter coordinate $\vec{R}'$, we can then compute the function $\hat{K}(\vec{R}',\vec{R}'')$ of eq. (7). For this it will be necessary to carry out integrations over all the nucleon coordinates and the generator coordinates $\vec{S}'$ and $\vec{S}''$, but not the parameter coordinates $\vec{R}'$ and

$\vec{R}''$. These integrations can be performed in a relatively straightforward manner, if one makes use of the freedom which is associated with the choice of the vector $\vec{C}_k$ appearing in the argument of h_j (for details, see ref. [11]).

Finally, it should be mentioned that the complex-generator-coordinate technique described here is not restricted to the case where the internal function of the cluster is chosen to have the simple form of eq. (30). It is, in fact, a straightforward matter to extend this technique to include also the case where the internal function is taken to be a sum of translationally-invariant, harmonic-oscillator shell-model functions, with each function characterized by a different width parameter for the corresponding oscillator well [6].

4. Bound-state, scattering, and reaction calculations

In this section, we show the results of some bound-state, scattering, and reaction calculations. These calculations are selected only for illustrative purposes; for further works using the resonating-group method, we refer the interested reader to other references [2,12-14].

4.1. Single-channel calculation without specific distortion

4.1a. $\alpha + {}^{16}O$ calculation

The first example we consider is the $\alpha + {}^{16}O$ problem [11]. In this example, the two clusters A and B are the α and the ${}^{16}O$ clusters, respectively. The antisymmetrized internal functions for these clusters are taken to have the form of eq. (30) with

$$\alpha_A = 0.514 \ \text{fm}^{-2} \qquad \alpha_B = 0.32 \ \text{fm}^{-2} \tag{41}$$

For the nucleon-nucleon potential, we use [15, 16]

$$V_{ij} = \left[V_R + \frac{1+P_{ij}^{\sigma}}{2} V_t + \frac{1-P_{ij}^{\sigma}}{2} V_s \right] \left(\frac{u}{2} + \frac{2-u}{2} P_{ij}^{r} \right) \cdot \frac{1+\tau_{i3}}{2} \frac{1+\tau_{j3}}{2} \frac{e^2}{r_{ij}} \tag{42}$$

where

$$V_R = V_{0R} \exp(-K_R r_{ij}^2)$$
$$V_t = -V_{0t} \exp(-K_t r_{ij}^2)$$
$$V_s = -V_{0s} \exp(-K_s r_{ij}^2) , \qquad (43)$$

with

$$V_{0R} = 200.0 \text{ MeV}, \quad K_R = 1.487 \text{ fm}^{-2}$$
$$V_{0t} = 178.0 \text{ MeV}, \quad K_t = 0.639 \text{ fm}^{-2}$$
$$V_{0s} = 91.85 \text{ MeV}, \quad K_s = 0.465 \text{ fm}^{-2} \qquad (44)$$

This particular nucleon-nucleon potential is chosen, since it yields a satisfactory description of not only the two-nucleon low-energy scattering data but also the essential properties of the deuteron, triton, and α particle.

The exchange-mixture parameter u in the nucleon-nucleon potential of eq. (42) is determined by solving eq. (11) with boundary conditions appropriate to an $\ell = 0$ bound state, and adjusting u until the calculation yields the experimental value [17] of 4.73 MeV for the α-particle separation energy in the ground state of ^{20}Ne. The value of u so determined is 0.881. With this value, we then calculate the energies of the $\ell = 2$ and 4 excited states and the phase shifts in the energy region from 0 to 30 MeV. The results are given in figs. 1 and 2, where $\ell = 6$

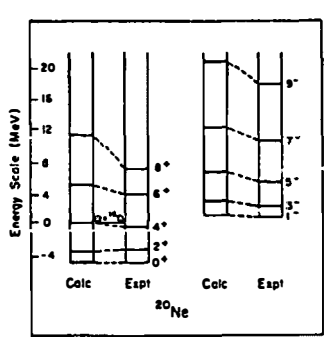

Fig. 1: Calculated and experimental spectra of ^{20}Ne.

