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Study of Cluster Structure in Light Nuclei and Cluster-Rearrangement Collisions Based on the Three-Cluster Resonating Group Method

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§1. Introduction and Summary

Recently, collisions between complex nuclei and the cluster structure of light nuclei have been extensively studied based on microscopic models.[1)~4)]

The RIFP research project

In Japan, a lot of works have been performed in the course of the project on "Alpha-Like Four-Body Correlations and Molecular Structures in Light Nuclei" organized by the Research Institute for Fundamental Physics, Kyoto university. Almost of these works are reported in Refs. 1,2,5 and 6. Main purposes of the project can be described as follows: (i) To establish comprehensive understanding of the structure of light nuclei on the basis of the cluster model, together with investigating the coexistence and interplay of the cluster structure and shell structure. (ii) To clarify, based on microscopic models, particular role of inter-cluster interactions in those structures and in collisions between clusters (complex nuclei), and to develop the microscopic models themselves for increasing the applicability of cluster model. (iii) To investigate molecular resonances not only as an important part of the heavy-ion reaction problem but also as a natural extension of the cluster-structure study to highly excited states of nucleus.[6)] Of course, as mentioned bellow, these themes correlate strongly to each other.

Cluster structure

In the recent studies in (i), not only the well-developed (two- and three-) cluster structure but also compact shell-model-like structure are both treated in the same framework of cluster model.[2),5)] As was concluded by Horiuchi in his talk of the Tokyo conference,[5)] nuclear levels which are understood by the cluster model are not some specific levels but almost all the low-lying levels, and in some nuclei the number of the levels which have cluster structure are comparable with or more than that of the levels which have shell structure (cf. Horiuchi's report of the present symposium).

Microscopic methods

In the progress of these studies in (i), an important role has been played by works[2),4),7)~9)] along (ii), especially by the development of resonating group method (RGM) and generator coordinate method (GCM) and by the introduction of

orthogonality condition model (OCM).[10] Application of OCM has been extended to coupled-channel two-cluster systems[11),12)] and to three-cluster systems.[13)∿16),21)] For rather heavy systems, an OCM based on the direct use of norm kernel (namely $\sqrt{1-K}$ rather than the eliminating operator Λ of Ref. 10) has been proposed[17)] and utilized.[18),11),19)∿21)] Varidity of OCM has been examined both in the two-cluster systems[22),23),19),20)] and in a three-cluster system[21),24)] by comparing the OCM result with the RGM result and by analyzing the dynamical particle-exchange effect which is not included in the orthogonality contraint and folding potentials for the inter-cluster relative motion:for example, in the $^{16}O+\alpha$ case, it is clarified by RGM study[22)] that the dynamical one-particle exchange effect in the surface contact region is comparable with (or rather exceed) the strength of folding potential and has a considerable parity dependence.

Very recently, a method of how to derive the analytical form of the GCM and RGM kernels for various systems has been greatly developed by Tohsaki-Suzuki[25)] together with making an ultra-high speed computing code named "Rendez-Vours" (cf. Tohsaki-Suzuki's report in the present symposium). It has been another essential problem of (ii) how to solve RGM equation under the scattering boundary condition with using the GCM technique; calculation of GCM kernel is more accessible to complicated systems than that of RGM kernel is. This method was solved in practically useful way by variational methods[26)∿32)] of Kohn-Hulthen-Kato type[33)∿35)] and by the microscopic R-matrix theory.[36)]

The computing code VAR-GCM

In §2, we shall show usefulness of our variational method[28),32)] for 2-cluster systems based on the use of GCM kernels; its computing cose is named"VAR-GCM". In this method it is not necessary at all to construct the Hill-Wheeler GC weight function which reproduce the regular and irregular Coulomb functions outside the nuclear-interaction region; the scattering boundary condition is imposed on the physical-space (RGM) wave function. What is required is the calculation of GCM kernels only in the nuclear-interaction region. In the application, for example, to the α-α scattering, the 6×6 GC matrix elements are quite enough to obtain sufficiently accurate phase shifts up to about 60 MeV of c.m. energy. It is discussed how to examine the accuracy of the calculation within the frame-work of the present method; it is to be stressed that in complicated calculations such as RGM and GCM, how to check the reliability of calculation is of great importance. Combined use of the codes Rendez-Vours and VAR-GCM is in progress for the microscopic study of interaction and scattering between various nuclei.

Three-cluster RGM

RGM study of three-cluster systems will prepare a new stage of progress in the study of cluster structure and collisions between complex nuclei. As

well as the three-cluster OCM[13)~16),21)] and GCM,[37)] the three-cluster RGM[38),39),24)] is suited to describe simultaneously both the loosely coupled cluster states and the compactly coupled shell-model-like states. This RGM is able to give a consideration on the validity of the three-cluster OCM and to supply the OCM with improved inter-cluster potentials in stead of the folding potentials.

An advantage of RGM study is generaly speaking that we can analyze precisely various roles of Pauli principle and nucleon-nucleon interactions in the process of scattering and reaction, in the formation of (three-cluster) molecular resonances and in the bound states. Especially in the theory of cluster transfer, proper treatment of cluster-spectroscopic factors and suitable choice of the interactions responsible for the cluster transfer is expected to be discussed by the three-cluster RGM study.

RGM study of the 3α system

Some difficulties in the three-cluster RGM are overcome based on the use of GCM kernels and certain GC basis functions.[24)] In §4, we shall show the successful result of 3α-RGM calculation by Fukushima and myself.[24)] Following observed quantities of ^{12}C are reproduced consistently: (a) Energy spectrum of 0_1^+, 2_1^+, 4_1^+, 1_1^-, 3_1^-, 0_2^+ and 2_2^+, (b) form factors of the elastic electron scattering and of the inelastic scattering to the 2_1^+ state and B(E2; $2_1^+ \to 0_1^+$), (c) electron scattering form factor of $0_1^+ \to 0_2^+$ and the monopole matrix element M($0_2^+ \to 0_1^+$), (d) the reduced α-decay width of the 0_2^+ state and B(E2; $0_2^+ \to 2_1^+$) which are important to understand the $3\alpha \to {}^{12}C$ reaction in helium burning stars and (e) the reduced α-decay widths of the 1_1^- and 3_1^- states, inelastic electron scattering form factors of $0_1^- \to 1_1^-$ and $0_1^+ \to 3_1^-$ and B(E3; $3_1^- \to 0_1^+$). It is shown that 3α clusters keep the same size of free α particle in the states of (a). An effect of the particle exchange between 3α clusters is discussed.

