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Clustering in Light Nuclei

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Abstract

Work on a simple model of clustering in light nuclei is outlined. The idea of the model is described along with methods for selecting the necessary parameters. The model is then developed further to take account of effects arising from intrinsic spins of the constituent clusters and results for alpha particle and trinucleon cluster states in several nuclei are presented.

1. The Model

This talk is intended as a necessarily rather condensed review of my work over the last few years on the cluster structures of light nuclei. The calculations have been performed with several collaborators whose names appear in the references.

The original ideas came from a consideration of the experimental results on three and four nucleon transfer reactions induced by heavy ions at incident energies of 10 MeV/nucleon.[1)] Such reactions are believed to favour the population of high spin states, but the extremely high selectivity observed in the spectra suggested that the preferentially excited states in the final nucleus might have some particularly simple structure. I guessed that what was happening was just the transfer of real helions, tritons, or alpha particles from the projectile on to the target nucleus to form loosely bound or resonant dinuclear states in which the light cluster, in its ground state, orbited the core or target cluster, which also remained in its ground state. It was hoped that the relative motion of the cluster and core could be described by means of a local attractive potential between the constituent nuclei. This picture seems very reasonable for the high spin states with large angular momentum of relative motion since the centrigugal barrier will tend to hold the clusters apart; the two nuclei will then have little overlap, making it plausible that their internal structures are not much perturbed.

We have gradually come to realise that this very simple model is a rather good description of many observed states and not only for those with high spin. Eventually we saw that the same type of configuration could be used to explain entire bands of states in various nuclei, a band being defined as those levels related by having the same value of the global quantum number Q = 2N + L, where N is the number of nodes in the relative motion wave function and L is the angular momentum. The node number N is, of course, a measure of the radial vibration energy in a two-body state. Thus for L = Q we have N = 0 and we recover the original picture of a state with constituents held apart by centrifugal effects. But if now we keep Q constant and vary N and L we clearly define a band of states with L changing by two units from level to level and as L decreases the radial vibration energy increases. Calculations show that this actually leads to an increase in the RMS separation of the clusters. Hence we have an explanation of the success of the model for low spin states also.

To turn the above picture into a calculable model we need two ingredients : (i) some way to fix the value of Q = 2N + L which defines a band, and (ii) a choice of potential to describe the cluster-core interaction. The first is disposed of by a simple recipe. We assume that our cluster states will have appreciable overlap with shell-model cluster configurations in which three or four nucleons move in low available shell orbitals above the Fermi surface of the core nucleus. If these orbitals are labelled by i, then each nucleon will carry $(2n_i + \ell_i)$ quanta of excitation so that the total number of quanta in the cluster is given by

$$Q(\text{shell}) = \sum_{i=1}^{3,4} (2n_i + \ell_i).$$

If we now suppose that, in the relevant states, the nucleons correlate so as to form a cluster structure with zero quanta of internal excitation, then all the excitation goes into the motion of the centre of mass of the cluster relative to the core. It is now reasonable to identify Q(shell) with the quantity Q = 2N + L of our model. Almost needless to say, it is very difficult to make this shell-model argument rigorous since in our picture the cluster is a real triton or alpha particle and the core is also moving around the common centre of mass of the dinuclear system. Thus a shell-model expansion of our states would require very many small components of higher orbital configurations to describe the tight spatial correlations and to make the mean separation of the centres of mass of the clusters larger. In addition, the shell description would need to include core excitations to take account of the displacement of the centre of mass of the core nucleus from that of the system. However, there would have to be very large single components of higher configurations in the shell-model to give a different effective $Q = 2N + L$ in the dinuclear model.

In the earlier work[2] we made use of the folding model to construct an effective interaction between the cluster and the core. This method represents the effective nucleus-nucleus potential by a double folding of the nuclear densities with some assumed internucleon force, i.e.

$$V(\underline{r}) \; \alpha \int \rho_1(\underline{r}_1)\, v(\underline{r} - \underline{r}_1 + \underline{r}_2)\, \rho_2(\underline{r}_2)\, d\underline{r}_1\, d\underline{r}_2.$$

In light nuclei this gives a rather gaussian-like smooth shape to the potential and we left the depth as a free parameter to be fitted to the cluster-core binding energy of some selected state of a band. In later work[3] we have parametrized the potential shape, using a symmetrized Woods-Saxon form given by

$$V(r) \; \alpha \; [\cosh(\tfrac{r}{a}) + \cosh(\tfrac{R}{a})]^{-1},$$

which is indistinguishable from a Woods-Saxon potential for $R/a >> 1$, but becomes similar to the folded shapes when R is less than or of the order of a. The potential depths required are quite large and lie between 100 and 200 MeV.

The curious thing about these potentials is that they give rise to almost perfect rotational band spectra. That is, all the bound states and many of the resonance energies fall naturally into bands of levels according to the formula

$$E(N, L) = f(2N + L) + \beta.L(L + 1),$$

where the function f(Q) gives the spacing between bands and the moment of inertia parameter is the same for all bands, i.e. β is independent of Q = 2N + L. We have now made considerable progress in understanding why this should be so, but that is another story. Whatever the reason, it is exactly the spectrum we need to fit the observed dinuclear level energies.

