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DWBA TREATMENT OF EXCHANGE EFFECTS IN NUCLEAR COLLISIONS

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1. Introduction. - In this lecture we shall concern ourselves with the consequence of the indistinguishability of nucleons on the transition amplitude for nuclear collisions. In particular, we shall only discuss the analyses of exchange effects in the framework of distorted wave Born approximation (DWBA). The role of exchange effects in nucleon-nucleus collisions and in the collisions of light ions (tritons, alpha) on nuclei has been discussed extensively in the literature[1]. But it is only with the advent of heavier projectiles (^{6}Li,^{16}O) that these effects have taken on a more important role.

In the collisions of nuclei of comparable sizes, the differential cross sections often exhibit a structure that is indicative of two different coherent reaction mechanisms contributing to the reaction. This has been interpreted as due to the interference between the direct reaction and an exchange reaction where a nucleon (or nucleons) is exchanged between the colliding partners. The exchange effects have been treated by the use of resonating group[2] or generator coordinate[3] methods wherein the full antisymmetry of the problem is exploited, or by the method of molecular orbitals[4] or DWBA[5] where specific exchange processes are included and the other ignored.

The DWBA is only applicable if the transition is predominantly a one-step transition. In the case where the exchange reaction involves an exchange of more than one nucleon, a cluster approximation is usually made because of the difficulties of performing a microscopic transfer calculation in DWBA. The predictions of DWBA[5] have been compared with the predictions of LCNO[4] in a few cases. We shall primarily review the DWBA approach.

2. Formalism. - Consider the reaction

$$a + A \rightarrow b + B$$

whe[illegible] contains n nucleons, A contains N-n nuc[illegible], b co[illegible] m nuc[illegible] and B contains (N-m) nucleons. If n = m, it refers to "direct" elastic scattering and if n = N-m, it is "exchange" elastic scattering. The initial asymptotic state $|\Phi_i\rangle$ and the final asymptotic state $|\Phi_f\rangle$ are defined by

$$|\Phi_i\rangle = (2\pi)^{-3/2} \exp(i\vec{k}_a,\vec{r}_{aA})\,|g_a(1\cdots n)\,g_A(n+1\cdots N)\rangle \qquad (2.1a)$$

$$|\Phi_f\rangle = (2\pi)^{-3/2} \exp(i\vec{k}_b,\vec{r}_{bB})\,|g_b(1\cdots m)\,g_B(m+1\cdots N)\rangle \qquad (2.1b)$$

where $g_\lambda(\xi)$ define the internal states of the nucleus λ. The symmetrized total wavefunction of the system is defined by

$$|\tilde{\Psi}_i^{(+)}\rangle = (N_i)^{-\frac{1}{2}} \sum_p \delta_p\, P|\Psi_i^{(+)}\rangle \qquad (2.2a)$$

where it is assumed that the internal states of a and A have been antisymmetrized with respect to their constituent nucleons. N_i is the number of permutation operations P which exchange nucleons between a and A,

$$N_i = \frac{N!}{n!\,(N-n)!} \qquad (2.2b)$$

and δ_p is the parity of P. The unsymmetrized total wavefunction $|\Psi_i^{(+)}\rangle$ is a solution of the integral equation

$$|\Psi_i^{(+)}\rangle = |\Phi_i\rangle + \frac{1}{(E^+ - H_{0i})}\, V_{aA}|\Phi_i^{(+)}\rangle \qquad (2.2c)$$

where H_{0i} is the initial asymptotic Hamiltonian and V_{aA} is the interaction between the nuclei a and A.