Coupled channel variational methods for rearrangement collisions

In order to study the cluster-rearrangement collisions (on and off the molecular resonances) by using the three-cluster RGM, it is strongly expected to establish the coupled channel (CC) variational method of Kohn-Hulthén-Kato type based on the use of GCM kernels. As is known, in very light two-cluster systems, CC-RGM calculations (without the use of GC kernels and basis functions) have been performed by a lot of authors (for example, Refs. 26, 40 and 41). For the study of heavier systems, however, it is desirable to introduce the GCM techniques. For the case of two-cluster representation, such CC method in the GCM framework have been proposed by Mihailovic et al.[29)] with using the variational method and by Baye et al.[36c)] with using the microscopic R-matrix theory.

However, we would like to treat the rearrangement collisions as a totally antisymmetrized "three-body" problem; namely as an extention of the usual direct nuclear reaction theory and of the three-cluster OCM for reactions. For the

multi-step direct nuclear reactions via rearrangement channels, a new treatment of CC variational method was proposed by Kawai, Mito, Takesako and myself[31]; we combined the GC variational method of Mito and Kamimura[28] with the CC framework of variational method proposed by Ohmura et al.[42] This new CC variational method is not yet applied to the full microscopic case, but was successfully applied to theoretical study of the (d,p) reaction[31]; due to the suitable choice of GC type basis functions and to the introduction of skilful procedure in treating the non-local kernels, the computing time of this method is about 1/20 of that of the usual CC method.[43] In §3, we shall introduce briefly this new CC method.[31]

It looks promising to extend this CC variational method to general multi-steap direct reactions including transfer proce:s es in the intermediate step and to their full microscopic study based on the three-cluster RGM.

§2. A variational method in RGM for scattering between complex nuclei based on the use of GCM kernels

In §2, we introduce the work of Ref 32, a new treatment of the Kohn-Hulthen-Kato type variational method, which is an improvement of the variational method of Mito and Kamimura[28] and an application of the work of Kawai, Kamimura, Mito and Takesako[31] to the microscopic case (single channel). This improvement enable us to adopt a variety of forms of the trial function based on the use of corresponding GC basis functions and GCM kernels.

The RGM equation of motion to be solved is

$$\langle \Phi_1 \Phi_2 Y_{L,M}(\hat{R}) \,|\, H - E \,|\, \mathcal{A}[\Phi_1 \Phi_2 u_L(R)/R \cdot Y_{LM}(\hat{R})] \rangle = 0 . \qquad (2.1)$$

which leads to the integro-differential equation

$$\mathcal{L}_L \cdot u_L = 0 \;\; ; \;\; \mathcal{L}_L(R,R') \equiv [T_L(R) + V^D(R) - E_{cm}]\delta(R,R') + \int_0^\infty dR'' W(R,R'') . \qquad (2.2)$$

Notation in (2.1) and (2.2) follows that of Ref. 28. The stationary expression (functional) of the S-matrix, S_{st}, is given by

$$S_{st} = S_t + \frac{i\mu}{\hbar^2 k} (u_t \mathcal{L}_L u_t) , \qquad (2.3)$$

$$(f \mathcal{L}_L g) \equiv \int_0^\infty \int_0^\infty f(R) \mathcal{L}_L(R,R') g(R') dR dR' . \qquad (2.4)$$

Here $u_t(R)$ is a trial function of $u_L(R)$ and has the asymptotic form

$$u_t(R) \sim u_L^{(-)}(k,R) - S_t u_L^{(+)}(k,R) , \quad (kR \gg 1) \qquad (2.5)$$

where $u_L^{(-)}$ and $u_L^{(+)}$ are incoming and outgoing wave functions that satisfy

$$\left[T_L(R) + V^D(R) - E_{cm} \right] u_L^{(\pm)}(R,R) = 0 \quad (R \geq R_0) \qquad (2.6)$$

In the above, S_t is the trial S-matrix.

Now the key point of the method (the point improved from Ref. 28 by Ref. 31) is to construct u_t by taking a linear combination

$$u_t(R) = \sum_{i=0}^{N} c_i \, u_i(R) \qquad (2.7)$$

of a given set of functions $\{u_i(R)\}$ each of which has the form

$$u_i(R) = \begin{cases} \alpha_i \, u_i^{(in)}(R) & (R < R_0) \\ u_L^{(-)}(R,R) - s_i \, u_L^{(+)}(R,R) & (R \geq R_0) \end{cases} \qquad (2.8)$$

Here we have regarded that $W_L(R,R')=0$ for $R \geq R_0$ and for $R' \geq R_0$; this is the definition of R_0. The complex coefficients α_i and s_i are determined by the matching of $u_i(R)$ at $R=R_0$. Imposing the normalization condition

$$\sum_{i=0}^{N} c_i = 1 \qquad \left(c_0 = 1 - \sum_{i=1}^{N} c_i \right) \qquad (2.9)$$

we confine c_0 (any one). From this condition we have

$$u_t = u_0 + \sum_{i=1}^{N} c_i \left(u_i - u_0 \right) \qquad (2.10)$$

This u_t behaves for $R \geq R_0$ as

$$u_t(R) = u_L^{(-)}(R,R) - S_t \, u_L^{(+)}(R,R), \quad S_t = \sum_{i=0}^{N} c_i s_i \qquad (2.11)$$

It is to be noted that S_t itself is not a variational parameter; $\{c_i;\ i=1 \cdots N\}$ are the variational parameters.

The stationary condition $\partial S_{st}/\partial c_i = 0$ $(i=1 \cdots N)$ gives

$$\left(\left[u_i - u_0 \right] \mathcal{L}_L \, u_t \right) = 0 \qquad (i = 1 \cdots N) \qquad (2.12)$$

which leads to N linear equations for c_i's:

$$\sum_{j=1}^{N} \mathcal{L}_{ij} \, c_j = \mathcal{M}_i \qquad (i = 1 \cdots N) \qquad (2.13)$$

$$\mathcal{L}_{ij} = K_{ij} - K_{i0} - K_{0j} + K_{00}, \quad \mathcal{M}_i = K_{00} - K_{i0} \qquad (2.14)$$

$$K_{ij} \equiv \left(u_i \mathcal{L} \, u_j \right) = \alpha_i \alpha_j \left\{ \left(u_i^{(in)} \mathcal{L}_L \, u_j^{(in)} \right) - \int_{R_0}^{\infty} u_i^{(in)}(R) \left[T_L + V^D - E_{cm} \right] u_j^{(in)}(R) \, dR \right\}. \qquad (2.15)$$