Our later work has been largely to elaborate on and identify these basic rotational level sequences in various nuclei, taking into account more and more secondary effects arising from intrinsic spins of the cluster and core nuclei. We have aimed at giving a unified theoretical explanation of many levels in the nuclei ^{15}N, ^{16}O, ^{18}O, ^{18}F, ^{19}F, ^{20}Ne and ^{24}Mg in terms of alpha particle or trinucleon clusters orbiting a core nucleus and, where possible, to calculate other observable quantities such as α-widths, nuclear moments and γ-decay rates.

Before describing these calculations I should briefly mention that considerable work has been done to justify the above model in more fundamental terms, in particular to attempt to derive it from the resonating group formalism. Friedrich, Wheatley and I[4)] gave an ansatz which reduces the resonating group equation to an ordinary Schrodinger equation with an effective local potential and which shows that the lowest bands required by the model are just the first states allowed by the Pauli principle. We then demonstrated that a local gaussian potential, independent of energy and angular momentum, will fit the phase shifts for α-α scattering and reproduce the properties of the 0^+ resonance in ^8Be.

Friedrich[5] has also shown how to construct effective local potentials from the energy surfaces of the generator coordinate method. Finally, Lomnitz[6] has developed a method of expanding matrix elements of many-body operators between antisymmetrized cluster states such that the first term is just the ansatz of ref.[4] and the higher terms necessarily involve internal excitations of the clusters. The conclusion seems to be that the effective local potential model, with appropriate choice of quantum numbers, will give results equivalent to the resonating group method whenever it is reasonable to consider that the clusters remain in their ground states.

2. Applications

In this section I will review our work to date in order of increasing complexity in the effects arising from spin interactions. The cases we have looked at so far include examples of the following combinations of cluster and core spins:

J (cluster)	J (core)	Example
0	0	$^{20}Ne = \alpha + {}^{16}O$
0	½	$^{19}F = \alpha + {}^{15}N$
½	0	$^{19}F = t + {}^{16}O$
0	1	$^{18}F = \alpha + {}^{14}N$
½	½	$^{18}O = t + {}^{15}N$

In addition to these we have performed calculations for systems in which one of the constituent clusters itself has a ground state rotational band, e.g.

$$^{16}O = \alpha + {}^{12}C(0^+, 2^+, 4^+)$$

$$^{24}Mg = {}^{16}O + {}^{8}Be(0^+, 2^+, 4^+).$$

These examples require the solution of up to nine coupled channel equations for a given J-value of the dinuclear system. The last case is rather speculative, but gives interesting results.

The first applications of the model, to the well known ground state band of ^{20}Ne and the excited 4p - 4h band of $^{16}O = \alpha + {}^{12}C$(g.s), were done in collaboration with Dover and Vary.[2] In both systems the constituent clusters have zero spin and the rotation spectra show up very cleanly. In Fig. 1 I show the model bands of ^{20}Ne which have $2N + L = 8, 9$ and the corresponding experimental states. As mentioned earlier, the calculated levels actually have a more accurately rotational spectrum than the observed states and, for example, the 8^+ level is found to be much lower than the predicted one. A likely explanation is that the alpha particle strength is split between two states whose centroid would be near the calculated energy. A similar phenomenon shows up in ^{19}F (see below).

Many of the states are above the alpha particle separation energy and so have to be calculated as resonances in the continuum. The same calculations give values for the α-decay widths which agree rather well with experiment under the assumption that the states are entirely $\alpha + {}^{16}O$ configurations, i.e. the α-spectroscopic amplitudes are unity.[2]

It is also possible to calculate RMS radii for ground and excited states. The ground state radius agrees with observation and the excited state radii actually decrease with increasing spin. This shows that the radial vibration motion is more effective in separating the clusters than the centrifugal effects. Finally, we have calculated the B(E2) γ-ray transition strengths connecting the rotation levels and find good agreement with the data using an effective charge much smaller than that required in the shell-model (see Table 1 for results on α-widths and B(E2) values). Very similar results are found for the 2N + L = 8, 9 bands in ^{16}O.

In the above reference[2)*] we also presented tentative calculations of levels in ^{15}N which we thought might have the configuration t+ ^{12}C(g.s). There seems now to be some evidence that our assignments were largely correct and the comparison of calculation with experiment indicated the presence of an appreciable spin-orbit component in the interaction of the triton with the ^{12}C core nucleus. But the occurrence of spin-orbit effects is better discussed in connection with ^{19}F where the evidence is much clearer.

In the next application of these ideas by Buck and Pilt[2)] we begin the job of sorting out the complicating effects coming from intrinsic spins of the clusters. In the nucleus ^{19}F many low lying states and also some highly excited states populated in multinucleon transfer reactions may be described by the shell-model configurations $(sd)^3$, $(sd)^2$ (fp), $(p)^{-1}$ $(sd)^4$ and $(p)^{-1}$ $(sd)^3$ (fp). In terms of our model the first two configurations would be the lowest shell-components of states represented as triton clusters orbiting ^{16}O with 2N + L = 6 and 7 respectively, while the second two suggest descriptions as an alpha particle interacting with an ^{15}N core nucleus and having 2N + L = 8 or 9. In the first pair the triton cluster has spin ½ and in the second two examples the ground state of ^{15}N has spin ½, so in all cases we expect to see spin-orbit effects in all cluster states with non-zero L of relative motion.

