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On the Relation between GCM and ATDHF

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All microscopic theories of collective motion have in common that they use a collective path $|q\rangle$ (or $|q,p\rangle$), which is a set of deformed Slater determinants depending on a classical collective parameter q (and its conjugate p). There are essentially two classes of approaches. One is the Generator-Coordinate-Method (GCM) which describes the many body wave functions as the superposition $|\Psi\rangle = \int dq\ f(q)\ |q\rangle$ and leads finally to a quantized collective Schrödinger-equation with the mass $M_{GCM} = \langle\ |\{P\{H,P\}\}|\ \rangle / 4\langle P^2\rangle^2$ where $P|\ \rangle = -i\partial_q|\ \rangle$. On the other side, there is the group of more classically minded approaches, thinking in terms of an explicit time evolution of the collective wave packets $|q(t),p(t)\rangle$ and deriving a classical Hamiltonian, $\mathcal{H}_C = \langle q,p|H|q,p\rangle$. This group embraces the static theories for potential-energy surfaces and all the various, more or less refined, versions of cranking, up to the fully consistent ATDHF description[1,2]. The collective mass is here $M^{-1}_{ATDHF} = \langle q|Q,|H,Q||q\rangle$ where $Q|q,p\rangle = -i\partial_p|q,p\rangle$. In general it is different from M_{GCM}. Making use of an explicit dynamical parameters p in the basis, M_{ATDHF} must be considered to be superior to M_{GCM}. One has to pay, however, for the better description of dynamics with the detour over the classical Hamiltonian $\mathcal{H}_C$ which requires a further quantization. Obviously it would be desirable to combine the better basis $|q,p\rangle$ with the thoroughly quantum mechanical description given by the GCM. This leads to

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a GCM with two conjugate parameters which, however, is not easy to handle. A straight-forward expansion, e.g., leads to a collective Hamiltonian $\mathcal{H}_C\ (q,\partial_q,p,\partial_p)$ which depends on too many quantities. It is a sort of double counting, expressing the fact that remnants of non collective motion are left in the basis $|q,p\rangle$ via imaginary parts of q and p (which are unavoidably taken into account by the GCM). One ought to reduce th two parameter GCM to a one parameter GCM, taking care, however, not to destroy the proper dynamics in $|q,p\rangle$[3]. To this end we consider small oscillations about each given point on the collective path. They are described by the RPA-algebra about $|q,p\rangle$, consisting in the four 1p1h operators $Q_0|\ \rangle = -i\partial_p|\ \rangle$, $P_0|\ \rangle = i\partial_q|\ \rangle$, $\mathcal{E}Q_1|\ \rangle = (iP_0 + \lambda Q_0)|\ \rangle$ and $\mathcal{E}P_1|\ \rangle = (iQ_0 - P_0/\mu)|\ \rangle$ (where the operators Q_1 and P_1 are introduced to complete the algebra). This amounts to diagonalize the RPA matrices $\langle[Q_i,[H,Q_j]]\rangle$, $\langle[P_i,[H,P_j]]\rangle$ and can be done analytically. The ground states of the local oscillations provide a new basis $|\tilde{q,p}\rangle = \int dq'dp'\ g(q,p;q',p')\ |q',p'\rangle$. The state $|\tilde{q,p}\rangle$ is the boson vacuum $b_0\ |\tilde{q,p}\rangle = (Q_0 + iP_0/\sigma)\ |\tilde{q,p}\rangle = 0$. Thus we have obtained a redundant basis, since $\partial_q|\ \rangle \sim \partial_p|\ \rangle$ which means that a p-progression is equivalent to a q-progression and therefore the p degree of freedom can be expressed by a purely statical superposition. For a redundant basis it can be shown that the GCM with $|\tilde{q,p}\rangle$ is completely equivalent to the one parameter GCM with $|\tilde{q,o}\rangle$. Thus, by prediagonalizing the local osciallations we have obtained a redundant basis which can reliably be treated in a one parameter GCM leading to a well defined collective Hamiltonian

$$H_{C_i} = +\ :\ \hat{p}^2/2M_{ATDHF}\ :\ + V - \frac{\Delta Q^2}{2} V'' - \frac{\Delta P^2}{2M_{ATDHF}}$$

which is identical to the one obtained from the consistent quantization within ATDHF[2]. The collective paths given by ATDHF or the local harmonic approach[4] are particularly suited to the above GCM treatment, since the local RPA matrices have already been diagonalized in those schemes.

To conclude: 1) From the GCM with two conjugate parameters a collective Schrödinger equation can be derived by means of a transformation to a redundant basis. 2) It is identical to the Schrödinger equation derived by means of consistency agruments purely within ATDHF. 3) Thus we have embedded the classical theories like ATDHF into the framework of the GCM.

References

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