As the trial form of $u_i^{(in)}(R)$, the followings are suited practically:

$$u_i^{(in)}(R)/R = 4\pi \, e^{-\lambda(R^2+R_i^2)} j_L(2\lambda R_i R) = \int Y_{LM}^*(\hat{R}) \, e^{-\lambda(R-S)^2} \delta(S-S_i)/S^2 \cdot Y_{LM}(\hat{S}) \, dS \, d\hat{R} \qquad (2.16)$$

$$u_i^{(in)}(R)/R = R^L L_{N_i}^{(L+1/2)}(2\lambda R^2)e^{-\lambda R^2} = \text{const.} \int Y_{LM}^*(\hat{R})\, e^{-\lambda(R-S)^2} \Phi_{N\cdot LM}(\lambda\lambda S)\, dS\, d\hat{R} \tag{2.17}$$

$$u_i^{(in)}(R)/R = R^L e^{-\nu_i R^2} = \text{const.} \int Y_{LM}^*(R)\, e^{-\lambda(R-S)^2} \Phi_{0LM}(\lambda\nu_i S)\, dS\, d\hat{R}. \tag{2.18}$$

Here $\lambda=N_1N_2\beta/2(N_1+N_2)$, $\beta=m\omega/\hbar$ being the single-particle H.O. parameter; N_1 and N_2 are the mass numbers of the colliding nuclei. The GC harmonic oscillator functions $\Phi_{NLM}(\lambda\nu S)$ are introduced in Refs. 44 and 45. The matrix elements $(u_i^{(in)}\mathcal{L}_L u_j^{(in)})$ are then given, interms of GCM kernels, by

$$(u_i^{(in)}\mathcal{L}_L u_j^{(in)}) = \text{const} \times \int \phi_i^*(S)\left[H^{(GCM)}(S,S') - E\cdot N^{(GCM)}(S,S')\right]\phi_j(S')\, dS\, dS'. \tag{2.19}$$

where $\phi_i(S)$ represents $\delta(S-S_i)/S^2\cdot Y_{LM}(\hat{S})$, $\Phi_{N_iLM}(\lambda\lambda S)$ or $\Phi_{0LM}(\lambda\nu_i S)$. Even in the cases of (2.17) and (2.18), the integral (2.19) is given in a simple form.[45] The additional integral appearing (2.15) is easy to calculate.

Approximate S-matrix is given by S_{st} and S_t. The stationary value S_{st} is more accurate than the stationary point S_t; the error of S_{st} (S_t) is the second (first) order with respect to the error contained in the trial function.[42] The check of the quantities $|S_{st}|-1$, $|S_t|-1$ and $|S_{st}-S_t|$ is a very good tool to examine how accurate the calculation is; this is an advantage of Kohn-Hulthen-Kato type variational method. It is to be stressed that in complicated calculations such as RGM and GCM, how to examine the accuracy of calculation is of great importance.

The present method is applied to the α-α scattering (the method of Ref. 28 was also applied to ^{16}O-α and ^{16}O-^{16}O scattering). Parameters are all the same as Ref. 28. Here use is made of the type (2.16) for the trial form of $u_i^{(in)}$. We set the GC mesh points S_i at a common interval ΔS. The matching radius R_0 is taken as $R_0=S_N$ (the outermost of S_i's) outside which we can regard $V^D(R)=Z_1Z_2e^2/R$. Let us denote the discretization points S_i's simply in the form $S_i[N, \Delta S; S_1\sim S_N]$.

Table I shows calculated result of δ_{st}, δ_t, $|S_{st}|$ and $|S_t|$ for L=0; δ is defined by $S=|S|e^{2i\delta}$. The smallness of the quantities $|S_{st}|-1$, $|S_t|-1$ and $\delta_{st}-\delta_t$ assures high accuracy of the calculation; the present result agrees very well with that of Ref. 28 and with another independent RGM result (the third-line phase shifts of Table I). Even the case of only six discretization points S_i gives quite satisfactory accuracy up to 60 MeV of c.m. It is found that the present method works slightly better than the method of Ref. 28 and is rather easier to make the programing. Extension of the present method and the method of Ref. 31 to the coupled channel three-cluster RGM is given in Ref.46.

§3. A coupled channel variational method for multi-step direct nuclear reactions

In this section we review the work of Kawai, Kamimura, Mito and Takesako[31] on the above title. The one-channel case of §2 should be refered to for the convenience of analogous consideration.

The importance of multi-step processes has been recognized in a variety of direct nuclear reactions. Among the theoretical methods for treating such processes, the method of coupled channels (CC) is probably the most suited for a general framework of the (phenomenological) analysis of experimental data.

The drawback of CC has been its difficulty in computation when applied to multi-step processes via rearrangement channels because of the complexity of the non-local coupling kernels and the coupled integro-differential equations that have to be solved. We then proposed[31] the use of a variational method for such CC calculations.

The method is a generalization of the Kohn-Hulthén-Kato type variational method of Ref. 42 with using the variational principle, $\delta I=0$ with

$$I[\Psi_t^{(\beta)}, \Psi_t^{(\alpha)}] = S_{\beta\alpha,t} + \frac{(-)^{J-M}}{\hbar}\int \tilde{\Psi}_t^{(\bar{\beta})\dagger}(H-E)\Psi_t^{(\alpha)} d\tau \qquad (3.1)$$

where $\Psi_t^{(\alpha)}$ ($\Psi_t^{(\beta)}$) is a trial total wave function with a definite total angular momentum JM and with the incident wave in the channel $\alpha(\beta)$ plus outgoing scattered waves. Here '~' stands for time reversal, $S_{\beta\alpha,t}$ is the trial S-matrix element contained in $\Psi_t^{(\alpha)}$, H is the total Hamiltonian and E is the total energy. $\bar{\beta}$ stands for the same channel as β except the sign of M. It can be shown that this variational principle is equivalent to the Schrödinger equation for $\Psi^{(\alpha)}$ and $\Psi^{(\beta)}$ and that the stationary value of I is the S-matrix element of the transition $\alpha \to \beta$, $S_{\beta\alpha}$. The Ψ's are assumed to be of CC type, i.e., of the form of a sum of the wave functions of the "strongly coupled channels". The trial functions are constructed with the assumption that the radial part of the wave function of relative motion in each of those channels is a linear combination of a proper set of basis functions (cf. Eq.(2.7)). The variation of the trial functions is then made by varying the coefficients of the linear combination (cf. the statement above Eq.(2.12)). The variational principle then leads to a set of linear inhomogeneous coupled equations for the coefficients (cf. Eq.(2.13)), the solutions of which give the stationary wave functions and the stationary value of I, namely $S_{\beta\alpha,st}$. The error in $S_{\beta\alpha,st}$ is of the second power of the errors in the Ψ_t's.

The radial basis functions are so chosen that (a) the correct radial wave functions can be well approximated by a small number of them, (b) each of them

have the correct asymptotic form of a linear combination of the incoming and outoutgoing Coulomb wave functions (cf. (2.5)) and (c) the coefficients (cf. $\mathcal{L}_{ij}$ and $\mathcal{M}_1$ of (2.13)) of the linear equations for the variational parameters can be calculated as analytically as possible (cf. (2.19)). The last condition is very important for a speedy computation which is essential for the practical applicability of CC to a reaction with many coupled rearrangement channels.

