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Three-cluster Problem in Light Nuclei Based on the Microscopic Theory

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§1. Introduction

Recently much attention[1)~3)] has become to be paid to the many-cluster problem. There are various subjects which are treated in the multi-cluster framework. Among them I can quote the following three themes as the important ones; (i) Study of the excited states with the well-developed multi-cluster structure, (ii) Study of the coexistence phenomena of the shell and cluster structures and the mechanism of the structure-change between the two different structures. (iii) Study of the rearrangement reaction processes with cluster transfer.

In this talk I discuss the first two problems (i) and (ii) a little in detail. There it is seen that the proper treatment of the Pauli principle is very important. The multi-cluster orthogonality condition model[4)~7)] is shown in my talk to be promising even for heavy and complicated systems as a tractable framework. This model takes into account the effect of the Pauli principle in a proper way and is of course useful also for the treatment of the problem (iii).

In §2 we first discuss the difference of the present multi-cluster treatment from the classical α-particle model.[17] We present in §3 the formulation of the three-cluster orthogonality condition model, especially the construction procedure[8)~(10)] of the basis states allowed by the Pauli principle. In §4 we discuss the results of the studies of the 3α problem for ^{12}C.[3)~(5), (11)~(16)] The proper treatment of the α-α interaction is emphasized to be important especially for the reproduction of the coexistence phenomena of the cluster and shell structures. §5 discusses the application of the $^{12}C+2\alpha$ orthogonality condition model to the study of ^{20}Ne structure, where our argument is on the hierarchy of the truncation in the cluster model. Finally in §6 concluding remarks are made.

§2. Classical α-particle model

Classical α-particle model[17] is a multi-cluster model. But now it is regarded to be unsuccessful, and is very different from the present cluster model viewpoint. Like as Brink [19] we can indicate two unsatisfactory points of the classical α-particle model. The first is the inadequacy[20)(21)] of the postulation of the small vibration of α-particles around (or the rotation of) the equilibrium arrangement of the α's of the ground state for all the low-lying excited states. This assumption can not explain the appearance of the low-lying anomalous excited states,

which are on the other hand successfully understood in the present cluster model by the cluster configulation with large structure-change from the ground state structure as is seen in ^{16}O(18) and in ^{12}C as will be discussed later in my talk. Another unsatisfactory point is about the treatment of the effect of the Pauli principle. The Pauli principle effect can not be treated by simple repulsion between clusters, especially when we investigate the coexistence problem of the shell and cluster structures, which will also be seen in ^{16}O and ^{12}C cases discussed later.

In relation to these defects we can indicate the peculiar fact that the α-α system has to be regarded as an exceptional case in this model. In the present cluster model, the α-α system is one of the basic systems of the model. We will see below that with use of the proper α-α force which describes the α-α system well, we can reproduce the 3α system (^{12}C) data fairly well by 3α model, even the coexistence phenomena of the cluster and shell structures. Similarly in discussing ^{20}Ne structure later by $^{12}C+\alpha+\alpha$ model, we use the $^{12}C-\alpha$ interaction which is based on the microscopic theory and which reproduces the ^{16}O data fairly well.

§3. Multi-Cluster Orthogonality Condition Model

Since the effect of the Pauli principle is very important to treat properly, it is desirable for us to rely upon the microscopic frameworks such as the resonating group method (RGM)[22] or the generator coordinate method (GCM)[23] with Brink type wave functions. Recent technical development[22][24][25] of the RGM and GCM has made the application of these methods possible in sd and pf shell region. However when the complicated systems are concerned such as in the coupled-channel or many-cluster calculations in sd shell region, there still remain difficulties in practical evaluation of kernels like as the angular momentum projection. I can quote two reasons to adopt the orthogonality condition model (OCM) in parallel with the microscopic frameworks RGM and GCM. One is its relative simplicity of treatment compared to RGM and GCM and indeed we show later that we can treat the $^{12}C+\alpha+\alpha$ system in this framework where the rotational excitation of ^{12}C cluster to its ground band member states 2^+ and 4^+ is allowed. The other is that the OCM is useful to get the insight of what is happening in fully microscopic calculations by RGM or GCM since the effect of the Pauli principle is taken into account only through the projection operator which rejects the Pauli-forbidden states from the relative motion while others are the same as the ordinary Schrödinger equation with local interactions.

The OCM[26][30] has been introduced by Saito[26] for the α-α system in order to extract the essential roles of the exchange kernels of the RGM which has succeeded not only in reproducing the α-α data but also in giving the microscopic interpretations of[27]~[29] the characteristic features of the phenomenological local potentials proposed to fit the experiments.

In OCM, the relative motion between two-clusters C_1 and C_2 is described by the following Schrödinger equation,

$$\left\{ -\frac{\hbar^2}{2\mu}\left(\frac{\partial}{\partial \vec{r}}\right)^2 + V_{eff}(r) \right\} \omega(\vec{r}) = E\,\omega(\vec{r}) , \qquad (3.1)$$

under the subsidiary condition of Eq.(3.2) which demands that the relative motion should be orthogonal to the forbidden states $\omega_F^i(\vec{r})$ by the Pauli principle,

$$\langle \omega(\vec{r}) \,|\, \omega_F^i(\vec{r}) \rangle = 0 , \quad i = 1, \sim n_F . \qquad (3.2)$$

The effective potential $V_{eff}(r)$ is usually assumed to be similar to the folding potential. Forbidden states $\omega_F^i(\vec{r})$ are defined as the solution of

$$\mathcal{A}\left\{ \omega_F^i(\vec{r})\,\phi(c_1)\,\phi(c_2) \right\} \equiv 0 . \qquad (3.3)$$

When we adopt the harmonic oscillator (H.O.) shell model (S.M.) wave functions with common oscillator parameter $\nu = m\omega/2\hbar$ for $\phi(c_i)$ and when $\phi(c_i)$ are SU_3 scalar like as in the case of the os-shell clusters, ^{16}O or ^{40}Ca, the solutions of $\omega_F^i(\vec{r})$ of Eq.(3.3) are given[31] by the H.O. functions $U_{N\ell m}(\vec{r}) \equiv R_{N\ell}(r) Y_{\ell m}(\hat{r})$

with number of the H.O. quanta $N=2n+\ell$ belonging to some definite set $N^F(1, 2)$; namely $\omega^i_F(\vec{r})=U_{N\ell m}(\vec{r})$ with $N\in N^F(1, 2)$.

Due to the orthogonality condition to the forbidden states, the relative wave function $\omega(\vec{r})$ has the almost energy-independent inner oscillation for a wide energy range. Since inner oscillation of $\omega(\vec{r})$ causes a large amount of kinetic energy, it works like as the repulsive force in this inner region. In the α-α case, the position of the outermost nodal point of $\omega(\vec{r})$ was found[27] to be just near the radius of the repulsive core of the phenomenological potentials.[32] This is the explanation by Tamagaki-Tanaka,[27] Okai-Park[28] and Saito[26] why we need the repulsive core in the phenomenological potentials. Fig. 1 shows the good fit of the OCM calculation by Saito to the observed α-α scattering phase shifts.