The exact transition amplitude is evaluated by first calculating the amplitude for a transition to a final channel where the N nucleons are distributed among a set of m and (N-m) nucleons in a specified order. The total transition amplitude is then obtained by considering all such equivalent partitions, i.e.:

$$\mathcal{T}_{fi} = (N_f)^{\frac{1}{2}} \langle\Phi_f|V_{bB}|\tilde{\Psi}_i^{(+)}\rangle \qquad (2.3a)$$

where

$$N_f = \frac{N!}{m!(N-m)!} \tag{2.3b}$$

Let us consider some specific cases.

i) Elastic Scattering

$$\mathcal{T}_{e\ell} = (N_i)^{\frac{1}{2}} \langle \Phi_i(\vec{k}_f)|V_{aA}|\tilde{\Psi}_i^{(+)}(\vec{k}_i)\rangle$$

$$= \sum_p \delta_p \langle \Phi_i(\vec{k}_f)|V_{aA}\ P|\Psi_i^{(+)}\rangle \tag{2.4}$$

We can label the permutations according to the number of nucleons exchanged between the colliding nuclei. If I is the identity, P(i,n+1) refers to the exchange of the first and (n+1)th nucleons and P(1•••ℓ, n+1•••n+ℓ) refers to the exchange of the set of nucleons (1•••ℓ) with the set (n+1•••n+ℓ), the total elastic transition amplitude becomes

$$\mathcal{T}_{e\ell} = \sum_{\ell=0}^{n}(-1)^{\ell}\binom{n}{\ell}\binom{N-n}{\ell}\langle \Phi_i(\vec{k}_f)|V_{aA}P(1\cdots\ell,n+1\cdots n+\ell)|\Psi_i^{(+)}\rangle \tag{2.5}$$

where we have assumed that $n \le (N-n)$.

ii) Rearrangement Collisions

In the case of rearrangement collisions, there is a mass transfer between the colliding nuclei. One has to interpret the meaning of direct and exchange terms. In the case of stripping, n>m, the direct term is defined as the case where the m nucleons of the nucleus b emerge from the projectile a. The number of ways of achieving this is to include all the permutations P which leave the first m nucleons unchanged, i.e.:

$$\mathcal{T}_{fi}^{direct} = \left(\frac{N_f}{N_i}\right)^{\frac{1}{2}}\binom{N-m}{n-m}\langle \Phi_f|V_{bB}|\Psi_i^{(+)}\rangle$$

$$= \left[\binom{n}{m}\binom{N-m}{n-m}\right]^{\frac{1}{2}}\langle \Phi_f|V_{bB}|\Psi_i^{(+)}\rangle \tag{2.6a}$$

The ℓ nucleon exchange term corresponds to one where (m-ℓ) nucleons of b emerge from the projectile a and the remaining ℓ nucleons of b emerge from the target nucleus A. Hence

$$\mathcal{T}_{fi}^{(\ell)} = (-1)^{\ell} \left(\frac{N_f}{N_i}\right)^{\frac{1}{2}} \binom{N-m}{n-m+\ell} \binom{m}{\ell} \langle \Phi_f | V_{bB} P(1\cdots\ell, n+1\cdots n+\ell \; | \Psi_i^{(+)} \rangle$$

$$= (-1)^{\ell} \left[\binom{N-m}{n-m+\ell} \binom{n}{n-m+\ell} \binom{N-n}{\ell} \binom{m}{\ell} \right]^{\frac{1}{2}}$$

$$\times \langle \Phi_f | V_{bB} \; P(1\cdots\ell, n+1\cdots n+\ell | \Psi_i^{(+)} \rangle \qquad (2.6b)$$

The total rearrangement amplitude is thus

$$\mathcal{T}_{fi} = \sum_{\ell=0}^{m} (-1)^{\ell} \left[\binom{N-m}{n-m+\ell} \binom{n}{n-m+\ell} \binom{N-n}{\ell} \binom{m}{\ell} \right]^{\frac{1}{2}}$$

$$\times \langle \Phi_f | V_{bB} \; P(1\cdots\ell, n+1\cdots n+\ell) | \Psi_i^{(+)} \rangle \qquad (2.7)$$

where the $\ell = 0$ term yields the direct amplitude.