The present method was tested in two cases in which "exact" values of the S-matrix elements are known. In both cases, the basis functions were assumed to be the type of (2.16); the type of (2.18) was also examined and we obtained the same result as the use of (2.16) type.

(I) A one-body four channel problem with square-well interaction potentials Four channels (i=1,2,3 and 4) for a neutron, each with a distorting potential $V_{ii}(R)$ (i=1∿4), are coupled to each other by square-well potentials $V_{ij}(R)$ (i≠j=1∿4). All the V's have the same radius 6.0 fm. The depths are V_{ii}=-41 MeV for all i and V_{ij}=-5 MeV for all i≠j. This problem is analytically soluble. The results of a variational calculation with 10 basis functions per channel are compared with the exact value of the L=0 S-matrix elements of the transiton 1 → i, for i=1∿4 in Table II; The incoming energy is E_{11}= 20 MeV and the Q-values are Q_1= 0, Q_2=2, Q_3=13, Q_4=17 MeV. The agreement is seen to be extremely good.

(II) $^{16}O(d,p)^{17}O(2s, 3.266\ MeV)$. E_d=10.490 MeV, E_p=11.536 MeV
A finite range CC calculation of the S-matrix elements of this process was carried out in Ref. 43 assuming the coupling of the d and p channels by a Gaussian potential $V_{np}(r)$ which also binds the deuteron. A variational calculation with exactly same parameters of the distorting potentials and V_{np} etc. was performed with 15 basis functions in each channel. The essentially important point for speedy calculation is that the deuteron internal wave function and the bound-state wave function of the transferred neutron were expanded with very sufficient accuracy in a set of 8 and 11 Gaussian functions[46] respectively. Then, an analytic treatment of the non-local kernels is possible in the calculation of the coefficients of the linear equations for the variational parameters. Table III shows the absolute magnitude of the calculated S-matrix elements, S_{pd}, S_{pp} and S_{dd} for the (d,p), (p,p) and (d,d) processes respectively for each L. They are compared with the "exact" values of Ref. 43. The agreement is seen to be excellent. The difference in phases is 0.1° to 0.5°. The computing time of the variational calculation was about 1/20 of that of Ref. 43 on the same computer.

Extension of the present method to the three-cluster RGM framework is given in Ref. 47. Application to the heavy-ion reaction (via molecular resonances) is in progress.

§4. The 3α-RGM study of the structure of ^{12}C

In this section we review the work of Fukushima and Kamimura[24] for the 3α-RGM study of the structure of ^{12}C nucleus.

4.1 The 3α RGM based on the use of GCM kernels

The RGM wave function of the 3α syatem is assumed to be

$$\Psi_{JM}(3\alpha) = \mathcal{A}\left[\phi_1(\xi_1)\phi_2(\xi_2)\phi_3(\xi_3)\, \mathcal{U}_{JM}(r,R)\right], \qquad (4.1)$$

where ϕ's are the intrinsic wave functions of α clusters with the $(0s)^4$ H.O. configurations in which the oscillator parameter is defined by $\beta=m\omega/\hbar$. U_{JM} is the unknown relative wave function of r and R (Fig. 1) and is to be solved by $\langle\phi_1\phi_2\phi_3|H-E|\Psi_{JM}\rangle=0$, where H is Hmiltonian with including Coulomb interaction:

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{12}\nabla_i^2 - T_G + \sum_{i<j}^{12} v_{ij} . \qquad (4.2)$$

For bound or quasi bound states, we solve U_{JM} by expanding it in terms of certain basis functions:

$$\mathcal{U}_{JM}(r,R) = \sum_{i\ell,j\mathcal{L}} C_{i\ell,j\mathcal{L}}^{(J)} \left[u_\ell^{(i)}(r)\otimes\chi_{\mathcal{L}}^{(j)}(R)\right]_{JM} \qquad (4.3)$$

We adopt following RGM basis functions (4.4∿7) together with their GCM representations. It is very convenient to assume the form of $u_{\ell m}^{(i)}(r)$ as

$$u_{\ell m}^{(i)}(r) = r^\ell e^{-\nu_i r^2} Y_{\ell m}(\hat{r}) = const\times\int e^{-\lambda(r-t)^2}\Phi_{0\ell m}(\lambda\nu_i t)\,dt \qquad (4.4)$$

with $\lambda=\beta$ and $\nu_i<\lambda$. The GC harmonic oscillator functions $\phi_{NLM}(\lambda\nu\vec{r})$ are introduced in Refs. 44 and 45. As for $\chi_{LM}^{(j)}(R)$, the following three are suited for practical calculations:

$$\chi_{LM}^{(j)}(R) = R^L e^{-\mu_j R^2} Y_{LM}(\hat{R}) = const\times\int e^{-\lambda'(R-S)^2}\Phi_{0LM}(\lambda'\mu_j;S)\,dS , \qquad (4.5)$$

$$\chi_{LM}^{(j)}(R) = R^L e^{-\lambda' R^2} L_{N_j}^{(L+\frac{1}{2})}(2\lambda' R^2) Y_{LM}(\hat{R}) = const\times\int e^{-\lambda'(R-S)^2}\Phi_{N_jLM}(\lambda'\lambda';S)\,dS , \qquad (4.6)$$

$$\chi_{LM}^{(j)}(R) = e^{-\lambda'(R^2+S_j^2)} j_L(2\lambda' S_j R)\, Y_{LM}(\hat{R}) = const\times\int e^{-\lambda'(R-S)^2}\delta(S-S_j)/S^2\, Y_{LM}(\hat{S})\,d\hat{S} , \qquad (4.7)$$

where $\lambda'=4\beta/3$ and $\mu_j<\lambda'$. The set (4.4) and (4.5) in the RG (r- and R-space) representation is the same that was used in RGM of Ref.38. The set (4.4) and (4.7) is useful in scattering or reaction problems.[31] Energy matrix elements are then given by

$$\langle\mathcal{A}[\Phi_1\Phi_2\Phi_3\{u_\ell^{(i)}\otimes\chi_{\mathcal{L}}^{(j)}\}_{JM}]|H|\mathcal{A}[\Phi_1\Phi_2\Phi_3\{u_{\ell'}^{(i')}\otimes\chi_{\mathcal{L}'}^{(j')}\}_{JM}]\rangle$$

$$= const\times\int\left[\Phi_\ell^{(i)}(t)\otimes\Phi_{\mathcal{L}}^{(j)}(S)\right]_{JM} H^{(GCM)}(tS,t'S')\left[\Phi_{\ell'}^{(i')}(t')\otimes\Phi_{\mathcal{L}'}^{(j')}(S')\right]_{JM} dt\,dS\,dt'\,dS' , \qquad (4.8)$$

where $H^{(GCM)}$ is the well known GCM kernel before angular-momentum projections; and similarly for overlap matrix elements. Here $\phi^{(j)}_{LM}(\$)$ represents $\phi_{0LM}(\lambda'\mu_j\$)$, $\phi_{N_jLM}(\lambda'\lambda'\$)$ or $\delta(S-S_j)/S^2\cdot Y_{LM}(\hat{\$})$; and $\phi^{(i)}_{\ell m}(\natural)=\phi_{0\ell m}(\lambda\nu_i\natural)$. Even in the case of (4.5) or (4.6) adopted, the analytical integration of (4.8) can be easily performed.[45)]