The basic cluster-core interaction potentials needed to fit the absolute energies and rotational spacings for the triton and alpha particle cluster levels in ^{19}F are given in Fig. 2. These shapes are both of the parametrized form mentioned earlier with, of course, different values of R and a. Also shown is a comparison of the analytical shape for the alpha potential with the folded potential used in the calculations of $^{20}Ne = \alpha + {}^{16}O$. They are very similar in both shape and depth.

The next diagram, Fig. 3, shows the ground state rotation spectrum of ^{20}Ne alongside the levels of ^{19}F which we treat as $\alpha + {}^{15}N$(g.s) with $2N + L = 8$. The state with $J^\pi = \frac{1}{2}^-$ is the first excited state of ^{19}F at 0.11 MeV. The picture clearly indicates the spin-orbit splitting of the rotation levels caused by the spin $J^\pi = \frac{1}{2}^-$ of ^{15}N ground state. Furthermore we learn from this comparison that when the heavier core nucleus carries spin the effective vector spin-orbit force appears to be rather small. I do not know any simple explanation of this, but in later work we have consistently assumed that vector spin-orbit interactions of the heavier cluster are small, and this seems to be supported by the results. In Fig. 4 I give a more complete alpha cluster spectrum for $2N + L = 8$ and 9 along with experimental data on spins and parities and on levels seen appreciably in alpha transfer reactions. The model clearly provides a theoretical candidate for every level seen in the transfer experiments.

The bands of levels in ^{19}F which we represent as triton states with $2N + L = 6$ and 7 are illustrated in Fig.5 along with data on states seen in triton transfer on to ^{16}O. The level with $J^\pi = \frac{1}{2}^+$ is the ground state of ^{19}F. The diagram also shows the expected positions of bands with $2N + L = 8$ and 9, but there is no experimental evidence for these.

To fit the observed spectra with 2N + L = 6, 7 we need to introduce an appreciable triton spin-orbit force, of the same sign and magnitude as deduced from triton elastic scattering studies. This is, of course, easy to include in the model, with the results shown in Fig. 5. The main complicating feature is that, experimentally, there are two low lying $\frac{13}{2}^+$ states, while our model predicts only one, which happens to lie at the centroid of the observed states as measured by the transfer strengths. Possible reasons for this splitting are discussed in our paper.[3)]

In the framework of this model of two nuclei in their ground states orbiting each other it is straight-forward to write down expressions for the various electrmagnetic operators in terms of the masses, charges and intrinsic magnetic moments of the constituent clusters as pictured in Fig.6. For example, the magnetic dipole and electric quadrupole operators are given by the formulas

$$\underline{\mu} = \mu_0[g_1 \underline{J}_1 + g_2 \underline{J}_2 + \frac{A_1^2 Z_2 + A_2^2 Z_1}{A_1 A_2(A_1 + A_2)} \underline{L}],$$

$$\mathcal{M}(E2, \lambda) = \frac{A_1^2 Z_2 + A_2^2 Z_1}{(A_1 + A_2)^2} r^2 Y_2^\lambda (\hat{\underline{r}}).$$

From expressions like these and the relative motion wave functions provided by solving the radial Schrodinger equation for the given potentials, it is easy to calculate various electric and magnetic static moments and γ-transition strengths. As illustration, results for several static moments of states in ^{19}F and ^{19}Ne are given in Tables 2 and 3 and compared with available data. Similar good agreement is found for γ-decay rates.[3)]

Encouraged by the success of the model for ^{19}F, Friedrich, Pilt and I[7)] attempted next to construct a similar model for the states in ^{18}F which are represented in the shell-model by the configurations $(p)^{-2}\ (sd)^4$, i.e. 4p - 2h states. The lowest level of this nature is the famous intruder state with $J^\pi = 1^+$ at 1.7 MeV excitation. In our description we assume these states to consist of an alpha particle, in orbits given by 2N + L = 8, bound to the nucleus ^{14}N in its 1^+ ground state. In this band we have L = 0, 2, 4, 6 and 8 which in ^{18}F gives rise to the intruder state with quantum numbers L = 0, $J^\pi = 1^+$, while the coupling of the spin 1 core to higher orbital states of the alpha particle clearly leads to a series of triplets such as e.g. L = 2, $J^\pi = 1^+, 2^+, 3^+$ etc. Many of these states can be identified quite well in α-transfer studies and the experimental levels are shown in Fig. 7 which provides some evidence for the existence of the lowest triplets.

We are now faced with the problem of accounting for the pattern of triplet splitting observed. The first thought that comes to mind is the possible presence of a vector spin-orbit interaction between the core with $\underline{S} = \underline{1}$ and the orbital angular momentum $\underline{L}$ of the alpha particle. But we already have evidence from the system $^{19}F = \alpha + {}^{15}N$ that spin-orbit effects are small when the spin is carried by the heavy core nucleus. Even more to the point, a spin-orbit term $\underline{L}.\underline{S}$ would give the wrong splitting pattern, i.e. with the usual sign for such an interaction the levels for given L would have spins J = L + 1, L, L - 1 in order of increasing excitation energy and the splitting would increase markedly with L. Hence we must assume that the vector spin-orbit interaction is very small and seek an alternative explanation. This, however, is quite easy to find since a spin 1 particle can also have a tensor interaction with its partner nucleus of spin 0.