When a cluster C_1 is deformed, it is desirable to allow the rotational excitation of C_1 in treating the system including C_1. This is the channel coupling problem. For the channel coupling two-cluster system C_1-C_2, the forbidden relative states $\{\omega^i_{Fj}(\vec{r})\}$ are defined as follows[33]

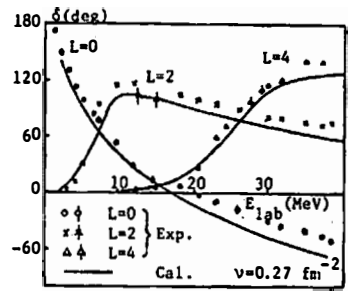


Fig. 1 α-α phase-shifts by OCM. V_{eff} is the folding potential calculated with use of Schmidt-Wildermuth two-nucleon force with Serber mixture.

$$\mathcal{A}\Big\{\sum_j \big[\omega^i_{Fj}(\vec{x})\phi_{Lj}(C_2)\big]_J \Phi(C_1)\Big\} = 0 \,, \tag{3.4}$$

where j specifies the channel (ℓ, L) with angular momenta L and ℓ of ω^i_{Fj} coupling to the total angular momentum J. If we adopt for $\phi_L(C_1)$ the SU_3 shell model wave function belonging to the non-scalar representation $(\sigma_0, \tau_0) \neq (0, 0)$, the solution of the above Eq.(3.4) is again very simple. $\omega^i_{F\ell}(\vec{r})$ are proportional to H.O. functions $U_{N\ell}$ and the quantum number i of $\omega^i_{F\ell}$ is specified by the SU_3 label (σ, τ) which results from the coupling of $(N, 0)$ of $U_{N\ell}$ with (σ_0, τ_0) of $\phi_L(C_1)$; $(N, 0)\times(\sigma_0, \tau_0) \to (\sigma, \tau)$. Namely,[33]

$$\begin{aligned}
&\sum_j \big[\omega^i_{Fj}(\vec{x})\phi_{Lj}(C_2)\big]_J \\
&= \sum_j \langle (N,0)\ell_j, (\sigma_0,\tau_0)L_j \| (\sigma\tau)\kappa J\rangle \big[U_{N\ell_j}(\vec{x})\phi_{Lj}(C_1)\big]_J \\
&= \big[U_{(N,0)}(\vec{x})\,\phi_{(\sigma_0,\tau_0)}(C_1)\big]_{(\sigma,\tau)\kappa J} \,.
\end{aligned} \tag{3.5}$$

In channel coupling case, usualy the H.O. quanta N is divided into three sets $N^F(1, 2)$, $N^{PF}(1, 2)$ and $N^A(1, 2)$. For $N \in N^F(1, 2)$ all (σ, τ) resulting from $(N, 0)\times(\sigma_0, \tau_0)$ are forbidden and for $N \in N^{PF}(1, 2)$ some special (σ, τ) are forbidden while for $N \in N^A(1, 2)$ all (σ, τ) are allowed. Table 1[33] shows the allowed (σ, τ) labels of the $^{12}C+\alpha$ system where ^{12}C is described by p-shell shell model wave function with (0, 4) SU_3 symmetry, $(0S)^4(0p)^8$ [4] (0, 4).

The coupled channel OCM equation has the same form as the ordinary coupled channel Schrödinger equation except that the wave function $\{\omega_j(\vec{r})\}$ must satisfy the subsidiary orthogonality condition to the forbidden states,

N	$(\sigma,\tau)+(N,0)\times(0,4)$
$N\leq 3$	all forbidden
4	(0, 0)
5	(2, 1)
6	(2, 0) (3, 1) (4, 2)
7	(3, 0) (4, 1) (5, 2) (6, 3)
$N\geq 8$	all allowed

Table 1. SU_3 classification of allowed states of $^{12}C+\alpha$ system.

$$\left\langle \sum_k \left[\omega_k(\vec{r})\phi_{L_k}(c_1)\right]_J \middle| \sum_j \left[\omega^i_{Fj}(\vec{r})\phi_{L_j}(c_1)\right]_J \right\rangle$$

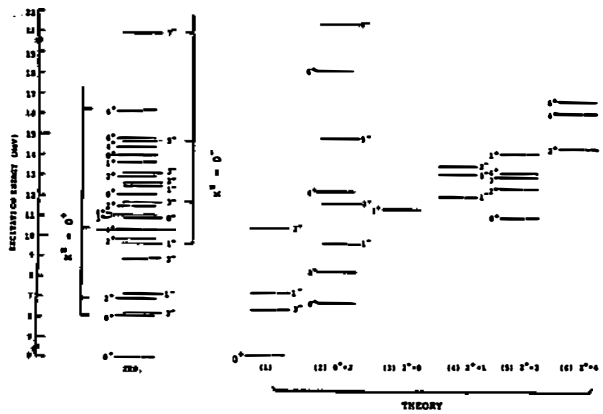
$$= \sum_k \left\langle \omega_k(\vec{r}) \middle| \omega^i_{Fk}(\vec{r}) \right\rangle = 0 \ , \quad i = 1, \sim n_F \ . \tag{3.6}$$

Fig. 2 shows the excellent reproduction of the observed ^{16}O spectra by $^{12}C+\alpha$ coupled channel OCM done by Suzuki.[11] It is remarkable that besides the excitation energies good reproduction was also obtained for the electric transition and α-decay values.[11] In this calculation, the effective potentials (elastic and coupling) are the folding potential with a little modification.

The multi-cluster OCM is nothing but the many-body Schrödinger equation where the interaction process between any pair of constituent clusters is described by the two-cluster OCM discussed above.