4.2 Truncation of the basis functions

In the present paper, use is made of the basis functions (4.5) for $\chi^{(j)}_{LM}$; but we verified that the use of GC type (4.7) gives the same results. It is to be stressed that due to the total antisymmetrization effect, a subspace where we take $\ell=0$ in $u^{(i)}_{\ell m}$ is to be expected to reproduce very well the result of full-space calculation in the case of low-lying states of ^{12}C considered.*) Actually, as for the energies of 0^+_1, 2^+_1, 4^+_1, 1^-_1, 3^-_1, 0^+_2, 2^+_2, states, this approximation (with further truncation below) gives very good agreement with the result of strong coupling 3α-GCM calcualtion by Uegaki et al.[37)] in the case of using the same parameters.

We adopt four values of ν_i and also four values of μ_j; $\nu_i/\beta=\{0.8, 0.45, 0.15, 0.05\}$ and $\mu_j/(4/3\cdot\beta)=\{0.8, 0.45, 0.15, 0.05\}$.**) The total number of the basis functions are then sixteen for each J^π; we examined that to increase the number of ν_i and μ_j more did not give countable energy gain to the states we mentioned above.

4.3 Cluster structure of ^{12}C

The ^{12}C nucleus has extensively studied by various 3α models, for example, by boson model,[48),49)] by Brink-Margenau model with hybridizing the shell model,[50)] by OCM,[13),14)] by GCM[51),37)] and by RGM.[38)] Here we would like to add a 3α-RGM calculation[24)] to them with showing successful analysis of various kinds of experimental data of ^{12}C.

First of all, we choose effective nulceon-nucleon interactions so as to reproduce well the saturation property of single α particle and observed α–α

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*) Note that, for example, the 0^+ state with the [4](04)SU_3 label is equivalent to any of three $\{\ell Q L\}_{0^+}$ configurations ($\ell=L=0$, 2 and 4) with $4\text{-}\hbar\omega$ quanta both in $u_{\ell m}$ and χ_{LM}; such three configurations are all dependent to each other due to the total antisymmetriztion effect. When we diagonalize the harmonic oscillator Hamiltonian (except the c.m. coordinate) with using the basis functions given in §4.2, we reproduce very well the eigen-energies of the several lowest shell-model states for each J^π with correct degeneracy (cf. Table I of Ref.13a).

**) Only for $J=4^+$, we put $\ell=0$ in the case of $\nu_i<3/4\cdot\mu_j$ and $L=0$ in the case of $\nu_i\geq 3/4\cdot\mu_j$; but we keep the total number sixteen. This procedure gives about 4-MeV gain; but no gain to the case of $J=0^+$ and 2^+.

scattering phase shifts. As an example we adopt Volkov No.2 potential with m=0.59. We use the oscillator parameter $\beta=m\omega/\hbar=0.55$ fm^{-2} which is very colse to the experimental value 0.57 fm^{-2}; calculated binding energy of α particle is 27.3 MeV. In Fig.2, with using the same parameters, the observed α-α scattering phase shifts are well reproduced by 2α RGM.

If we use the same interaction and size parameter in the 3α system, we obtain the energy spectrum of Fig.3, in which the calculated 3α break-up threshold energy is adjusted to the observed one. Here we confine our interest*) to the lowest level[52] of each spin 0 to 4 and to the second 0^+ and 2^+ levels.[53] Many authors have pointed out[13),37),48),49] that the former levels are rather compact shell-model like states, while the latter two levels are loosely coupled clustering states. Such observed level structure is fairly well reproduced both for compact states and for clustering states. But we have to check first the saturation property of the 3α system. Namely, in Fig.3 we used the same size of the free α perticle, but we examine whether this size is best or not in the 3α system. Fig.4 shows the total energy of each level as a function of the size parameter β. The dash-and-dotted curve shows the saturation property of separated 3α particles. We see clearly that all the states here saturate almost at the same size of the free α particle; note that the value of β of the usual shell model is about 0.42 fm^{-2}, but it is far from the saturation point. In the following calculations, we always use $\beta=0.55$ fm^{-2} at the saturation. It is interesting that 3α clusters keep almost the free-α size, but the obtained wave functions should be checked crucially.

First we check the charge distribution by analyzing electron scattering form factors. Calculated and observed form factors of the elastic scattering and of the inelastic scattering to 2_1^+ state are given in Fig.5. The $B(E2; 2_1^+ \to 0_1^+)$ strength is listed in Table IV. Our wave functions of 0_1^+ and 2_1^+ are so good to reproduce correct charge and transition charge densities.

Next we discuss the 0_2^+ states at 7.66 MeV excitation. Fig.6 shows the reduced $\alpha+^8\mathrm{Be}(0^+)$ width amplitudes of the ground and 0_2^+ states. It is seen that the α-clustering in the 0_2^+ state is almost complete and much stronger than that in the ground state as was pointed out many authors.[13),37),48),49] So it is interesting to see the transition quantities between the 0_1^+ and 0_2^+ states which have so different characters. The electron scattering form factor of $0_1^+ \to 0_2^+$ is given in Fig.5, and the monopole matrix element $M(0_2^+ \to 0_1^+)$ is listed in Table IV. Agreement between calculation and observation is very satisfactory.

This excited 0_2^+ level is famous from astrophysical interest in the production

*) There are four other levels of $E_x<15$ MeV. But all the levels of $E_x<15$ MeV except 1^+ at 12.7 MeV is well predicted by 3α OCM[13c)] and by 3α GCM.[37)]

of ^{12}C nucleus in stars. Namely, in helium burning stars, 3α particle can gather to make this 0_2^+ resonance, but it decays almost back to $^8Be+\alpha$ again. With very weak branching ratio, however, the resonance decays to the 2_1^+ level with emitting E2 gamma ray, ^{12}C nucleus being produced. The reduced α-decay width of this 0_2^+ is given in Table V, while the fusion $B(E2; 0_2^+ \to 2_1^+)$ strength is listed in Table IV. We see that the α-decay and fusion processes are well understood in the framework of 3α RGM.