It is in fact possible to construct three different interactions each of which is a scalar product of a second rank spin tensor and a second rank tensor constructed from just one of the following quantities - the relative coordinate of the centres of mass of the two clusters, their relative linear momentum or their relative angular momentum. The third one is sometimes called the quadratic spin-orbit term and we neglect it essentially on the same grounds as for neglect of the vector spin-orbit interaction. It is difficult to make distinction between the first two tensor interactions since they have very similar matrix elements. However, a simple microscopic model we are currently working on shows that only the interaction constructed out of the core spin $\underline{S} = \underline{1}$ and the relative coordinate of the clusters is likely to be appreciable (in our calculations the other two terms vanish identically). The model consists of calculating the matrix elements of a nucleon-alpha interaction using the states of an alpha particle orbiting a core state with two holes in the p-shell.

The surviving tensor interaction has the form

$$V_T = [(\underline{S}.\hat{\underline{r}})^2 - \tfrac{2}{3}]\, F(r),$$

where $\underline{S} = \underline{1}$ is the spin of ^{14}N and $\hat{\underline{r}}$ is the direction of the alpha particle relative to the core. The function F(r) is a calculable form factor, but in the first calculations we have taken the radial matrix elements to be independent of L and have assumed that V_T does not couple different L-values strongly. These assumptions remain to be checked by means of more detailed work.

The resultant splittings of the rotational energy levels are given by the expressions

$$\langle JL|V_T|JL\rangle = \gamma \cdot \frac{L}{2L+3} \quad : J = L + 1$$

$$-\gamma \quad : J = L$$

$$\gamma \frac{L+1}{2L-1} \quad : J = L - 1,$$

with the radial matrix element γ left as a single fitting parameter. With positive γ in each triplet the formula predicts that the state with J = L moves down and the other two states move up in rather good agreement with the available data shown in Fig. 7.

We have also calculated the energies of a band of states in ^{18}O which we represent as $^{18}O = \alpha + {}^{14}C(gs)$ with 2N + L = 8 and which has some experimental support from α-transfer studies. This example is much simpler than the case of ^{18}F since the ^{14}C ground state has spin 0. Hence we see a clean rotation band as in ^{20}Ne. In Fig. 8 I have collected the results of calculation for α-bands in ^{18}O, ^{18}F, ^{19}F and ^{20}Ne to show that the underlying rotation spectra are very similar in all four nuclei when spin effects are allowed for.

As yet another example of the complications caused by spin effects Pilt and I[8)] have considered the trinucleon cluster states in mass 18 nuclei which in the shell-model are described by $(p)^{-1}\,(sd)^3$ configurations. We have used the cluster structures

$^{18}O = 3H + {}^{15}N$; T = 1 only, and

$^{18}F = ({}^3He + {}^{15}N) \mp ({}^3H + {}^{15}O)$;

with T = 0, 1 respectively. The cluster nucleus ground states have spins $\underline{s}_1 = \underline{s}_2 = \frac{1}{2}$ and we have the possible presence of the interactions:

(i) Spin-Spin : $\underline{s}_1 \cdot \underline{s}_2$

(ii) Spin-Orbit : $\underline{L} \cdot \underline{s}_1$, $\underline{L} \cdot \underline{s}_2$

(iii) Tensor : $[3(\underline{s}_1 \cdot \hat{\underline{r}})\ (\underline{s}_2 \cdot \hat{\underline{r}}) - \underline{s}_1 \cdot \underline{s}_2]$,

each with its own radial form factor and there is also the usual central interaction responsible for the fundamental rotational structure of the spectrum.

A suitable set of basis states may be constructed by first coupling together the cluster spins to total spin $\underline{S}$ and then coupling this to the relative orbital angular momentum $\underline{L}$ to give the final J-value, i.e. $|J\ M : L\ S(s_1\ s_2)\rangle$. This yields a rich spectrum of negative parity states based on the L = 0, 2, 4 and 6 levels of a trinucleon band with 2N + L = 6 and the states of definite total angular momentum J arise from the coupling of S = 0 or 1 to the above L-values. Taking account of the various interactions listed above we are able to reproduce much of the low lying negative parity spectra in ^{18}F(T = 0) and in ^{18}O(T = 1) very well. The model also provides definite theoretical candidates for most of the higher excited states which are seen strongly in three nucleon transfer on to ^{15}N.

As before we have neglected the coupling between different L-values induced by the tensor interaction and spin-orbit effects arising from the spin of the mass 15 core nucleus. But we have included the spin-orbit coupling of the mass 3 cluster which mixes some pairs of the basis states given above. With these assumptions we can reproduce the level positions in both nuclei with essentially one set of four parameters which are the rotational level spacing and the radial matrix elements of the form factors of the above interactions (which we assume to be independent of L).