For a system composed of the three SU_3-scalar clusters, the OCM equation is written as follows

Fig. 2. Energy levels of ^{16}O by $^{12}C+\alpha$ coupled channel orthogonality condition model. Calculated levels denoted by (1) are levels with shell-model-like structure and others (2)∿(6) are those with cluster structure. Levels denoted by (2) $0^+\times J$ have the main configulation $^{12}C(0^+)$ coupled with the relative-motion angular momentum $\ell=J$ and so forth.



$$\left\{ \sum_{i}^{3} T_i - T_{c.m.} + \sum_{i>j}^{3} V_{eff}(\vec{x}_i - \vec{x}_j) \right\} \chi(\vec{s}, \vec{t}) = E \chi(\vec{s}, \vec{t}) ,$$

$$< \chi(\vec{s}, \vec{t}) \mid \begin{matrix} \text{Two-cluster forbidden states} \\ \text{between any pair of clusters} \end{matrix} > = 0 ,$$

$\vec{s}, \vec{t}$ = Jacobi coordinates . (3.7)

The procedure to handle the orthogonality condition in multi-cluster system is not unique and as an different approach from ours here I only quote the name of Ref.(34) by Neudatchin and his coworkers. We note that the functional space of the multi-

cluster system can be divided into two mutually orthogonal subspaces; one is composed of the multi-cluster wave functions which satisfy the above orthogonality condition in Eq.(3.7) and is called as the allowed space, while the other is the subspace orthogonal to this allowed space and is called as the forbidden space. Any function $\chi_F(\vec{s},\vec{t})$ in the forbidden space satisfies the equation $\mathcal{A}\{\chi_F(\vec{s},\vec{t})\phi(C_1)\phi(C_2)\phi(C_3)\} = 0$. Our treatment of the orthogonality condition in the multi-cluster probrem is that we first construct the complete basis states of the allowed space and then we solve the ordinary many-body Schrödinger equation only with use of these basis states of the allowed space.[4][5]

The construction of the allowed states of the multi-cluster space can be simplified by the introduction of the (Elliott) SU_3 group.[4][5][10] We sketch the procedure for the three SU_3-scalar cluster system below.[10] We can expand the allowed three-cluster state $\chi_{N(\lambda,\mu)\kappa J,p}$ with use of the allowed two-cluster states $U_{N_2}(\vec{t}_{12})$, $N_2 \notin N^F(1,2)$ of the C_1-C_2 sub-system as follows

$$\chi_{N(\lambda,\mu)\kappa J,p} = \sum_{N_2 \notin N^F(1,2)} A^{N(\lambda,\mu)}_{p,N_2(2)} \left[U_{N_1}(\vec{s}_3)\, U_{N_2}(\vec{t}_{12}) \right]_{(\lambda,\mu)\kappa J},$$

$$\vec{s}_3 = \vec{x}_3 - (M_1\vec{x}_1 + M_2\vec{x}_2)/(M_1+M_2) \ , \quad \vec{t}_{12} = \vec{x}_1 - \vec{x}_2 \ . \tag{3.8}$$

Here $[U_{N_1} U_{N_2}]_{(\lambda,\mu)}$ denotes the SU_3 vector coupling $(N_1,0)\times(N_2,0) \to (\lambda,\mu)$. The expansion coefficients $A^{N(\lambda,\mu)}_{p,N_2(2)}$ can be regarded

as the coefficient of fractional parentage (cfp) and they are obtained by the remaining orthogonality requirement for $\chi_{N(\lambda,\mu)\kappa J,p}$ to the two-cluster forbidden states of C_1-C_3 and C_2-C_3 systems. The equation to determine $A^{N(\lambda,\mu)}_{p,N_2}(2)$ is given by

$$\sum_{N_2 \in N^F(1,2)} A^{N(\lambda,\mu)}_{p,N_2}(2) \langle N_3 N_4 \mid N_1 N_2, (\lambda,\mu) \rangle_{\theta_\alpha} = 0 \ ,$$
$$\text{for } N_4 \in N^F(\beta,\gamma) \ , \ (\alpha,\beta,\gamma) = \begin{cases} (1,2,3) \\ (2,3,1) \end{cases} \ , \qquad (3.9)$$

which is equivalent to finding the eigen-vectors $A^{N(\lambda,\mu)}_{p,N_2}(2)$ with the eigen-value $q_p=0$ of the following secular equation

$$\sum_{N_2 \in N^F(1,2)} Q^{N(\lambda,\mu)}(N_1' N_2' \mid N_1 N_2) A^{N(\lambda,\mu)}_{p,N_2}(2) = q_p A^{N(\lambda,\mu)}_{p,N_2'}(2) \ ,$$
$$Q^{N(\lambda,\mu)}(N_1' N_2' \mid N_1 N_2) = \sum_{\alpha=1}^{2} \sum_{N_4 \in N^F(\beta,\gamma)} \langle N_3 N_4 \mid N_1' N_2', (\lambda,\mu) \rangle_{\theta_\alpha} \langle N_3 N_4 \mid N_1 N_2, (\lambda,\mu) \rangle_{\theta_\alpha} . \qquad (3.10)$$

$\langle N_3 N_4 | N_1 N_2, (\lambda,\mu) \rangle_{\theta_\alpha}$ is the reduced Talmi-Moshinsky-Smirnov coefficient which is defined by

$$[U_{N_1}(\vec{r}_3) U_{N_2}(\vec{r}_{12})]_{(\lambda\mu)\kappa J} = \sum_{N_3+N_4=N_1+N_2} \langle N_3 N_4 \mid N_1 N_2, (\lambda,\mu) \rangle_{\theta_\alpha} \times [U_{N_3}(\vec{r}_\alpha) U_{N_4}(\vec{r}_{\beta\gamma})]_{(\lambda\mu)\kappa J} . \qquad (3.11)$$

By using the quasi-spin introduced by Bargmann-Moshinsky[35], this coefficient is easily calculated as follows

$$\langle N_3 N_4 \mid N_1 N_2, (\lambda,\mu) \rangle_{\theta_\alpha} = d^{j}_{m',m}(\theta_\alpha) \ ,$$
$$j = \frac{\Lambda}{2} \ , \ m' = \frac{1}{2}(N_3 - N_4) \ , \ m = \frac{1}{2}(N_1 - N_2) \ , \qquad (3.12)$$

where $d^{j}_{m',m}(\theta) \equiv \langle jm' | e^{-i\theta J_y} | jm \rangle$ is the familiar rotation matrix, and angle θ_α is defined by using the H.O. creation operators, as follows

$$\begin{pmatrix} \vec{a}^{+}(\vec{\sigma}_\alpha) \\ \vec{a}^{+}(\vec{t}_{\beta\delta}) \end{pmatrix} = \begin{pmatrix} \cos\theta_\alpha/2 & -\sin\theta_\alpha/2 \\ \sin\theta_\alpha/2 & \cos\theta_\alpha/2 \end{pmatrix} \begin{pmatrix} \vec{a}^{+}(\vec{\sigma}_3) \\ \vec{a}^{+}(\vec{t}_{12}) \end{pmatrix}, \qquad (3.13)$$

Since the SU_3 Clebsch-Gordon coefficients $\langle (N_1\,0)\ell_1,\ (N_2\,0)\ell_2 \| (\lambda,\mu)KJ \rangle$ are easily obtained by a recursion formular,[36] all the calculations to construct the three-cluster allowed states reduce to the primitive ones.