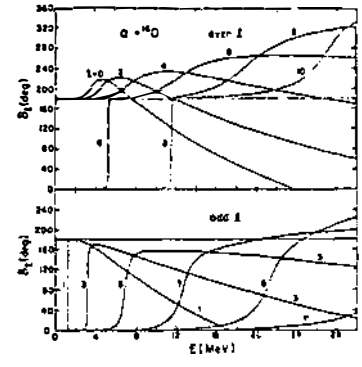

Fig. 2: Calculated phase shifts for α + ^{16}O scattering.

and 8 resonance states are also shown. From fig. 1 it is noted that there is a reasonable agreement between calculation and experiment [18,19], although the calculated excitation energies are generally too large. The reason for this discrepancy is probably that, in our calculation, the specific distortions of the clusters have not been explicitly taken into consideration and a simplified nucleon-nucleon potential has been used.

A comparison between calculated and experimental [20] results for the ratio $\sigma(\theta)/\sigma_c(\theta)$ at 19.2 MeV is shown in fig. 3. To obtain the calculated values, we have introduced into the formulation a phenomenological imaginary potential to take approximate account of reaction effects. This imaginary potential is chosen to have a Woods-Saxon derivative form with geometry parameters $R_s = 4.2$ fm and $a_s = 0.6$ fm. The depth parameter is then adjusted to yield an over-all best fit to the experimental data; the resultant value for this parameter is 2.0 MeV, which yields a total reaction cross section of 927 mb. As is seen from fig. 3, the agreement so obtained is indeed quite satisfactory, with the only discrepancy being in the detailed structure of the interference minimum near 80^c.

Many resonating-group calculations for systems heavier than $\alpha + {}^{16}O$ have also been performed. These are reported in refs. [5,21-26].

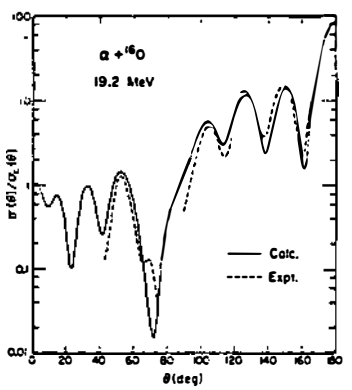

Fig. 3: Comparison of calculated and experimental differential cross sections for $\alpha + {}^{16}O$ scattering at 19.2 MeV.

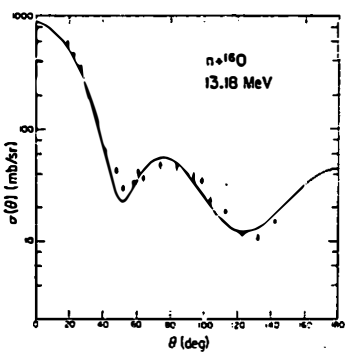

Fig. 4: Comparison of calculated and experimental differential cross sections for $n + {}^{16}O$ scattering at 13.18 MeV.

4.1b. n + ^{16}O calculation

To show that the complex-generator-coordinate technique is useful even in the case where flexible cluster internal functions are employed, we discuss here a recent calculation on the n + ^{16}O system [6]. In this calculation, the ^{16}O internal function is assumed as a sum of two functions, with each of them chosen to be a translationally-invariant antisymmetrized product of single-particle functions of the lowest configuration in a harmonic oscillator well of width parameter α_j, i.e.,

$$\phi(^{16}O) = \sum_{j=1}^{2} b_j \phi_j \qquad (45)$$

with

$$\phi_j = \mathcal{A}_{16}\left[\xi_{16}\prod_{i=1}^{16} \phi_i(\vec{r}_i - \vec{R}_{16}) e^{-\frac{1}{2}\alpha_j(\vec{r}_i - \vec{R}_{16})^2}\right] \qquad (46)$$

The parameters b_j and α_j are then adjusted to yield the experimentally determined rms matter radius of ^{16}O (2.6 fm) and a best over-all agreement with the form-factor data for q^2 up to about 7 fm^{-2}.

The nucleon-nucleon potential used has the same form as that given by eqs. (42)-(44). The exchange-mixture parameter u is determined by fitting the neutron separation energy of 3.26 MeV in the $\ell = 0$, $J^\pi = 1/2^+$ first excited state of ^{17}O. The resultant value of u is 0.945, which is somewhat larger than but still reasonably close to the value of 0.881 required in the α + ^{16}O case discussed above.

The n + ^{16}O differential cross section calculated at 13.18 MeV is shown in fig. 4, where a comparison with experimental data [27] is also made. To obtain the calculated result, we have adopted a phenomenlogical imaginary potential which is adjusted to yield a good agreement with the measured value for the total reaction cross section. From this figure one sees that all the features exhibited by the experimental data are well reproduced, thus fully demonstrating the usefulness of the resonating-group method in scattering calculations.