This fit is achieved even though the negative parity level orderings in ^{18}F and ^{18}O are quite different. The observed rearrangement is entirely reproduced merely by changing the sign of the spin-spin interaction on going from T = 0 states to T = 1 states or, in other words, multiplying this interaction by an isospin exchange operator. I have not been able to think of any simple reason for this, but the result itself is certainly very striking. It is also interesting, and necessary for the consistency of the model, that the rotational level spacing parameter and the mass 3 spin-orbit strength turn out to be very similar to those for the triton states in ^{19}F.

The final results for negative parity spectra in ^{18}F and ^{18}O are shown in Fig. 9 and 10 along with the definite and tentatively assigned observed states. I should also mention that for both the alpha particle and trinucleon states in mass 18 nuclei the model gives an excellent account of experimental data on M1 and E2 γ-transitions.[7,8]

I have not time to go into great detail about the applications of the model to systems in which one of the clusters has itself a rotation band, but I must at least briefly mention the results. A graduate student, C. Wheatley, has computed the states of the model for the system $^{16}O = \alpha + {}^{12}C(0^+, 2^+, 4^+)$, where the alpha particle moves in the deformed quadrupole field of the ^{12}C core nucleus.[9] For a given J-value of the system the calculation involves the solution of many coupled channel equations for the bound and scattering states. This yields a much richer spectrum than the simple $\alpha + {}^{12}C(gs)$ model for the excited rotation band of ^{16}O which begins at 6 MeV. In fact, the calculation gives almost the entire low energy T = 0 spectrum known up to about 14 MeV except only that 0^- states are absent in the model. Wheatley has also generated phase shifts for the scattering of alpha particles by ^{12}C

which agree well with experiment. All this is achieved with essentially only two parameters - the depth of the central potential, chosen so as to position the levels correctly, and an effective deformation parameter to describe the quadrupole coupling.

The code developed to obtain the above results can be used with trivial modifications to treat a model of ^{24}Mg represented as $^{16}O(gs) + {}^{8}Be(0^+, 2^+, 4^+)$. This may seem a somewhat exotic idea, but the results shown in Fig. 11 indicate that it should be taken seriously. The calculated spectrum is at least as good as the shell-model results also given in Fig. 11.

To sum up, the ideas described here give a remarkably simple and unified account of a wide range of experimental data in light nuclei and many other applications are possible. It would, for instance be instructive to reinvestigate the spectrum of ^{15}N described as $t + {}^{12}C(0^+, 2^+, 4^+)$. The model can be extended to even lighter nuclei such as ^{10}B and ^{11}B and a start has also been made on nuclei in the region of ^{40}Ca.

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TABLE 1

ALPHA WIDTHS IN ^{20}Ne

J^{π}	Γ_{α}^{exp}(keV)	Γ_{α}^{calc}(keV)
6^+	0.11	0.21
8^+	0.035	0.108
1^-	> 0.013	0.021
3^-	8.0	6.7
5^-	141	81
7^-	280	183

GAMMA DECAYS IN ^{20}Ne

TRANSITION	$B(E2)_{EXP}$	$B(E2)_{TH}$
$2^+ \rightarrow 0^+$	57.3	57.3
$4^+ \rightarrow 2^+$	71.0	71.9
$6^+ \rightarrow 4^+$	66.0	60.0
$8^+ \rightarrow 6^+$	24.0	34.6

UNITS : $e^2.fm^4$

TABLE 2

CHARGE RADIUS AND QUADRUPOLE MOMENTS IN ^{19}F

QUANTITY	STATE	CALCULATED	EXPERIMENT
$\langle R^2 \rangle^{\frac{1}{2}}$	GS ($\frac{1}{2}^+$)	2.90	2.90
Q	$J^\pi = \frac{5}{2}^+$	-8.83	±(11 ± 2)
Q	$J^\pi = \frac{3}{2}^+$	-6.51	-

TABLE 3

MAGNETIC DIPOLE MOMENTS IN ^{19}F AND ^{19}Ne

NUCLEUS	LEVEL	CALCULATED	EXPERIMENT
^{19}F	$\frac{1}{2}^+$, 0.0 MeV	2.98	2.63
	$\frac{5}{2}^+$, 0.197	3.70	3.61
	$\frac{3}{2}^+$, 1.554	-1.14	-
^{19}Ne	$\frac{1}{2}^+$, 0.0 MeV	-2.13	-1.89
	$\frac{5}{2}^+$, 0.238	-0.85	-0.74
	$\frac{3}{2}^+$, 1.536	2.43	-

IN UNITS OF $\mu_0 = e\hbar/2Mc$

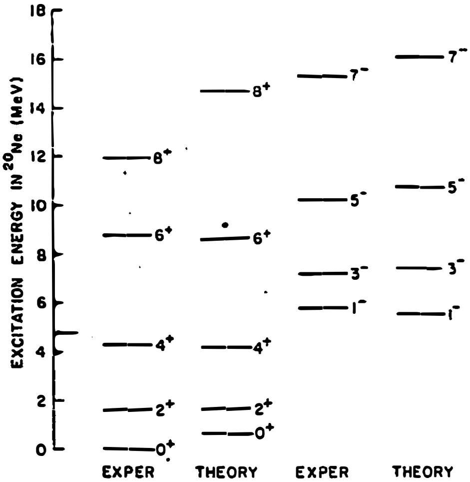


Fig. 1: Rotational bands with $K^{\pi} = 0^{+}$ and 0^{-} in ^{20}Ne. Theoretical spectra were calculated using a folded potential. The arrow indicates the $\alpha + {}^{16}O$ threshold in ^{20}Ne.