When one cluster C_1 has a non-SU_3-scalar wave function, the construction procedure of the allowed three-cluster states needs one more process accumulated on the above-mentioned prescription. This is due to the existence of the sets $N^{PF}(1,\alpha)$ for the two-cluster systems C_1-C_α ($\alpha = 2, 3$). What we do first is to regard $N^{PF}(1,\alpha)$ as if it were contained in $N^{A}(1,\alpha)$ and construct $\chi_{N(\sigma,\tau)\rho,\,p}$ following the prescription stated above. The truely-allowed three-cluster wave function $\Psi_{N(\lambda,\mu)KJ,\,r}$ can be expanded by using these $\chi_{N(\sigma,\tau)\rho,\,p}$ as follows

$$\Psi_{N(\lambda,\mu)KJ,\,r} = \sum_{(\sigma,\tau)\rho} H^{N(\lambda\mu)}_{r,\,(\sigma\tau)\rho} \left[\chi_{N(\sigma\tau),\rho}\, \phi_{(\sigma_0\tau_0)}(C_1)\right]_{(\lambda\mu)KJ}, \qquad (3.14)$$

where $[\chi_{(\sigma\tau)}\phi_{(\sigma_0\tau_0)}]_{(\lambda\mu)}$ denotes the SU_3 vector coupling $(\sigma,\tau)\times(\sigma_0,\tau_0)\to(\lambda,\mu)$. The coefficients $H^{N(\lambda,\mu)}_{r,\,(\sigma,\tau)\rho}$ are determined by requiring the orthogonality of $\Psi_{N(\lambda\mu)KJ,\,r}$ to the two-cluster forbidden states $[U_{N_2}(\vec{t}_{1\alpha})\phi_{(\sigma_0\tau_0)}(C_1)]_{(\sigma,\tau)}$ with

$N_2 \in N^{PF}(1,\alpha)$, $(\sigma,\tau) \in \Omega_{N_2}(1,\alpha)$ which are regarded as if allowed in constructing $\chi_{N(\sigma\tau)\rho P,p}$. By the notation Ω_{N_2}, we mean the set composed of those (σ,τ) which gives the forbidden state $[U_{N_2}(\vec{t}_{1\alpha})\phi_{(\sigma_0\tau_0)}(C_1)]_{(\sigma\tau)}$ for $N_2 \in N^{PF}(1,\alpha)$. $H^{N(\lambda\mu)}_{r,(\sigma\tau)p}$ are obtained as the solution of the following equation

$$\sum_{(\sigma\tau)p} H^{N(\lambda\mu)}_{r,(\sigma\tau)p} X_\alpha\left((\sigma\tau),p,N_2,(\sigma'\tau')\right) = 0 ,$$

$$X_\alpha\left((\sigma\tau),p,N_2,(\sigma'\tau')\right) \equiv A^{N(\sigma\tau)}_{p,N_2}(\alpha) \times U\left((N_1,0)(N_2,0)(\lambda,\mu)(\sigma_0\tau_0);(\sigma,\tau)(\sigma'\tau')\right),$$

$$\text{for } N_2 \in N^{PF}(1,\alpha),\ (\sigma'\tau') \in \Omega_{N_2}(1,\alpha),\ (\alpha = 2,3),$$

(3.15)

where $A^{N(\sigma\tau)}_{p,N_2}(\alpha=3)$ are the cfp of the expansion of $\chi_{N(\sigma\tau)\rho P,p}$ by $[U_{N_1}(\vec{s}_2)U_{N_2}(\vec{t}_{13})]_{(\sigma\tau)\rho p}$ like as $A^{N(\sigma\tau)}_{p,N_2}(\alpha=2)$ in Eq.(3.8). Eq.(3.15) is equivalent to finding the eigen-vector $H^{N(\lambda\mu)}_{r,(\sigma\tau)p}$ with eigen-value $q_r=0$ of the following equation

$$\sum_{(\sigma\tau)p} Q^{N(\lambda\mu)}\left((\bar{\sigma}\bar{\tau})\bar{p} \mid (\sigma\tau)p\right) H^{N(\lambda\mu)}_{r,(\sigma\tau)p} = q_r H^{N(\lambda\mu)}_{r,(\bar{\sigma}\bar{\tau})\bar{p}} ,$$

$$Q^{N(\lambda\mu)}\left((\bar{\sigma}\bar{\tau})\bar{p} \mid (\sigma\tau)p\right) = \sum_{\alpha=2,3} \sum_{N_2 \in N^{PF}(1,\alpha)} \sum_{(\sigma'\tau') \in \Omega_{N_2}(1,\alpha)} X_\alpha\left((\bar{\sigma}\bar{\tau}),\bar{p},N_2,(\sigma'\tau')\right) \times X_\alpha\left((\sigma\tau),p,N_2,(\sigma'\tau')\right) .$$

(3.16)

In Eqs. (3.15), (3.16) $U((N_1,0)(N_2,0)(\lambda,\mu)(\sigma_0,\tau_0);(\sigma,\tau)(\sigma',\tau'))$ denotes the SU_3 recoupling coefficient.

§4. 3α model for ^{12}C

A basic three-cluster problem is a 3α problem. Fig. 3 shows all the observed levels of ^{12}C[39] below the first T=1 level at 15.11 MeV. Among them the 1^+ level at 12.71 MeV is known to have broken orbital symmetry [4431] as a main configuration and so it is the level out of the scope of the present 3α model without symmetry breaking. The 7.65 MeV 0^+ and 10.3 MeV 2^+ (2^+ assignment is due to Morinaga[37] and is consistent with the calculation given below) are observed to have very large α-decay widths exceeding the Wigner limit values and were postulated by Morinaga[37] to be the α-clustering levels with large structure change from the ground state shell model structure.

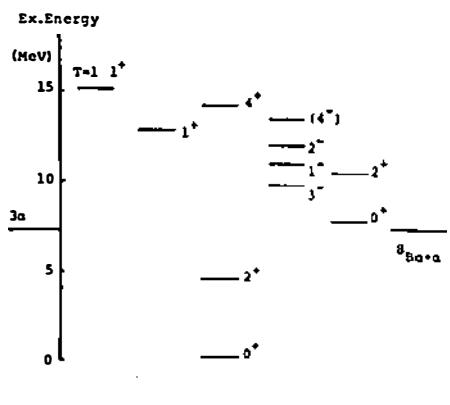


Fig. 3 Observed energy levels of ^{12}C.

The allowed states of the 3α system calculated by the prescription of §3 are given in Table 2. This table shows a good correspondence of the 3α allowed states with the shell model classification; for

$N<8$ there is no state allowed by the Pauli principle and for $N=8$ only. $\Phi\{(0s)^4(0p)^8;\ (0,4)J\}$ is allowed as $[4^3]$ symmetry shell model states. The explicit relation between our 3α allowed states $\chi_{N(\lambda,\mu)KJ}$ and the ^{12}C shell model wave functions can be seen by constructing the 12-nucleon wave function $\mathcal{A}\{\chi_{N(\lambda,\mu)KJ,p}\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$; for example $n_J\mathcal{A}\{\chi_{8(0,4)J}\phi^3(\alpha)\}\equiv\Phi\{(0s)^4(0p)^8;\ (0,4)J\}$.