The differential cross section is defined in the usual way, i.e.:

$$\frac{d\sigma}{d\Omega} = \frac{\mu_{aA} \; \mu_{bB}}{(2\pi h^2)^2} \frac{k_b}{k_a} \, \Sigma | \mathcal{T}_{fi} |^2 \qquad (2.8)$$

where Σ refers to the summing and averaging over the initial and final states, i.e.:

$$\Sigma | \mathcal{T}_f |^2 = \frac{1}{(2J_A+1)(2S_{a+1})} \sum_{\substack{M_A \mu_a \\ M_B \mu_b}} |\mathcal{T}_{fi}|^2 \qquad (2.9)$$

where J_A, J_B are the spins of the nuclei A and B, and S_a, S_b are the spins of a and b and M_A, M_B, μ_a, μ_b are the respective Z-components of J_A, J_B, S_a and S_b.

3. DWBA. - The DWBA expression is obtained from the exact transition amplitude (2.3a) and has the form

$$\mathcal{T}_{fi}^{DWBA} = \left(\frac{N_f}{N_i}\right)^{\frac{1}{2}} \Big[\delta_{if} \langle \chi_f^{(-)} | U_{bB} | \Phi_i \rangle$$

$$+ \sum_p \delta_p \langle \chi_f^{(-)} | (V_{bB} - U_{bB}) P | \chi_i^{(+)} \rangle \Big] \qquad (3.1)$$

where U_{bB} is an auxiliary potential chosen to reproduce the elastic scattering of the nuclei b and B, $|\chi_f^{(-)}\rangle$ representing the corresponding elastic scattering wavefunction, and $|\chi_i^{(+)}\rangle$ is the elastic scattering wavefunction for the pair of nuclei a and A.

(i) Elastic Scattering

$$\mathcal{T}_{fi}^{e\ell} = \langle \chi_f^{(-)} (\vec{k}_f) | U_{aA} | \Phi_i (\vec{k}_i) \rangle$$

$$+ \sum_{\ell=0}^{n} (-1)^{\ell} \binom{n}{\ell} \binom{N-n}{\ell} \langle \chi_f^{(-)} (\vec{k}_f) | V_{aA} \, P(1-\ell, n+1-n+\ell) | \chi_i^{(+)} \rangle \qquad (3.2)$$

The total direct term consists of the potential scattering term and the $\ell=0$ (no nucleon exchange) part of the second term: All the exchange contributions arise from the second term on the right hand side of eq. (3.2). The usual practice is to consider only one exchange process corresponding to the exchange of the cores i.e.; exchange of n nucleons, i.e.:

$$\mathcal{T}_{fi}^{e\ell} \simeq \Big[\langle \chi_i^{(-)} (\vec{k}_f) | U_{aA} | \Phi_i (\vec{k}_i) \rangle + \langle \chi_i^{(-)} (\vec{k}_f) | (V_{aA} - U_{aA}) | \chi_i^{(+)} (\vec{k}_i) \rangle$$

$$+ (-1)^n \langle \chi_i^{(-)} (\vec{k}_f) | V_{aA} - U_{aA}) P(1 \cdots n, n+1 \cdots 2n) | \chi_i^{(+)} (\vec{k}_i) \rangle \qquad (3.3)$$

If one ignores the mass of the valence nucleons relative to that of the core (no-recoil approximation), the effect of the exchange of nucleons is equivalent to making a parity transformation on the relative center-of-mass vector $\vec{r}_{aA} \simeq \vec{r}_{aa} \simeq \vec{R}$. The transition amplitude then has the form[6)]

$$\mathcal{T}_{fi} \simeq \Big[\langle \chi_i^{(-)} (\vec{k}_f) | U_{aA} | \Phi_i (\vec{k}_i) \rangle + \langle \chi_i^{(-)} (\vec{k}_f) | (V_{aA} - U_{aA}) | \chi_i^{(+)} (\vec{k}_i) \rangle \Big]$$

$$+ (-1)^n \langle \chi_i^{(-)} (\vec{k}_f, \vec{R}, \vec{r}_f) | V_{ax} (\vec{r}_i) | \chi_i^{(+)} (\vec{k}_i, -\vec{R}, \vec{r}_i) \rangle \qquad (3.4)$$

where x refers to the cluster of the (N-2n) valence nucleons, and the coordinates are defined in fig. 1.