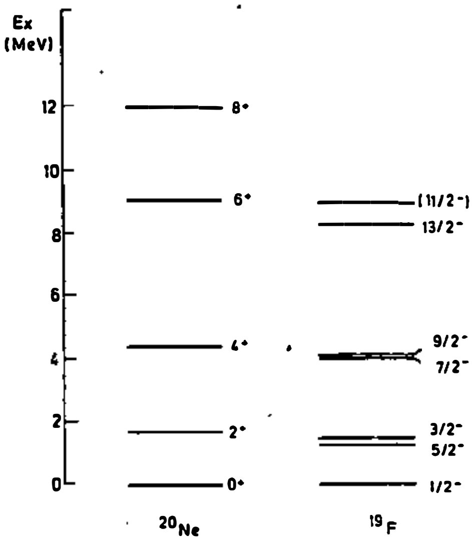


Fig. 3: Spectrum of states of the ground state band of ^{20}Ne and the lowest $K^{\pi} = \frac{1}{2}^{-}$ band in ^{19}F. The close correspondence of levels implies that similar α-cluster potentials should describe both sets of states.

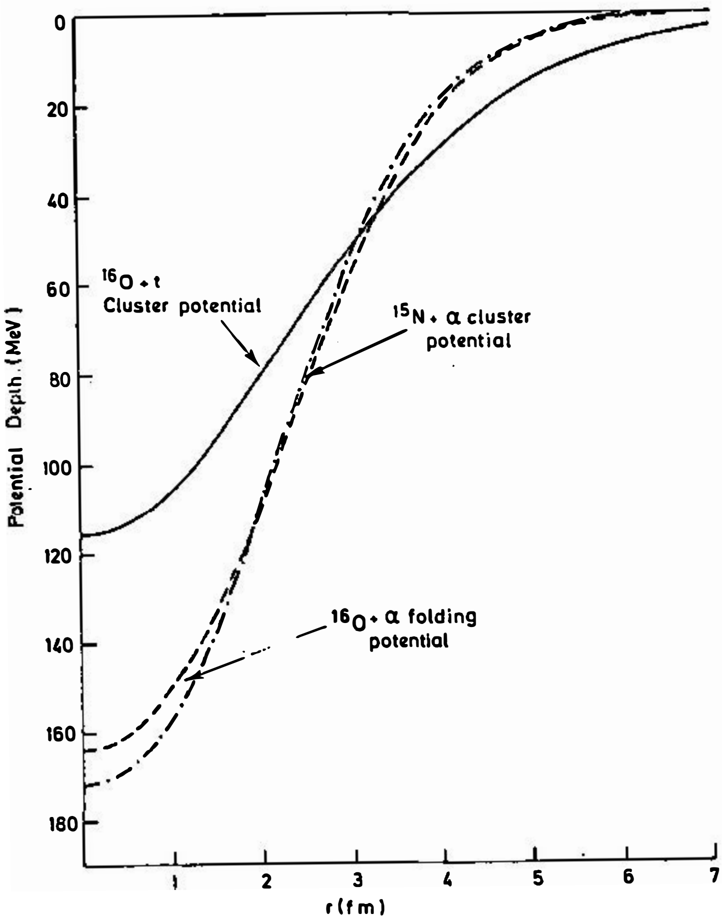


Fig. 2: Potentials used to describe the t + ^{16}O and α + ^{15}N cluster states in ^{19}F. The folded potential for α + ^{16}O from ref.[2] is also shown for comparison.

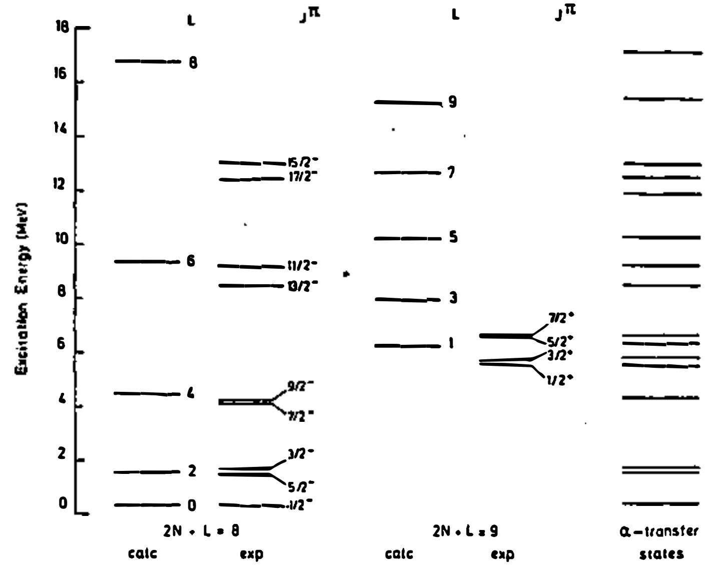


Fig. 4: Calculated and experimental α + ^{15}N cluster states in ^{19}F. States strongly populated in α-transfer reactions are also shown.