N	$(\lambda,\mu)^n$
8	(0,4)
9	(3,3)
10	(6,2),(2,4)
11	(9,1),(5,3),(3,4)
12	(12,0),(8,2),(6,3),(4,4),(0,6)
13	(11,1),(9,2),(7,3),(5,4),(3,5)
14	(14,0),(12,1),(10,2),(8,3),$(6,4)^2$,(2,6)
15	(15,0),(13,1),(11,2),$(9,3)^2$,(7,4),(5,5)
	(3,6)

Table 2. SU_3 classification of 3α allowed states.

The calculated spectra are given in Fig. 4 where the same $\alpha-\alpha$ interaction as Saito[26] is adopted ($\nu=0.275$ fm^{-2} adopted here is slightly different from Saito's $\nu=0.27$ fm^{-2}.) What is noted first is the reproduction of all the observed levels below the first $T=1$ level at 15.11 MeV except 12.71 MeV 1^+ mentioned above. (The 13.35 MeV level has unnatural parity but J assignment is uncertain experimentally. This calculation and Ref.38 both predict this level to be 4^-, forming a $K^\pi=3^-$ band together with the 3^- at 9.64 MeV.) Next, to see the character of the second excited 0^+ near 3α threshold, we show in Fig. 5 the reduced width amplitude (RWA) of the $\alpha+^8Be(0^+)$ break-up of the calculated 0_1^+ and 0_2^+ levels. (Here $^8Be(0^+)$ wave function needed for RWA is obtained by solving 2α OCM.) We clearly see that 0_2^+ is the state with large $\alpha+^8Be(0^+)$ clustering at

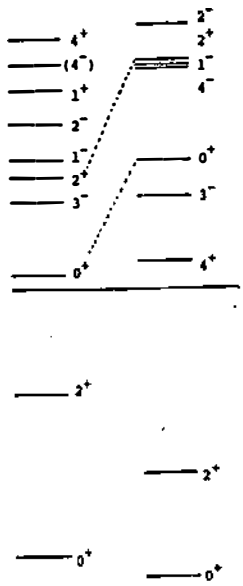


Fig. 4 Energy spectra of ^{12}C by $\hat{\pi}3\alpha$ OCM.

surface. This is consistent with large observed α-decay reduced width of the observed 0_2^+.[39] Since $^8Be(0^+)$ itself has a well-developed cluster structure of α-α, we can say that 0_2^+ has a well-developed three-cluster structure. To see the characters of 0_1^+ and 0_2^+ further, the squared norms of the expansion components by the number of H.O. quanta are given in Table 3. The ground state

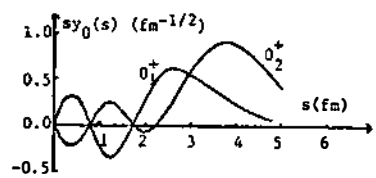


Fig. 5 Reduced $\alpha+^8Be(0^+)$ width amplitudes of 0_1^+ and 0_2^+. S^2 values are 0.62 and 1.40 for 0_1^+ and 0_2^+, respectively

(0_1^+) is seen to have a large N=8 (0,4) component, showing a good correspondence with the p-shell shell-model description of the ground state. On the contrary, the expansion coefficients of 0_2^+ spread over many

N	8	10	12	14	16	18	20	22
0_1^+	0.80	0.08	0.08	0.02	0.01	0.01	<0.01	<0.01
0_2^+	0.13	0.13	0.12	0.17	0.14	0.12	0.08	0.05

Table 3. Squared norms of the expansion components by the number of H.O. quanta of 0_1^+ and 0_2^+ by OCM.

higher N, which indicates a well-developed clustering character of this level. In spite of this large difference between two 0^+, the monopole transition matrix element between them is obtained to be 6.1 fm^2, close to the observed value 5.8 fm^2.

One of the characteristic features of 3α OCM results is its large binding energy of the ground state 0_1^+, which is remarkably different from the previous 3α calculations[40] with phenomenological α-α forces with repulsive core. The reason of this difference is considered to be interpreted in the following way.[5] When 3α's come near each other, the effective number of interaction bonds increases than that of kinetic energy terms by about 1.5 times. This causes the strengthening of the repulsive effect for the case of the potential with inner repulsive core. On the contrary, in the OCM case, the (deep) attractive (folding-type) potential is deepened while orthogonality condition is not so strengthened since it allows the existence of the compact 3α configuration of N=8, (0, 4). The increase of the effect of the attractive potential in the inner region overwhelms the large kinetic energy due to the inner oscillation of the α-α relative wave function of OCM. This leads to the result that in 3α system the existence of the compact 3α configuration become possible for which the amplitude of the wave function in the inner region is so large and which has the large binding energy.

The microscopic GCM treatments[41,42] of the 3α system

yield the almost similar results as OCM including the coexistence of shell and cluster structures. Thus we can say that the OCM is not only simpler to treat but also useful to get the insight about what is happening in the fully microscopic treatment, especially about the important and subtle behaviour of the Pauli principle effect.

§5. $^{12}C+\alpha+\alpha$ model for ^{20}Ne

In recent years, the structures of the sd-shell nuclei especially those of the excited states have been studied extensively by cluster model with many successes accumulated.[1] [3] A typical example is ^{20}Ne. In Fig. 6 we give the observed energy levels of ^{20}Ne classified into rotational bands. Among them the three bands with $K^{\pi}=0_1^+$, 0_1^-, 0_4^+ have been investigated by $^{16}O+\alpha$ cluster model[1][2] and this model has succeeded to reproduce well the observed

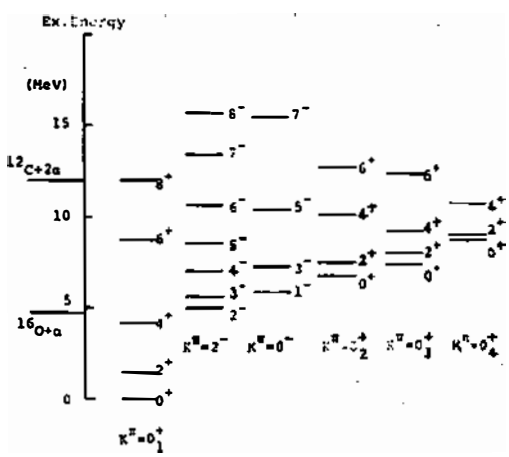


Fig. 6 Overlapping rotational bands in ^{20}Ne

quantities, excitation energies, E2 transition values and α-decay widths. The ground band $K^{\pi}=0_1^+$ has a shell-model-like structure and has a large overlap with the shell model configuration $\Phi\{(sd)^4;\ [4](8,0)J\}$ (which is equivalent to the cluster model wave function $n_J \mathcal{A}\{U_{N=8,J}(\vec{r})\,\phi(^{16}O)\,\phi(\alpha)\}$). The two bands with $K^{\pi}=0_1^-$ and 0_4^+ have proved to have the well-developed ^{16}O-α di-cluster structure. The ^{16}O-α relative wave functions of the $K^{\pi}=0_4^+$ band levels have one more nodal points than those of the $K^{\pi}=0_1^+$ band.