The no-recoil approximation, eq. (3.4), has been used in the analysis of elastic scattering of ^{12}C and ^{13}C and that of ^{28}Si and ^{29}Si where the exchange process was a neutron transfer between identical cores. The success of this analysis seems surprising in view of the fact that recoil effects in single nucleon transfer reactions are known to be very important[7)]. It should be noted that only the L = 0 (L = transfer orbital angular momentum) part of the transfer amplitude interferes with the direct elastic scattering amplitude, and all other L add incoherently. If the interference structure is successfully reproduced in the no-recoil approximation, the recoil effects should not change the phase of the transfer amplitude. It is worthwhile investigating the importance of recoil effects on the L = 0 part of the elastic transfer amplitudes.

There have been several calculations of elastic transfer processes including two nucleon and four nucleon (α particles) transfer which have been considerably successful. This raises the question of the possibility of multiple transfer. The answer to this question has been partially answered[7)]. If the optical potentials are strongly absorbing potentials, the probability of multiple transfer becomes small and DWBA becomes adequate.

We show below a DWBA analysis of elastic scattering of ^{6}Li and d with the inclusion of exchange effects[9)]. The exchange in this case is an alpha transfer reaction. The results are shown in Fig. 2 and 3. Unlike the case of heavy ion induced transfer reactions, the light ion induced transfer reaction is volume process and not a peripheral process.

ii) Rearrangement Collisions

Once again, the only exchange process usually considered is one where the cores are exchanged, i.e., in the reaction a + A → b + B, the exchange process is one where m nucleons are exchanged between a and A. The transition amplitude is thus

$$\mathcal{T}^{DW}_{fi} \cong \mathcal{T}^{D}_{fi} + \mathcal{T}^{ex}_{fi} \quad (3.5a)$$

$$\mathcal{T}_{fi}^{D} = \left[\binom{n}{m} \binom{N-m}{n-m} \right]^{\frac{1}{2}} \langle \chi_f^{(-)} | V_{bB} - U_{bB} \rangle | \chi_i^{(+)} \rangle \tag{3.5b}$$

and

$$\mathcal{T}_{f}^{ex} = (-)^m \left[\binom{N-m}{n} \binom{N-n}{m} \right]^{\frac{1}{2}} \langle \chi_f^{(-)} | (V_{bB} - U_{bB}) P(1\cdots m, n+1\cdots n+m) \chi_i^{(+)} \tag{3.5c}$$

The exchange term is a sum of a knock-out term and a heavy particle stripping term[10].

The analysis[11] of the reactions $^{11}B(^{16}O,^{15}N)^{12}C$, $^{12}C(^{6}Li,\alpha)^{14}N$ and $^{16}O(^{6}Li,\alpha)^{18}F$ are shown in Fig. 4.

4. Other Symmetry Effects. - In the special case where the pair of nuclei in the final (or initial) state are members of the same symmetry multiplet, one can make simple predictions on the symmetries of the angular distribution. In particular if the final nuclei are members of the same isospin multiplet and if the total isospin of the system is unique, then one expects a symmetry of the differential cross section around 90° [12,13] provided isospin is conserved in the reaction. The reaction $^{13}C(^{15}N,^{14}N)^{14}C$ was analyzed[14] where the expected symmetry around 90° was obsefved and the small asymmetry was explained in terms of the interference of the proton and neutron transfer where an isospin violation is possible. The reaction $^{12}C(^{14}N\ ^{13}N)^{13}C$ [15], however, exhibited features which could not be accounted for by DWBA. The energy dependence of the interference pattern could not be reproduced nor could the strong asymmetry around 90°. The asymmetry in the angular distribution could possibly be obtained by a careful assessment of the Coulomb interactions in the reaction but it seems unlikely that this could at the same time account for the strong energy dependence of the interference pattern.