4.2. Single-channel calculation with specific distortion

As an example of a calculation with the specific distortion effect taken into account, we consider the α + α problem [15]. In this calculation, three distortion-function terms are included in the

resonating-group formulation for the purpose of improving the behavior of the trial wave function in the strong-interaction region. The nucleon-nucleon potential used is again that of eqs. (42) - (44), with the value of u chosen such that the resonance energy of the $\ell = 0$ ground state is correctly obtained. The result is u = 0.950, which is very close to the value required in the n + ^{16}O case.

In fig. 5 we show $\ell = 0$, 2, and 4 phase shifts, calculated at c.m. energies from 0 to 20 MeV. The solid curves show the result obtained with the specific distortion effect taken into consideration, while the dashed curves show the no-distortion result. The empirical data points are those of ref. [28] . As is seen, there is a good over-all agreement between the calculated result with distortion functions and the empirical result. In the $\ell = 4$ states, the calculated phase shifts are generally somewhat too small. This can probably be attributed to the fact that, in this calculation, a rather simple nucleon-nucleon

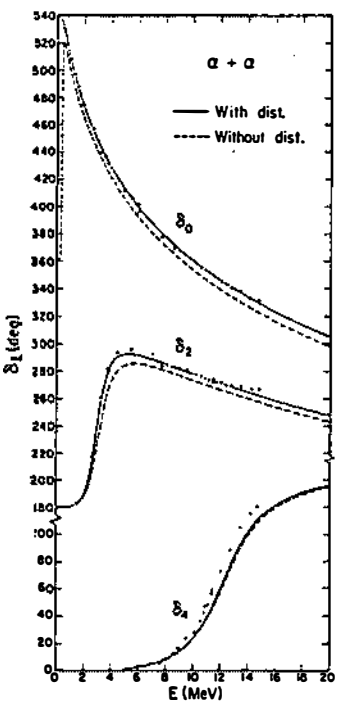

Fig. 5: α + α scattering with and without specific distortion effect.

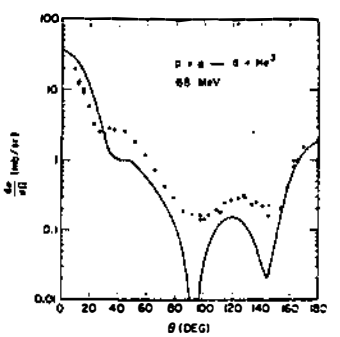

Fig. 6: Comparison of calculated and experimental differential reaction cross sections for the process α (p,d)^{3}He.

potential containing only a weakly repulsive core has been adopted.

From fig. 5 one further notes that, even though the specific distortion effect has only a minor influence on the phase shift in the $\ell = 4$ state, it does have a significant influence on both $\ell = 0$ and $\ell = 2$ phase shifts. Therefore, for a proper description of the behavior of the $\alpha + \alpha$ system in the strong-interaction region, it is important that this effect should be carefully taken into consideration.

4.3. Coupled-channel calculation

In this subsection, we discuss a calculation in the five-nucleon system where both the $d + {}^3He$ channel (channel f) and the $p + \alpha$ channel (channel g) are included [29]. For this calculation, the formulation of subsect. 2.3 is used and reaction effects, due to the presence of other open channels, are approximately taken into account by the use of phenomenological imaginary potentials.

The calculated $\alpha(p,d)^3He$ differential reaction cross section at $E_g = 68$ MeV is shown by the solid curve in fig. 6. Here one sees that the agreement with experimental data [30] is fairly satisfactory. The presence of deep minima and the slight underestimate in the calculated cross section are likely due to the fact that in the nucleon-nucleon potential employed there are no noncentral components.

From fig. 6 it is also noted that the differential reaction cross section has a decreasing trend in the forward angular region, but begins to increase when θ passes about 90^0. As has been determined in careful detail [31], the reason for this is that, at a relatively high energy, the reaction proceeds mainly through different mechanisms in the forward and backward angular regions. Thus, in the forward angular region it proceeds mainly through a one-nucleon pickup process, while in the backward angular region it proceeds mainly through a two-nucleon pickup process. Both of these processes are automatically included in the two channel-calculation described here, because in this calculation a totally antisymmetrized wave function is employed.