The $K^{\pi}=0_2^+$ band is considered[42] to have a main configuration $(sd)^4[4](4,2)$, but the rather strong mixing with the $K^{\pi}=0_4^+$ band has been suggested experimentally.[43] In order to reproduce the above mentioned characteristics of the four bands $K^{\pi}=0_1^+$, 0_1^-, 0_2^+, 0_4^+, Nemoto, Yamamoto, Suzuki, Ikeda and myself[44] have adopted the α-^{12}C-α model. This model uses the GCM framework and the intrinsic state is the linearly arranged configuration of α-^{12}C-α. This configuration of the model is introduced as the extension of the ^{16}O-α model, by allowing the dissociation process of the ^{16}O cluster into ^{12}C+α configuration. Good reproduction of the observed energy levels by this model is seen in Fig. 7.

The hierarchy of the truncation scheme such as $^{16}O+\alpha \rightarrow {}^{12}C+2\alpha \rightarrow {}^{8}Be+3\alpha$ represents a process of the activation of the clustering degree of freedom in unit of α-cluster. The structures of the ^{20}Ne levels shown in Fig. 6 are strongly suggested to be understood

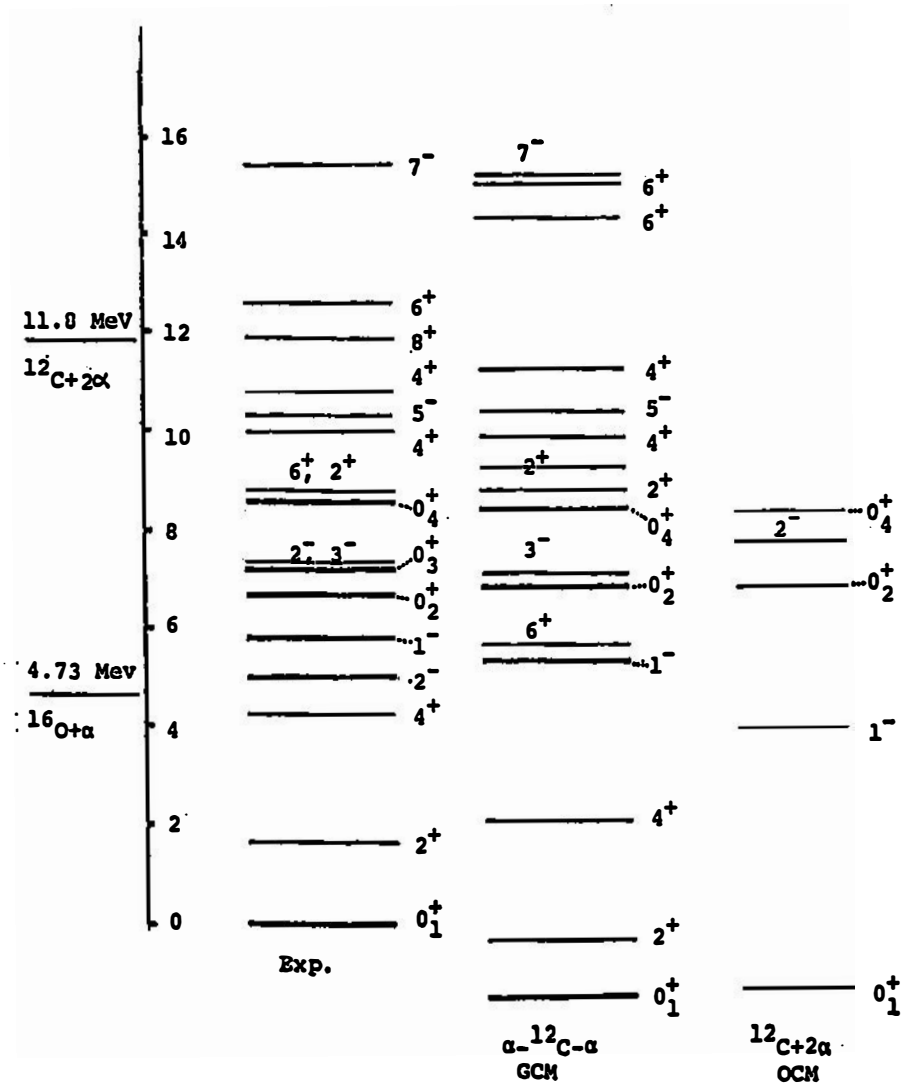


Fig. 7 Calculated energy levels of ^{20}Ne. The levels indicated by bold line are the band head levels.

within $^{12}C+2\alpha$ space. This is because the $K^{\pi}=0_3^+$ band seems to have $^{12}C+^{8}Be$ -like structure in view of the strong excitation by the ^{8}Be -transfer reaction[45] and because the $K^{\pi}=2_1^-$ band is considered to have a main configuration $(0p)^{-1}(sd)^5[4^5](8,2)$ which is contained in $^{12}C+2\alpha$ space as is shown below in Table 4.

The full treatment of the $^{12}C+2\alpha$ model without the restriction of the linear chain arrangement of clusters is thus desirable. Ikeda, Kato and myself have recently tackled this problem and have found that the OCM is applicable also for such a complicated system. This $^{12}C+2\alpha$ OCM permits the rotational excitation of ^{12}C cluster and the allowed states calculated by the prescription given in §3 are shown in Table 4. Band head levels of the five bands $K^{\pi}=0_1^+$, 2_1^-, 0_1^-, 0_2^+, 0_4^+ are also given in Fig. 7. In this calculation, the effective force between ^{12}C and α

N	$(\lambda,\mu)^n$
12	$(8,0)\ (4,2)\ (0,4)$
13	$(9,0)\ (8,2)\ (7,1)\ (6,3)\ (5,2)^2(4,4)\ (3,3)^2(2,5)\ (1,4)$
14	$(11,1)(10,0)^3(9,2)^2(8,1)^3(8,4)\ (7,3)^3(6,2)^5(6,5)\ (5,4)^3$ $(4,3)^3(4,6)^2(3,5)^2(2,4)^3(1,6)\ (0,8)$
15	$(12,1)^2(11,0)^5(11,3)(10,2)^4(9,1)^6(9,4)^2(8,3)^5(7,2)^7(8,6)$ $(7,5)^3(6,4)^5(5,3)^6(6,7)\ (5,6)^3(4,5)^4(3,4)^4(4,8)\ (3,7)^2$ $(2,6)^2(1,5)^2(2,9)\ (1,8)\ (0,7)$

Table 4. SU_3 classification of the coupled channel $^{12}C+\alpha+\alpha$ model space.