Next the wave functions of 1_1^- and 3_1^- are examined. The reduced α-decay widths of those states are given in Table II. Form factors of inelastic electron scattering to those states are shown in Fig.5; the $B(E3; 3_1^- \to 0_1^+)$ strength is listed in Table IV. Agreement between observation and calculation is quite satisfactory.

Table VI gives the $\alpha+^8Be(0^+)$ spectroscopic factors and expectation values of the kinetic energy of the relative motion between 3α clusters. Smallness of the kinetic energy is a good measure to see the degree of looseness of the coupling between 3α clusters.

Thus our wave functions have passed through all the above-examined tests for comparison with obsevations. So this fact gives a reality to the statement that 3α clusters keep almost the same size of free α particle in the here-considered states of ^{12}C, and gives a support to a 3α boson model and a 3α OCM in which the intrinsic state of α clusters is regarded to be unchanged (structureless) and the forbidden states of relative motion are constructed based on the use of almost same size of free α particle.

4.4 Comparison between 3α RGM and 3α OCM

Kamimura, Fukushima and Tohsaki-Suzuki proposed[21] a kind of 3α OCM

$$\sqrt{1-K}\left(\sum_{i=1}^{3} T_i - T_G + \sum_{i<j}^{3} V_{ij}^{(eff)} - E_{rel}\right)\sqrt{1-K}\, U_{JM} = 0 \qquad (4.9)$$

as an approximation of 3α-RGM equation (cf.§4.1). They solved Eq.(4.9) with using the same basis functions of §4.2 and the same parameters of §4.3. Their result in the case of using the direct potential as $V_{ij}^{(eff)}$ are compared in Fig.7 with the present energy spectrum (§4.3) of RGM. The role of dynamical particle exchange between clusters which is not taken in this OCM causes attractive effect considerably; but state dependence of the effect is seen. This short range attractive effect between three clusters seems to prevent the clusters from their swelling of size in the compact states (cf.Fig.4). Further investigation of difference between the RGM and OCM will supply the OCM with proper effective potential $V_{ij}^{(eff)}$ instead of the direct potential.

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Table I

Illustration of accuracy of the present method for the α-α scattering. For each E_{cm}, the first line desplays δ_{st} and $|S_{st}|$, while the second line (in parenthesis) δ_t and $|S_t|$; the third line (in N=6) exhibits the phase shift obtained by another RGM. The phase shifts are given in degrees between 0° and 180°. Use is made of three kinds of the trial functions with the GC discretization points S_i[N=6, 0.55 fm; 2.5 fm∿5.25 fm], S_i[N=8, 0.5 fm; 2.5 fm∿6.0 fm] and S_i[N=10, 0.45 fm; 2.0 fm∿6.05fm].

E_{cm}	N=6	(L=0)	N=8	(L=0)	N=10	(L=0)
(MeV)	150.21	1.0000	150.30	1.0000	150.30	1.0000
1.0	(150.22	1.0000)	(150.33	0.9999)	(150.31	1.0000)
	150.14					
	63.19	1.0001	63.57	1.0000	63.60	1.0000
4.0	(63.47	1.0017)	(63.68	1.0045)	(63.63	1.0012)
	63.77					
	169.44	1.0001	170.99	1.0000	171.03	1.0000
10.0	(169.59	1.0121)	(170.99	0.9994)	(171.03	0.9996)
	171.15					
	111.14	1.0004	111.50	1.0000	111.50	1.0000
20.0	(111.42	0.9652)	(111.60	1.0001)	(111.51	1.0001)
	111.44					
	47.08	1.0001	47.79	1.0000	47.82	1.0000
40.0	(46.87	0.9829)	(47.87	0.9957)	(47.86	0.9978)
	47.71					
	9.39	1.0015	9.70	1.0003	9.84	1.0000
60.0	(8.48	1.0462)	(9.58	0.9980)	(9.85	1.0054)
	9.70					

Table II

Comparison of the S-matrix elements between the variational method (V)[31] and the exact calculation (E) : Problem (I).

	S_{11}	S_{21}	S_{31}	S_{41}
E	(-.0754, .0123)	(-.1122, -.8722)	(.3880, .0159)	(.0747, .2537)
V	(-.0756, .0124)	(-.1124, -.8722)	(.3880, .0158)	(.0748, .2537)

Table III

Comparison of the S-matrix elements between the variational method (V)[31] and "exact" calculation (E)[43] :Problem (II).

		$\lvert S_{pd}\rvert$	$\lvert S_{pp}\rvert$	$\lvert S_{dd}\rvert$
L=0	E	.1251	.6122	.2764
	V	.1291	.6103	.2793
L=2	E	.2538	.6730	.2104
	V	.2541	.6723	.2116
L=4	E	.2363	.9437	.2379
	V	.2393	.9432	.2410
L=6	E	.0988	.9947	.8654
	V	.1011	.9945	.8662

Table IV. Calculated (3α-RGM) and experimental (EXP)[52] values of reduced transition matrix elements in ^{12}C.

	3α-RGM	EXP	
$B(E2;2_1^+ \to 0_1^+)$	9.3	8.5	(e^2fm^4)
$B(E2;0_2^+ \to 2_1^+)$	5.6	8.9	(e^2fm^4)
$B(E3;3_1^- \to 0_1^+)$	124	107	(e^2fm^6)
$M(0_2^+ \to 0_1^+)$	6.7	5.7	(fm^2)

Table V. Calculated (3α-RGM) and experimental (EXP)[52] values of reduced α-decay widths $\theta_\alpha^2(a)$ in ^{12}C.

J^π	$\theta_\alpha^2(a)$		
	a (fm)	3α-RGM	EXP
0_2^+	6.0	0.98	0.78
	7.0	0.56	0.41
3_1^-	5.0	0.171	0.182
	6.0	0.071	0.087
1_1^-	6.0	0.081	0.097
	7.0	0.113	0.103

Table VI. Calculated $\alpha + {}^8Be(0^+)$ spectroscopic factors S_α^2 and expectation values of the relative-motion-kinetic energy.

J^π	$E_{total} \cdot E_{3\alpha}$ (MeV)	S_α^2	K.E. (MeV)
0_1^+	-7.41	0.80	51.3
2_1^+	-4.64	0.20	56.0
0_2^+	0.33	1.80	19.7
3_1^-	0.73	0.67	35.2
1_1^-	3.39	0.31	25.0

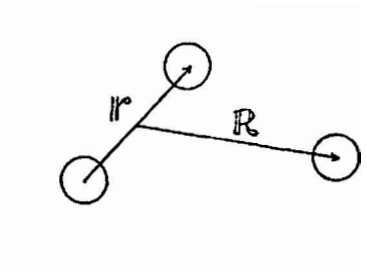

Fig. 1. Relative coordinates of the 3α system.