5. Effects of the Pauli principle

In this section, we discuss the effects of the Pauli principle

on the effective interaction between two clusters. This discussion will be relatively brief, since a similar discussion on this subject has recently been given elsewhere [32]. For effects of this principle in other types of nuclear problems, the readers are referred to refs. [2,31,33,34].

One of the important findings from resonating-group calculations is that the phase-shift behavior generally exhibits a distinct odd-even dependence on the orbital angular momentum. This is demonstrated in fig. 7, where one sees that while the degree of this dependence is very weak in the n + ^{16}O and n + ^{40}Ca systems, it is quite strong in the ^{3}H + α system.

The occurrence of the odd-even ℓ-dependence is a consequence of the Pauli principle. This is so, since it is well known that if this principle were not taken into consideration or, in other words, if the operator $\mathcal{A}$ in eq. (7) were set as unity, then the effective intercluster potential would just be equal to the direct potential V_D [see eqs. (11) and (12)] which has no ℓ-dependence and which will yield phase-shift points following a smooth trend with respect to the orbital angular momentum.

To demonstrate the presence of odd-even ℓ-dependence in a clearer way, we assume that the effective intercluster potential is given by

$$\tilde{V}_{\ell\lambda} = C_{\ell\lambda} V_{N\lambda} \tag{47}$$

with λ denoting the channel-spin multiplicity (the index λ may be dropped if the considered system has a single λ-value), $V_{N\lambda}$ being the direct

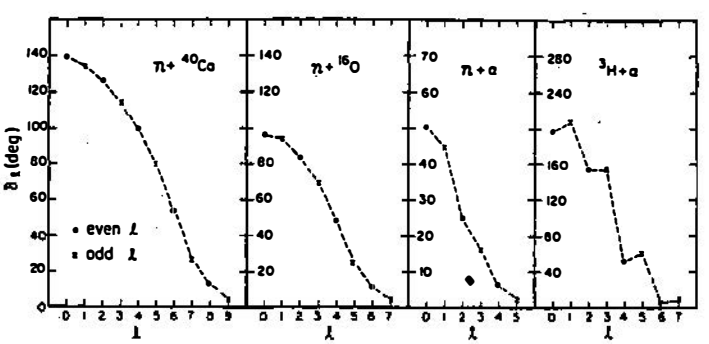

Fig. 7: Phase shifts as a function of ℓ for various systems, all calculated at a wave number of 1.52 fm^{-1}.

nuclear potential, and $C_{\ell\lambda}$ being an ℓ-dependent parameter adjusted to yield exactly the phase-shift values of the corresponding resonating-group calculation. In fig. 8, the results for $C_{\ell\lambda}$ in n + ^{40}Ca and ^{3}H + α systems at indicated energies are shown. As is seen, this parameter is only weakly ℓ-dependent in the n + ^{40}Ca system, but shows a strong zigzag pattern in the ^{3}H + α system. If one attempts to fit the values of $C_{\ell\lambda}$ with a formula

$$C_{\ell\lambda} = C_{a\lambda} + (-1)^{\ell} C_{b\lambda} , \tag{48}$$

then at 30 MeV the best values of $C_{a\lambda}$ and $C_{b\lambda}$ for these systems, together with the values determined in the n + ^{6}Li case [35] , are given in table 1. From this table, it is noted that the value of $C_{a\lambda}$ is significantly larger than 1 in every one of these cases, while the value of $C_{b\lambda}$ is not only strongly system-dependent but also depends on the channel spin. Indeed, one sees that, in the n + ^{6}Li system, even the signs of $C_{b\lambda}$ are not the same in the two channel-spin states. This is, in fact, again an effect of the Pauli principle; because of the presence of two nucleons in the nonclosed 1p-shell of ^{6}Li, there exists a blocking effect which causes the effective potential to behave differently in different channel-spin states [35] .

Since the amount by which $C_{a\lambda}$ deviates from 1 and the amount by which $C_{b\lambda}$ deviates from 0 is a quantitative measure of the effects of antisymmetrization, table 1 shows that the Pauli principle is generally important and must always be taken into consideration in nuclear scattering problems.