In the reaction $^{13}C(^{15}N,^{14}N)^{14}C$ [14], a similarity was observed in the transitions leading to ^{14}N in its ground state and first excited state. This seems to indicate that these states are members of a supermultiplet

5. Cluster Transfer Model - If more than one nucleon were transfered either in elastic or rearrangement collisions, the transfer amplitude is usually evaluated as if the nucleons are transferred as a cluster. The spectroscopic amplitude for the cluster decomposition is defined in the case

$$B = A + x$$

where B contains (N-m) nucleons, A contains (N-n) nucleons and x is a cluster of (n-m) nucleons, by

$$\binom{N-m}{n-m}^{\frac{1}{2}} \int d\xi_A \, \Phi^*_{J_B M_B}(\xi_B) \Phi_{J_A M_A}(\xi_A)$$

$$= \sum_{JM} \langle J_A M_A \; J M | J_B M_B \rangle$$

$$\times \sum_{qQSL} A_{AB}((J_A)(LS)J \; J_B) \left[\phi^{qS}(\xi_x) \phi^{QL}(\vec{R}_{xA}) \right]^J_M$$

where q and Q denote the oscillator quanta contained in the internal motion of the cluster and its motion relative to the center-of-mass of A respectively. S is the intrinsic spin of the cluster.

The binding between x and A usually simulated in the DWBA by choosing a Woods-Saxon potential which yields the empirical separation energy for their breakup. The amplitudes A_{AB} are obtained by treating them as parameters used to adjust the magnitudes of the theoretical cross-sections. The amplitudes thus obtained are compared with the predictions of structure calculations. Extensive studies of the α amplitudes in the 1p shell nuclei have been made and compared with Kurath's predictions.

(Wigner supermultiplet) and that the interaction causing the transition is spin and isospin independent. A more quantitative analysis would possibly yield an estimate of the degree of violation of the spin-isospin symmetry.

6. Discussion. - There are several points of interest that arise in the context of DWBA analyses of heavy ion collisions.

i) The most important question obviously relates to the validity of DWBA. This leads back to the question of elastic scattering potentials. Is it possible to determine whether they are strongly absorbing or weakly absorbing potentials?

ii) The question of recoil effects in elastic transfer reactions have to be investigated in more detail. The interference structure in elastic scattering is due to the coherence of the direct amplitude with the $L = 0$ part of the transfer amplitude. It should be of interest to study the effect of recoil on the phase and magnitude of the $L = 0$ part of the transfer amplitude. (The LCNO model implicitly makes a no-recoil approximation.)

iii) A well known inadequacy of DWBA is its failure to predict the angular distribution in the case of charged particle transfer when the Q matching is poor[16]. The discrepancy between DWBA and experiment becomes larger as the charge of the transferred particle increases. This effect should thus be present in the case of elastic transfer of a proton or alpha particle.

iv) The DWBA, unlike GCM or RGM, does not include all the possible exchange amplitudes. There are interesting reactions where the possibility of a partial exchange of the cores may be important. Consider the reaction $^{16}O + {}^{18}F \rightarrow {}^{16}O + {}^{18}F^*$. The usual DWBA calculation includes the direct inelastic scattering and the core exchange term

$$^{16}O_1 + \underbrace{(^{16}O_2 + d)}_{^{18}F} \rightarrow {}^{16}O_2 + \underbrace{(^{16}O_1 + d)}_{^{18}F^*}$$

There are some states of ^{18}F which have a large alpha cluster amplitude[17]. One can in this case have a possible reaction

$$\underbrace{({}^{2}C + \alpha_1)}_{{}^{16}O} + \underbrace{({}^{14}N + \alpha_2)}_{{}^{18}F} \rightarrow \underbrace{({}^{12}C + \alpha_2)}_{{}^{16}O} + \underbrace{({}^{14}N + \alpha_1)}_{{}^{18}F*}$$

which involves a four nucleon exchange. One would normally expect this process to be less important because it involves four spectroscopic factors instead of two in the case of a deuteron exchange. If, however, the particular final state has a large α width and a negligible deuteron width, the reaction proposed may be important. At the present state of DWBA technology, the reaction amplitude is very difficult to evaluate. In contrast, the RGM and GCM calculations would automatically include these processes. If the RGM and GCM techniques are extended to include many channel effects through the introduction of optical potentials and are modified to treat collisions of nuclei heavier than α particle, it would provide a more accurate treatment of exchange effects.