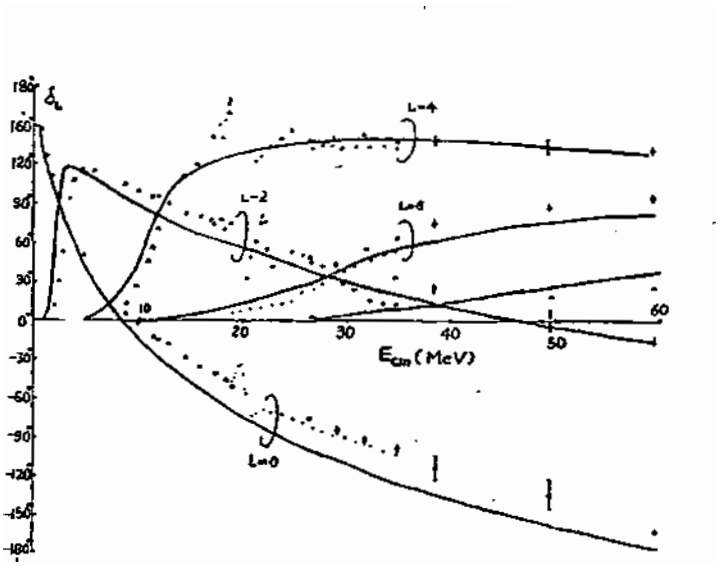

Fig. 2. α-α scattering phase shifts.

Solid curves are given by 2α RGM with Volkov No. 2 (m=0.59) and β=0.55 fm.$^{-2}$

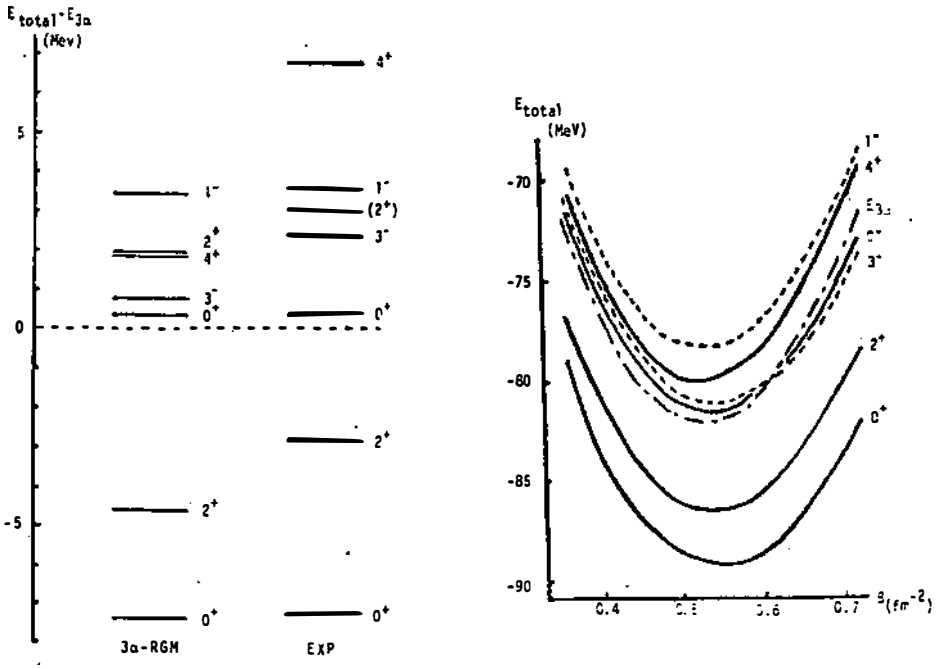

Fig. 3. Calculated (3α RGM) and experimental (EXP) energy levels of ^{12}C. Only the lowest level of each spin and the 0_2^+ and 2_2^+ levels are shown. For ground state BE.(3α RGM)=89.4 MeV, while BE.(EXP)=92.2 MeV.

Fig. 4. Saturation of the 3α system. Total energy of each level is given as a function of the oscillator size parameter $\beta=m\omega/\hbar$. $E_{3\alpha}(\beta)$ gives the calculated energy of separated 3α clusters as a function of β. Note that all the states here saturate almost at the same size of free α particle.

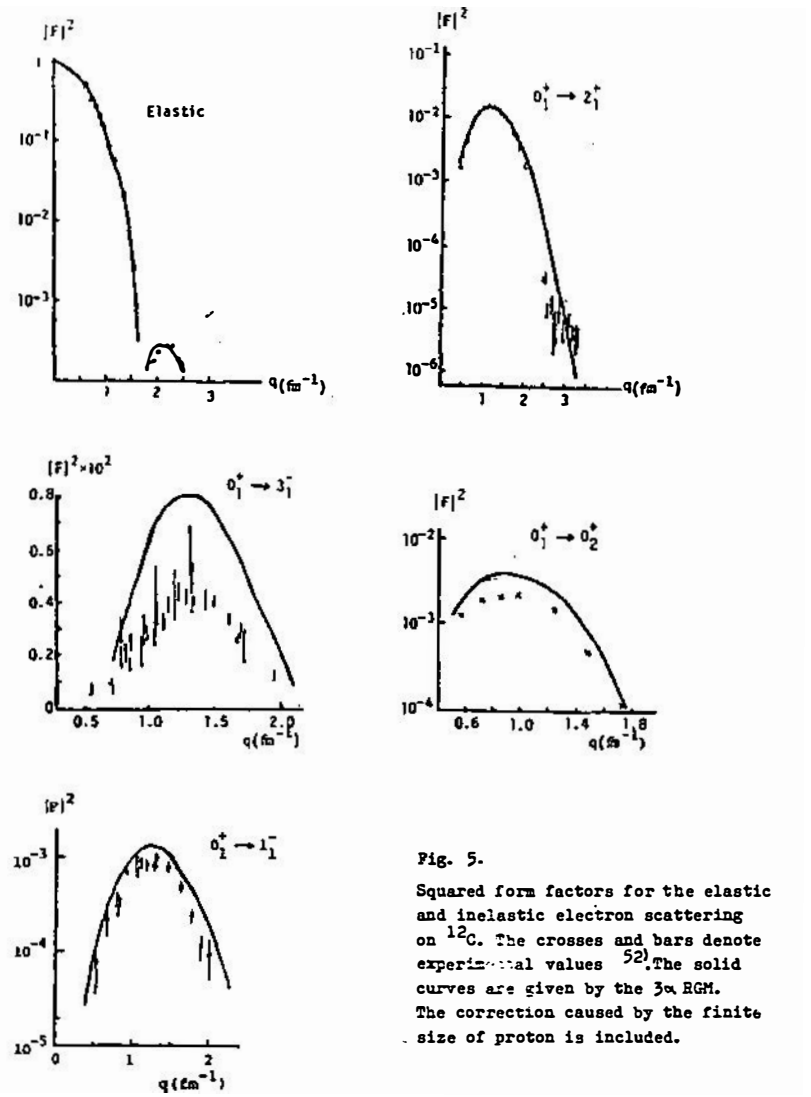

Fig. 5.