is the folding potential with use of the Volkov No.2 force[46] with m=0.60 and the H.O. size parameter of clusters ν=0.173 fm^{-2}. We have checked that the $^{12}C+\alpha$ OCM with this folding potential gives the almost the same good fit of the ^{16}O spectra as Suzuki's calculation shown in Fig. 2. The effective $\alpha-\alpha$ force is also the folding potential which is now calculated by using the Schmidt-Wildermuth force[22] with Serber type and ν=0.195 fm^{-2}. The H.O. size of the three-cluster wave functions is ν=0.173 fm^{-2}. The basis states adopted are similar to the extended shell model calculation by Tomoda and Arima;[21] For $K^{\pi}=0_1^+$, 0_2^+, 0_4^+, they are (N=12, (8,0), (4,2), (0,4))+(N=14, (10,0)^3) +(N=16, (12,0)^8)+(N=18 32, (N-4,0) of $^{16}O-\alpha$), for $K^{\pi}=0_1^-$ they are (N=13$\sim$29, (N-4,0) of $^{16}O-\alpha$) and for $K^{\pi}=2_1^-$ they are (N=13$\sim$19, (N-5,2)^n). Basis states for $K^{\pi}=2_1^-$ are still insufficient and the calculated energy for 2_1^- should be regarded as preliminary.

Besides the energy levels shown in Fig. 7, the characters of the wave functions for $K^{\pi}=0_1^+$, 0_1^-, 0_2^+, 0_4^+ are found to be similar to the microscopic GCM. Thus we can say that the OCM is reliable even in heavy and complicated systems and therefore promising. Improved treatment of $K^{\pi}=2_1^-$ band and the investigation of the $K^{\pi}=0_3^+$ band are now in progress by this OCM.

§6. Concluding remarks

In light nuclear system, much attension has become to be paid to the many-cluster problem. Three themes quoted in the introduction are considered to be important ones and in this talk the first two problems (i) and (ii) are discussed by taking the two examples, 3α model for ^{12}C and $^{12}C+2\alpha$ model for ^{20}Ne. Since the proper treatment of the Pauli principle is very important, it is desirable to apply the microscopic framework of RGM and GCM. However, still now the complicated multi-cluster systems are not so easy to access by fully microscopic frameworks. By this reason and by the usefulness in getting the insight about the subtle behaviour of the Pauli principle, I emphasized in this talk the promising feature of the simpler framework - the OCM. And I explained slightly in detail how to construct the multi-cluster allowed states in a simple and rapid way. This OCM framework is of course applicable for the problem (iii).

The semi-microscopic method OCM can be viewed as the semi-phenomenological framework, and can serve to link the phenomenological treatments with the microscopic ones. In view of the increase of the data in the wide region of light nuclei which need the cluster model analyses, the development of the OCM is necessary in parallel with RGM and GCM.

Acknowledgement

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References

(1) Proc. INS-IPCR Int. Symp. on Cluster Structure of Nuclei and Transfer Reactions Induced by Heavy Ions, Tokyo (1975).

(2) Proc. II Int. Conf. on Clustering Phenomena in Nuclei, Maryland (1975).

(3) H. Horiuchi, an invited paper of the Int. Conf. on Nuclear Structure, Tokyo (1977), which is to be in a Supplement to the Journal of the Physical Society of Japan, Vol. 44, 1978.

(4) Yu. Smirnov, I. Obukhovsky, Yu. Tchuvil'sky and V. Neudatchin: Nucl. Phys. A235 (1974), 289.

(5) H. Horiuchi, Prog. Theor. Phys. 51 (1974), 1266, 53 (1975), 447; Proc. VII Int. Conf. on Few-Body Problems in Nuclear and Particle Physics, Delhi (1975-1976), p.165.

(6) H. Nishioka, S. Saito and M. Yasuno, Proc. Int. Conf. on Nuclear Structure, Tokyo (1977), (Contr. Papers) p.157.

(7) K. Kato and H. Bando, ibid p.202.

(8) K. Kato and H. Bando, ibid p.195; Prog. Theor. Phys. 53 (1975), 692.

(9) H. Horiuchi, Prog. Theor. Phys. 55 (1976), 1448.

(10) H. Horiuchi, Prog. Theor. Phys. 58 (1977), 204.

(11) Y. Fujiwara and R. Tamagaki, Prog. Theor. Phys. 56 (1976); Prog. Theor. Phys. Suppl. No. 61 (1977), 229.

(12) N. de Takacsy, Nucl. Phys. A178 (1972), 469.

(13) E. Uegaki, S. Okabe, Y. Abe and H. Tanaka, Prog. Theor. Phys. 57 (1977), 1262.

(14) H. Hutzelmeyer and H.H. Hachenbroich, Z. Physik 232 (1970), 356.

(15) Y. Fukushima and M. Kamimura, Proc. Int. Conf. on Nuclear Structure, Tokyo (1977), (Contr. Papers) p.164.

(16) N. Takigawa and A. Arima, Nucl. Phys. A168 (1971), 593.

(17) D.M. Dennison, Phys. Rev. 96 (1954), 378; A.E. Glassgold and A. Galonsky, Phys. Rev. 103 (1956), 701.

(18) Y. Suzuki, Prog. Theor. Phys. 55 (1976), 1751, 56 (1976), 111.

(19) D. Brink, Proc. II Int. Conf. on Clustering Phenomena in Nuclei, Maryland (1975), p.3.

(20) D. Brink, Proc. 15th Solvay Conf. on Physics, Brussels (1970), p.1.

(21) G. Bertsch and W. Bertozzi, Nucl. Phys. A165 (1971), 199.

(22) K. Wildermuth and Y.C. Tang, A Unified Theory of the Nucleus (Vieweg, Braun-schweig, Germany, 1977).

(23) D. Brink, Proc. Int. School of Phys., Enrico Fermi, (1965), (C. Block, ed), Academic Press, New York and London (1966), p.247.

(24) M.V. Mihailovic and M. Rosina, ed, Generator Coordinate Method for Nuclear Bound States and Reactions, Fizika 5 Suppl. (1973).

(25) H. Horiuchi, K. Ikeda, M. Kamimura, S. Saito, R. Tamagaki and A. Tohsaki, Prog. Theor. Phys. Suppl. No. 62 (1977), to be published.

(26) S. Saito, Prog. Theor. Phys. 41 (1969), 705.

(27) R. Tamagaki and H. Tanaka, Prog. Theor. Phys. 34 (1965), 191.

(28) S. Okai and S.C. Park, Phys. Rev. 145 (1966), 787.

(29) R. Tamagaki, Prog. Theor. Phys. Suppl. Extra Number (1968), 242.

(30) S. Saito, S. Okai, R. Tamagaki and M. Yasuno, Prog. Theor. Phys. 50 (1973), 1561.

(31) H. Horiuchi and Y. Suzuki, Prog. Theor. Phys. 49 (1973), 1974.