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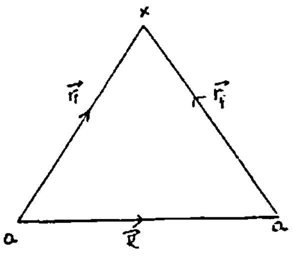

Fig. 1 Coordinate system for elastic transfer in the reaction

$a_2 + \underbrace{(a_1 + x)}_{A} \rightarrow a_1 + \underbrace{(a_2 + x)}_{A}$

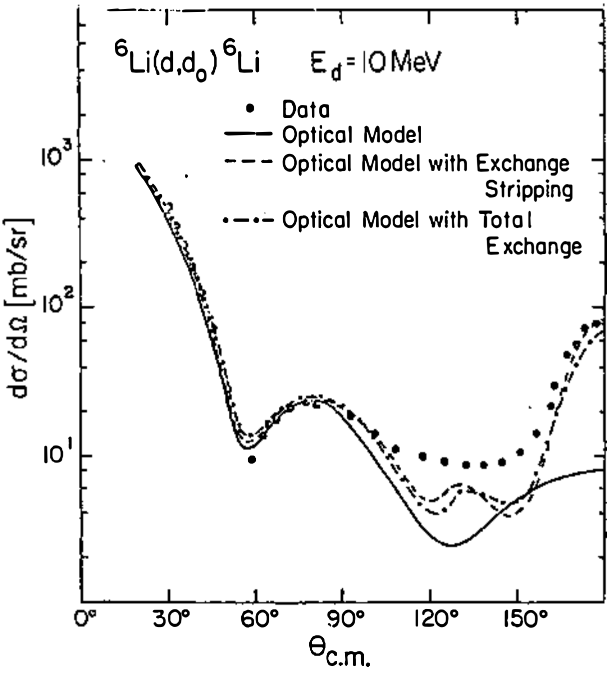

Fig. 2.

Comparison of optical model calculations, optical model calculations with exchange stripping and optical model calculations with total exchange with data at 10 MeV bombarding deuterons.

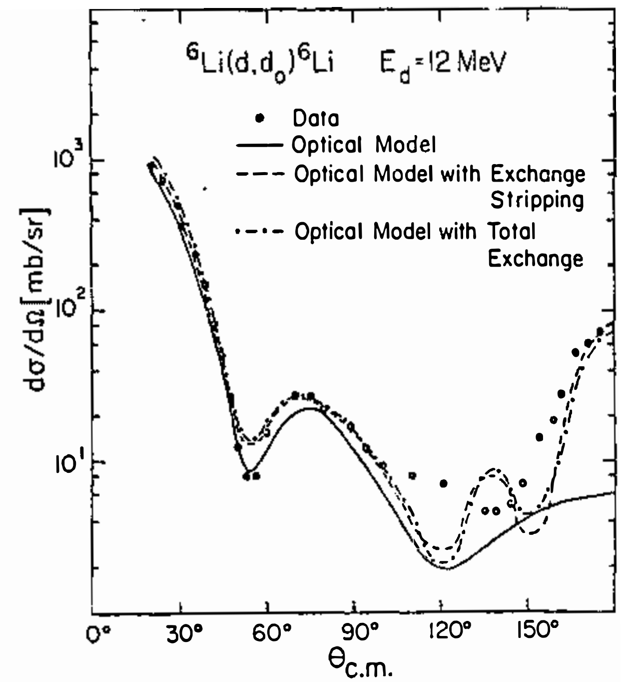

Fig. 3.

Comparison of optical model calculations, optical model calculations with exchange stripping and optical model calculations with total exchange with data at 12 MeV bombarding deuterons.