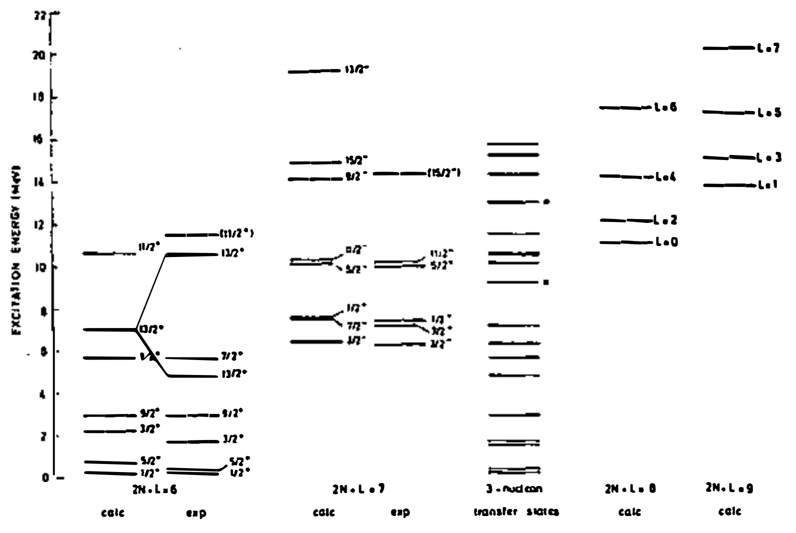


Fig. 5: Calculated and experimental t + ^{16}O cluster states in ^{19}F. States strongly populated in triton transfer reactions are also shown.

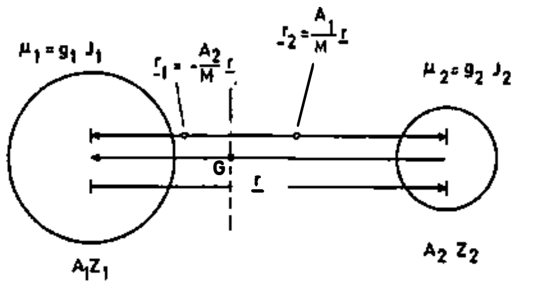


Fig. 6: Definitions of quantities used in constructing electromagnetic operators. The centre of mass of the cluster core system is denoted by G.

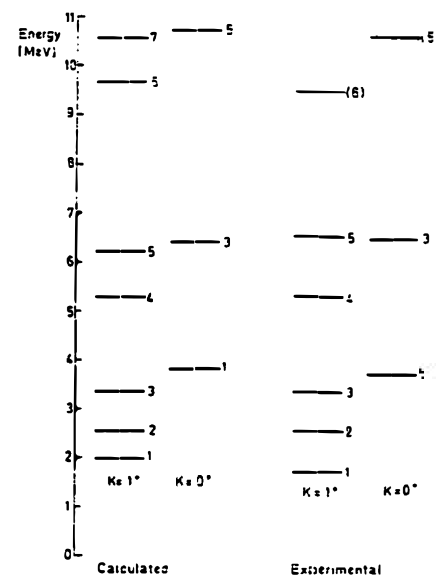


Fig. 7: Experimental and calculated states of the 2N + L = 8 alpha particle cluster bands in ^{18}F. The states have been divided into two bands denoted by $K^{\pi} = 0^{+}$ and 1^{+}

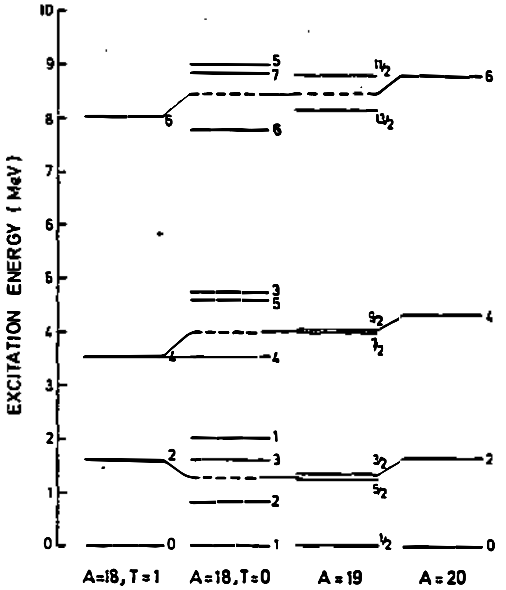


Fig. 8: Alpha particle cluster bands with 2N + L = 8 in the nuclei ^{18}O, ^{18}F, ^{19}F and ^{20}Ne, showing the stability of alpha-clustering as nucleons are added to the core.