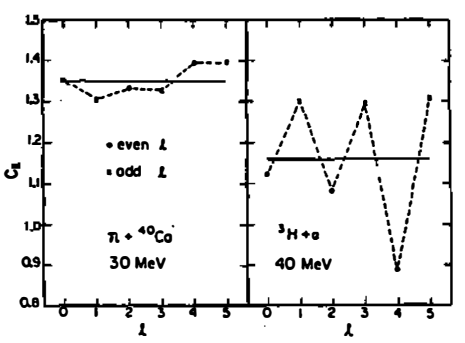

Fig. 8: C_ℓ as a function of ℓ.

Table 1

Values of $C_{a\lambda}$ and $C_{b\lambda}$

System	$C_{a\lambda}$	$C_{b\lambda}$
n + ^{6}Li (λ=2)	1.13	-0.05
n + ^{6}Li (λ=4)	1.20	0.05
^{3}H + α	1.16	-0.14
n + ^{40}Ca	1.35	≈0

At a relatively high energy, the behavior of the differential scattering cross section in the backward angular region is strongly correlated with the magnitude (not the sign) of $C_{b\lambda}$ [36]. In fig. 9, we show a comparison [37] between the results obtained for $^3He + \alpha$ scattering at 44.5 MeV using the resonating-group method and the potential model of eqs. (47) and (48) with $C_{a\lambda} = 1.15$ and $C_{b\lambda} = 0$. Here it is seen that, although the agreement in the forward angular region is rather satisfactory, there is a strong disagreement at angles larger than about 90^o. In the $n + {}^{40}Ca$ system, on the other hand, one sees from fig. 10 that, because of the small magnitude of $C_{b\lambda}$, the agreement between the resonating-group result (solid dots) and the result obtained using the potential model with $C_{a\lambda} = 1.35$ and $C_{b\lambda} = 0$ (solid curve) is fairly reasonable even at large angles (the dashed curve in this figure is obtained by omitting antisymmetrization effects, i.e., by setting $C_{a\lambda} = 1$ and $C_{b\lambda} = 0$ in the potential model).

As has been explained previously [36], the term $(-1)^{\ell} C_{b\lambda} V_{N\lambda}$ is introduced to approximately represent the effect of the pickup exchange process. By carefully examining the odd-even behavior in many systems, it was recently asserted [38] that $C_{b\lambda}$ has generally a large magnitude when the pickup process involves a relatively large value for the ratio M/m, where M is the mass of the nucleus which does the pickup and m is

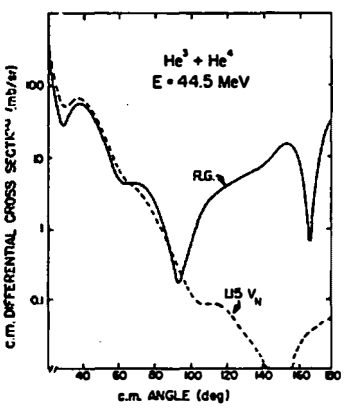

Fig. 9: Comparison of resonating-group and potential-model results for $^3He + \alpha$ scattering at 44.5 MeV.

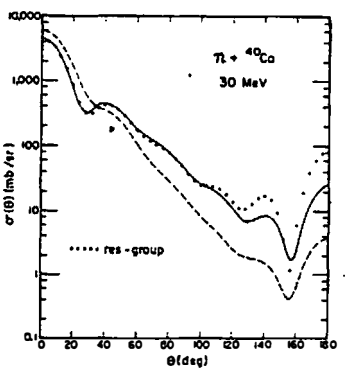

Fig. 10: Comparison of resonating-group and potential-model results for $n + {}^{40}Ca$ scattering at 30 MeV.

the mass of the cluster being picked up. This assertion certainly seems reasonable from an intuitive viewpoint and if it should turn out to be universally true, then one would expect the odd-even effect to have a large influence in heavy-ion scattering such as $^{12}C + ^{13}C$ [39], but to have a much smaller influence in light-ion scattering by medium- and heavy-weight nuclei such as nucleon-scattering on nuclei with $A \gtrsim 16$ [40].

6. Conclusion

Within the past few years, substantial progress has been made in utilizing the resonating-group method to study nuclear-structure and nuclear reaction problems. With the help of a complex-generator-coordinate technique to evaluate various matrix elements required in resonating-group calculations, it is now possible to perform realistic calculations for scattering systems which involve even rather heavy nuclei.

There are, however, still many problems which remain to be considered. These include: (i) the study of direct reactions involving relatively heavy clusters, (ii) the scattering of light and heavy ions by strongly deformed nuclei, (iii) the development of practical methods to use more realistic nucleon-nucleon potentials containing tensor and repulsive-core components, (iv) the problem of three- and more-cluster decays, and so on. Some of these problems are well within our present-day capabilities, while others may require further advance in mathematical techniques and computational innovations.