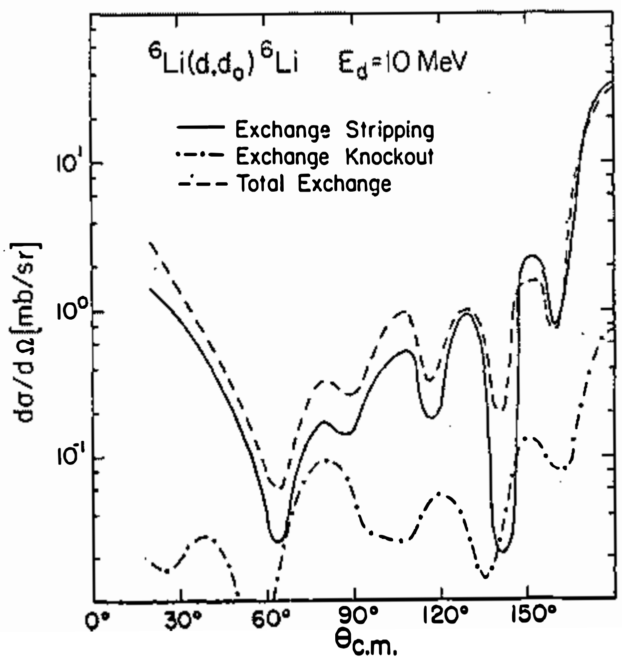

Fig. 4.

Comparison of exchange stripping, exchange knockout and total exchange contributions at 10 MeV bombarding deuterons.

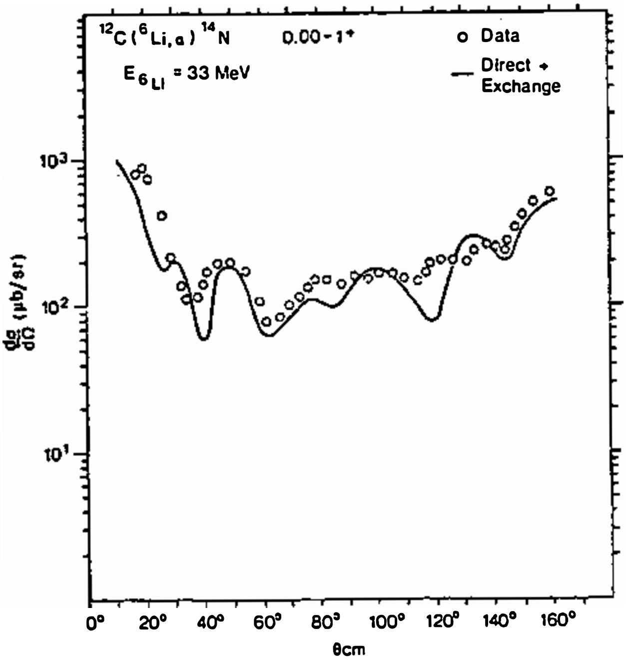

Fig. 5.

Comparison of experimental and calculated differential cross-section for $^{12}C(^{6}Li,\alpha)^{14}N$ ground state transition. The calculation was done including the direct and exchange mechanisms.

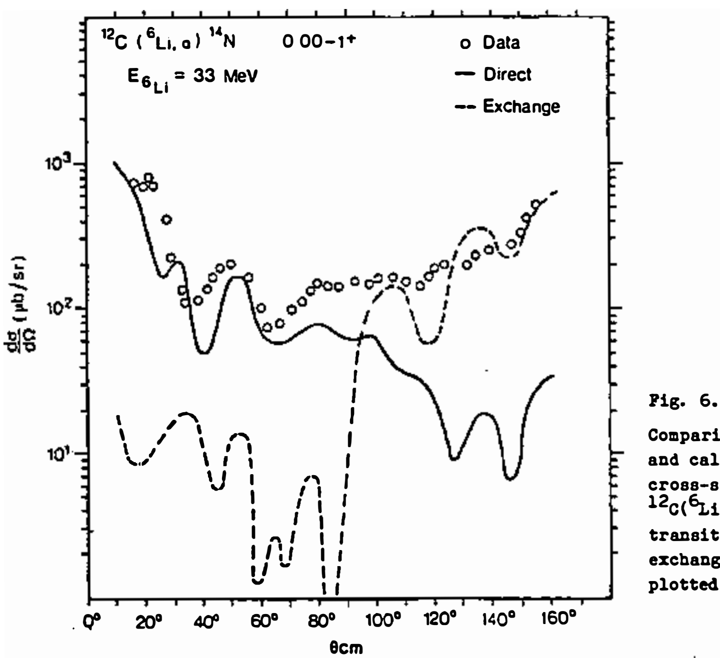

Fig. 6.