(32) S. Afzal, A. Ahmad and S. Ali, Rev. Mod. Phys. 41 (1969), 247; S. Ali and A. Bodmer, Nucl. Phys. 80 (1966), 787.

(33) H. Horiuchi, Prog. Theor. Phys. 51 (1974), 745.

(34) V.G. Neudatchin, Proc. II Int. Conf. on Clustering Phenomena in Nuclei, Maryland (1975), p.205; V.I. Kukulin, V.G. Neudatchin and V.N. Pomerantsev, Sov. J. Nucl. Phys. 24 (1977), 155.

(35) V. Bargman and M. Moshinsky, Nucl. Phys. 18 (1960), 697; 23 (1961), 177.

(36) B. Kaufman and C. Noack, J. Math. Phys. 6 (1965), 142; A. Gal, Ann. Phys. 49 (1968), 341.

(37) H. Morinaga, Phys. Letters 21 (1966), 78.

(38) E.W. Schmidt and K. Wildermuth, Nucl. Phys. 26 (1961), 463.

(39) F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A248 (1975), 1.

(40) See R. Tamagaki and Y. Fujiwara, Prog. Theor. Phys. Suppl. No.61 (1977), 229.

(41) A. Arima, Proc. II Int. Conf. on Clustering Phenomena in Nuclei, Maryland (1975), 38.

(42) Y. Akiyama, A. Arima and T. Sebe, Nucl. Phys. A138 (1969), 273.

(43) H.T. Fortune, R. Middleton and R.R. Betts, Phys. Rev. Lett. 29 (1972), 738.

(44) F. Nemoto, Y. Yamamoto, H. Horiuchi, Y. Suzuki and K. Ikeda, Prog. Theor. Phys. 54 (1975), 104.

(45) R. Middleton, J.P. Garrett and H.T. Fortune, Phys. Rev. Lett. 27 (1971), 950.

(46) A.B. Volkov, Nucl. Phys. 74 (1965), 33.

DISCUSSION

Y.C. Tang: Can you use your orthogonality-condition model to study the properties of liquid helium or alpha-cluster matter?

H. Horiuchi: This problem you pointed at is a very interesting one. I tried to apply the OCM to α-matter to see the difference of treatments of the repulsive force in the inside region of interaction between OCM and the usual approach with phenomenological local force with repulsive core. But I have not found any simple and reliable procedure to treat the OCM interaction in infinite matter.

A. Weiguny: How do you define your orthogonality condition if there are no strictly forbidden states as is the case when different oscillator lengths are used for different fragments.

H. Horiuchi: This problem was investigated by Saito et al. They divide the Hamiltonian into two parts, one is the Hamiltonian of OCM and the other is the perturbation term which is due to the existence of the almost redunant states. In the example they used, this perturbation has effect only at rather high excitation energy region. But in the heavy system this effect may have some influence at rather low excitation energy region. But I think that even in that case the orthogonality condition is a good expression of the Pauli principle and the effect of the almost redunant states can be treated as a perturbation.

B.G.Giraud: In my opinion, the foundation of the OCM could be that, when a state becomes quasi forbidden, renormalization by $(1-K)^{-1/2}$ pushes the state progressively out of the Hilbert space, provided one proves that $(1-K)^{-1/2}\, H(1-K)^{-1/2}$ is indeed a large positive number for that state.
Contrarily to the claim of Saito, I doubt anyhow that the state which would be pushed up away in this way would necessarily show up as a true resonance.

H. Horiuchi: I agree with the second comment. For the first question I would like first to make a comment; the OCM can be considered as a semiphenomenological model which treats the effect of the Pauli principle in a natural and physical way. Its validity has been shown by many applications at least for two-body systems. Your question is about the microscopic foundation. The matrix element $I = (1-K)^{-1/2} H(1-K)^{-1/2}$ has the value $\langle\phi_n|H|\phi_n\rangle$ with $\phi_n = \frac{1}{\sqrt{(\hat{A}_n)\mu_n}} \mathcal{A}\{\chi_n(\vec{r})\phi(A_1)\phi(A_2)\}$ in the harmonic oscillator basis representation ($\|\phi_r\| = 1$). In the example by Saito, the limit of ϕ_n when $\mu_n \to 0$ goes to the higher shell configuration in the shell model language and the diagonal part of I has a rather large value. But for heavier systems like $\alpha + {}^{16}O$, ϕ_n with $\mu_n \to 0$ can remain in the lowest shell configuration. In this case the diagonal part of I is not so large. Also the non-diagonal parts in this example are not large. However, we have no general proof whether this situation is general or not.

D.R. Thompson: Can you tell me what would happen if you were to replace your effective potential by another one which contains a weakly repulsive Pauli core so that the forbidden or almost forbidden states do not occur? Could you then neglect the OCM projection and obtain similar results?

H. Horiuchi: I don't think it is a good procedure. First the consideration that the forbidden states just correspond the lower bound states in rather deep potential makes such a potential unreasonable. Secondly the RGM matrix element V_{RGM} is similar to the OCM matrix element $\sqrt{1-K}\ V_{eff}\sqrt{1-K}$ when we adopt a deep potential like a direct (or folding) potential by using the two-nucleon force with not so strong Majorana exchange mixture.

D. Baye: We have performed an exact generator coordinate calculation of elastic $\alpha + {}^{12}C$ scattering using the same interaction as Suzuki. Our results are not in agreement with his OCM results. In this case, the OCM does not seem to be a good approximation of the RGM.

H. Horiuchi: I also did a microscopic calculation for $\alpha + {}^{12}C$ system in the RGM framework using the harmonic oscillator basis. The two-nucleon force I used is not the same as that of Suzuki. The result showed too deep binding energy for the ground state of ^{16}O. For other states, however, the results agree rather well with the results obtained by OCM. For treating the system in a fully microscopic framework we need to fit the binding energies of three clusters α, ^{12}C, ^{16}O with a single effective two-nucleon force. But at present we have no such effective forces. This is the main reason of the seeming discrepancy between OCM and RGM.

N. Mankoč-Borštnik: 1) How do your results depend on the choice of the parameters of the force?
2) Have you calculated electromagnetic properties? How do the electromagnetic properties look in comparison with experimental data (or with microscopic or other models) and how do they depend on the choice of parameters of the force?

H. Horiuchi: I would like to answer about the results of the special system $^{12}C + \alpha$.
I adopted the effective force between two clusters ^{12}C and α, a folding potential constructed by using Volkov No. 2 force with m = 0.60 in treating $^{12}C + \alpha + \alpha$ system.

This inter-cluster force gives almost the same wave functions for $^{12}C + \alpha$ system as Suzuki's. Suzuki constructed his inter-cluster force differently. In this $^{12}C + \alpha$ system I only calculated α-decay properties and found good similarity between ours and Suzuki's. But in view of the similarity between wave functions I think there is not much difference for other properties.