Squared form factors for the elastic and inelastic electron scattering on ^{12}C. The crosses and bars denote experimental values [52]. The solid curves are given by the 3α RGM. The correction caused by the finite size of proton is included.

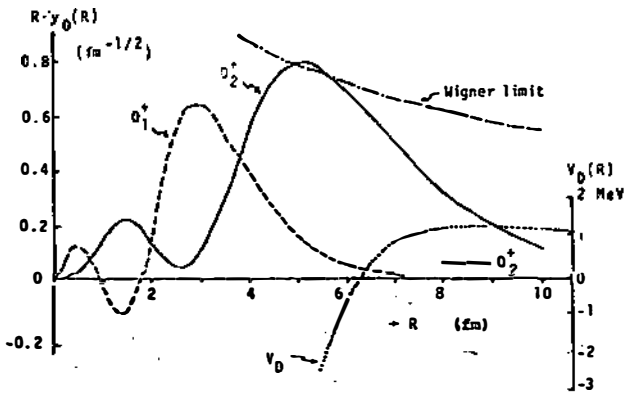

Fig. 6. Reduced width amplitudes of $\alpha+{}^{8}\mathrm{Be}(0^+)$ break up for the 0_1^+ and 0_2^+ states. The dotted curve gives the direct (folding) potential between α and ${}^{8}\mathrm{Be}(0^+)$.

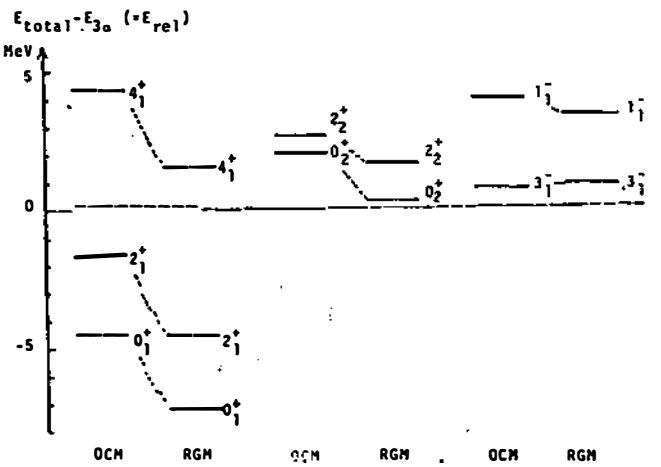

Fig. 7. Comparison between 3α RGM and 3α OCM (with the use of direct potential)[21] in the energy level of ^{12}C. Use is made of Volkov No. 2 (m=0.59) and β=0.55 fm^{-2} both in RGM and OCM.

DISCUSSION

A. Weiguny: When calculating reduced widths, do you renormalize your RGM wave function with respect to the overlap exchange kernel?

M. Kamimura: It is a matter of course in any RGM calculation to pay full consideration to such a problem. The reduced width amplitude $y_J(R)$ (then $\theta_\alpha^2(a)$) is always calculated with $y_J(R) = \langle \phi_1\phi_2\phi_3\, Y_{JM}(\hat{\underset{\sim}{R}}) | \mathcal{A}[\phi_1\phi_2\phi_3\, U_{JM}(\underset{\sim}{R})] \rangle$ with the normalization $\langle \mathcal{A}[\phi_1\phi_2\phi_3\, U_{JM}(\underset{\sim}{R}) | \mathcal{A}[\phi_1\phi_2\phi_3\, U_{JM}(\underset{\sim}{R})] \rangle = 1$.
Here overlap exchange kernel is treated correctly.

K. Goeke: The GCM and RGM are entirely intuitive methods. You need a preconceived notion of the process you are going to describe and from this imagination you select the proper clusters with the proper intrinsic wave function. The only way to check these assumptions consist in trial and error, i.e. by adding more clusters and comparing with previous results or with experiments. GCM and RGM do not provide a systematic way to determine the number and the structure of the clusters to be used. It would be desirable to put some effort in this direction.

M. Kamimura: The GCM and RGM studies are not done in simple intuitive way. Suppose, for example, a study of ^{20}Ne nucleus or twenty-nucleon problem. At the first stage, we naturally consider the case of $^{16}O + \alpha$ system, but this is not based on a simply intuitive way of assumption on the configuration. Before entering into the $^{16}O + \alpha$, a lot of studies have been performed on the α and the ^{16}O nuclei themselves. Through these microscopic studies with Pauli principle and the property of nucleon-nucleon interaction taken into account properly, we know why and how the α and ^{16}O can be considered as good constituent units. We further know when and how the α and ^{16}O clusters show other states of existence. If necessary, in the next stage we shall include them. Thus we begin to investigate

the $^{16}O + \alpha$ system using the accumulated knowledge from systematic studies and based on our physical intention and our philosophy. So the GCM, RGM and also OCM studies are quite systematic and physical. In such a philosophy to accumulate knowledge from systematic studies and to compose physical aspects, there is one of differences between the GCM, RGM and OCM studies and, for example, your TDHF study.

Y.C. Tang: It is commonly understood that in a Ritz variational calculation one obtains a better estimate of the energy than of the wave function. From your results one sees that the excitation energy of the first 2^+ state is about a factor of 2 too small. Yet, your (calculated) elastic and inelastic form factors agree very well with experiment. How do you explain this seeming contradiction?

M. Kamimura: It is common in all the 3α models to give somewhat too low energy of the first 2^+ state. However, as Arima and Takigawa showed in their model, we can expect that, if we include the effect of spin-orbit interaction, the energy of 2^+ will be improved without changing the part of wave functions which is responsible for the transition charge density and B(E2) strength.

G. Paić: Is your calculation of the second 0^+ state in ^{12}C consistent with its interpretation as an Efimov state i.e., resulting from the two body resonance in the α-α system.

M. Kamimura: No! Any 3α state here does not include such sharp 2α resonances of Efimov at rather high energies. Concerning the second 0^+ state, as I showed, the amplitude of $[^8Be(0^+)\otimes\alpha]$ configuration is quite large, and the $^8Be(0^+)$ state corresponds to the sharp 2α resonance just above the 2α threshold, namely at about 0.3 MeV.