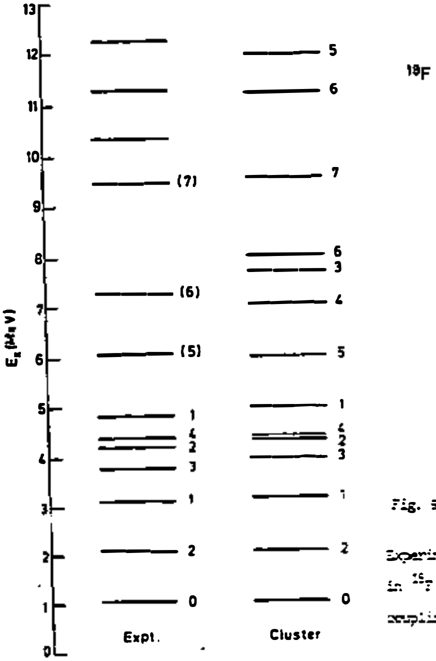
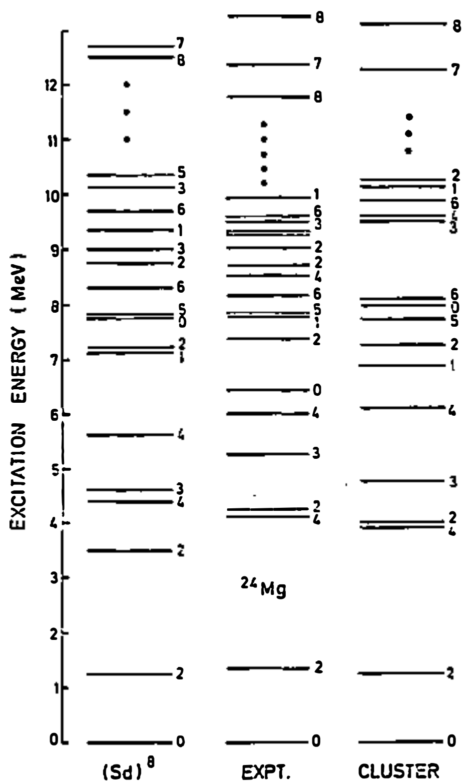


Fig. 9:

Experimental and calculated trinucleon negative parity levels in ^{19}F with 2N + L = 6. The rich spectrum results from the coupling of the spin ½ clusters to the orbital motion.



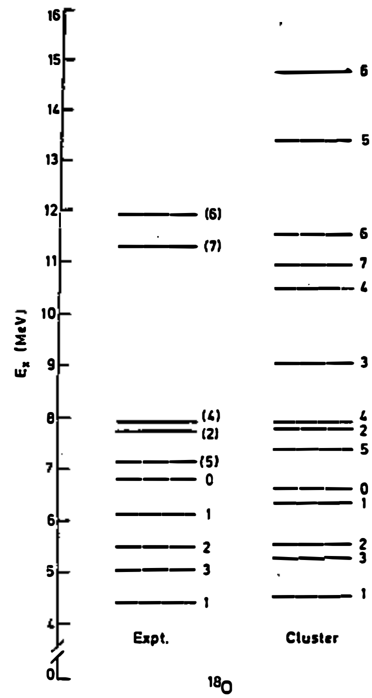


Fig. 10: Experimental and calculated negative parity levels of the system $^{18}O = t + {}^{15}N$.

Fig. 11: States in ^{24}Mg compared with the shell-model and the $^{16}O + {}^{8}Be(0^+, 2^+, 4^+)$ cluster calculation.

DISCUSSION

M.V. Mihailović: Are your results stable to mixing different fragmentations, for example: $\alpha + {}^{14}N$ and $d + {}^{16}O$?

B. Buck: I expect that there would be relatively little mixing between different cluster configurations unless the energies of states of the same spin + parity of the different cluster representations happened to be nearly the same. In this sense the model is quite stable.

Y.C. Tang: In ^{19}F the spin-orbit splitting of the $5/2^-$ and $3/2^-$ levels which have predominantly an $\alpha + {}^{15}N$ cluster configuration is expected to be much smaller than that of the $5/2^+$ and $3/2^+$ levels which have predominantly an $t + {}^{16}O$ cluster configuration. This is so, because the relative orbital angular momentum has to be shared among all 15 nucleons of the ^{15}N cluster and the intrinsic spins of the nucleons in ^{15}N are paired off to yield a total spin angular momentum of only 1/2.
Now, I have one question. In your $\alpha + {}^{15}N$ calculation, you choose your global quantum number to be 8. What is your reason for not choosing this number to be 7?

B. Buck: If the global quantum number were chosen to be 7 instead of 8 it would imply that one of the nucleons of the cluster would occupy the hole in the p-shell of ^{15}N. It would not then make sense to think of this nucleon correlating strongly with the other three nucleons in the (sd) shell. We would have in fact the closed shell of ^{16}O nucleus as core and the resulting configuration of three nucleons clustering outside this core is just the $t + {}^{16}O$ model that I have talked about.

H. Horiuchi: I have two questions concerning the ^{24}Mg calculation. The first is about the large observed α-spectroscopic factor of the second 0^+ level. I think the shell model calculations including the excitation to pf shell have not succeeded

to reproduce this $\propto s^2$-factor and also recent multi-cluster model has not succeeded. So I would like to know how is your wave function of 0_2^+. Is it not difficult to get the α amplitude because you are adopting the ^{16}O + ^{8}Be model?
The second question is how to treat the Pauli principle effect when excitations of the clusters are allowed.

B. Buck: 1) I have not brought with we the details of the structure of the states of ^{24}Mg produced by the model so I cannot answer this question.
2) I have not thought about this very much but I would guess off-hand that excitation of one or both of the clusters would lead to more extended Pauli blocking effects so that the lowest allowed values of the global quantum numbers would have to be chosen different. Also of course some of the excitation quanta would be lost to the relative motion of the centres of mass of the clusters.