In conclusion, my opinion is that the resonating-group method is a practical method which certainly deserves further considerations. It is perhaps not overly optimistic to say that, within the foreseeable future, one might be able to solve with this method most of these nuclear-physics problems which have heretofore been studied only by macroscopic, phenomenological means.

Acknowledgments

Much of the work reported here was performed in collaboration with

Dr. D. R. Thompson and Dr. M. LeMere. It is my sincere pleasure to thank them for their able assistance and valuable discussions.

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DISCUSSION

M.S. Weiss: In a regime where fusion was important, could you employ the distortion functions or would you have to introduce a new functional form?

Y.C. Tang: To describe nuclear fusion in an adequate manner, it will be necessary to use a large number of distortion functions. These distortion functions can be translationally-invariant shell-model functions or cluster-model functions. More appropriately, one might wish to use antisymmetrized products of single-particle orbitals in deformed wells characterized by deformation parameters which can describe elongation and neck constriction.

M.V. Mihailović: You have introduced the distortion channel in order to describe the compound system. How do you choose the wave functions for distortion channel? Are they related to the Hamiltonian of the system? How many channels have you used in some of your calculations?

Y.C. Tang: We choose our distortion functions by physical reasoning. In the case of $\alpha+\alpha$ scattering, for example, we adopt $\alpha+\alpha$ bound structures where the α cluster has a rms radius either larger or smaller than that of a free α particle. In this particular problem, it was found that use of 3 appropriately chosen distortion functions is sufficient. In the $d+\alpha$ case where an easily distortable deuteron cluster is involved, we have used a larger number of 9 distortion functions.

M. Kamimura: I am much interested in your pointing out the importance of particle-exchange effect between colliding clusters, because I also examined this effect in the $\alpha + {}^{16}O$ case and found this effect to be quite large in the surface-contact region and it almost exceeds the stength of the folding potential.
In your $n+{}^{6}Li$ case, the nonclosed nucleus ^{6}Li is described by usual shell-model configuration. But the $\alpha+d$ clusterization must be large. Your estimate of Pauli principle effect in the $n + {}^{6}Li$ case is rather small. But it seems to me that the particle-exchange effect may be under-estimated in the surface clustering region of ^{6}Li.

Y.C. Tang: In the n + ^{6}Li problem, our primary purpose is to study the blocking effect created by the presence of two nucleons in the 1p-shell of the target nucleus ^{6}Li. To make the interpretation as clear as possible, it is important to use a simple description for the ^{6}Li cluster. This is the reason why we choose for the ^{6}Li internal function the most space-symmetric state of the $(1s)^4 (1p)^2$ configuration in a harmonic-oscillator well. If one generalizes this function to describe better the surface clustering effect, then it is expected that the odd-even feature will become more pronounced in the channel-spin 1/2 state, but the blocking effect will be somewhat reduced in its importance.

B.G. Giraud: Could you comment on the influence of the intrinsic cluster wave-functions? They are sometimes poor eigenstates of the Hamiltonian, aren't they?

Y.C. Tang: The cluster internal function is usually chosen to yield reasonable values for the rms radius and the binding energy. In the d + d case where a detailed examination of the influence of the cluster internal function has been made, it was found that at relatively low energies the choice of a sum of three Gaussian functions for the deuteron cluster yields satisfactory results for the scattering cross section. It is certainly true that, for computational reasons, one occasionally uses internal functions which are relatively poor representations of the eigenfunctions of the cluster Hamiltonians. In our d + ^{16}O calculation for example, we used for the deuteron cluster a single Gaussian function with an appropriately chosen width parameter; this produces a correct value for the rms radius, but a binding energy which is too small.

G. Paić: What is the physical meaning of the odd-even effect? Is the blocking the cause of its different magnitude?

Y.C. Tang: The odd-even effect arises from the exchange of core nucleons. In a more pictorial language, one might say that it arises from a pick up process. For example, in the case of ^{3}He + ^{4}He scattering, this process refers to the pick-up of a neutron by the ^{3}He cluster. The blocking effect has also its origin in the Pauli exclusion principle; therefore, a proper consideration of this effect is important in determining the magnitude of the odd-even feature in the effective internuclear potential.