Comparison of experimental and calculated differential cross-section for $^{12}C(^{6}Li,\alpha)^{14}N$ ground state transition. The direct and exchange components are plotted separately.

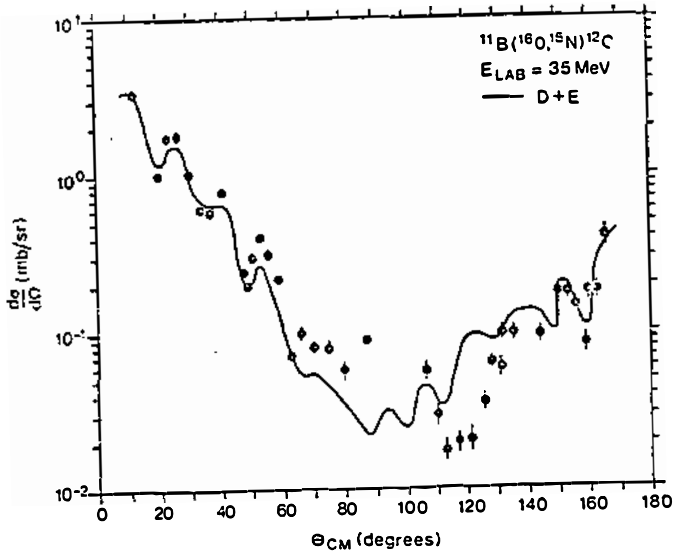

Fig. 7.

Comparison of experimental and calculated differential cross-section for $^{11}B(^{16}O, ^{15}N)^{12}C$ ground state transition. The calculated cross-section includes both direct (proton transfer) and exchange (α transfer) mechanisms.

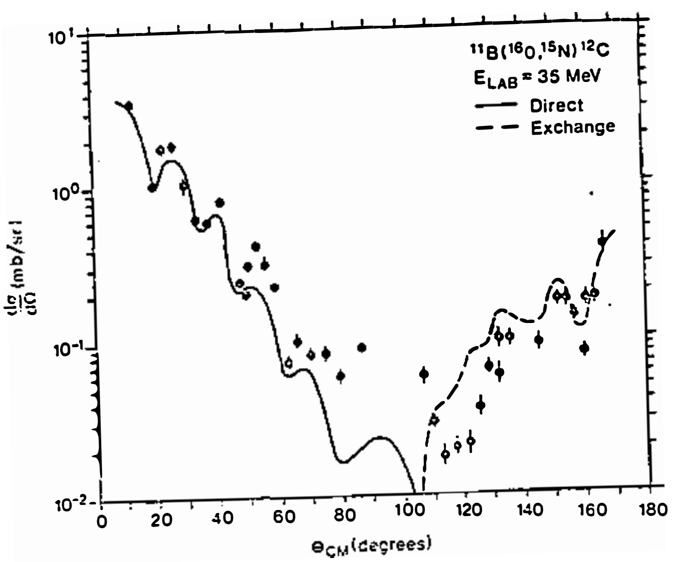

Fig. 8.

The direct and exchange contributions shown separately for the reaction $^{11}B(^{16}O, ^{15}N)^{12}C$ ground state transition.

DISCUSSION

K. Dietrich: I wish to say that I share your scepticism concerning a multiple exchange of nucleons or clusters of nucleons in these light ion reactions. If this multiple exchange mechanism were to exist I would expect that it is strongly energy-dependent since the penetrability of the barrier between the interacting nuclei depends strongly on the energy of relative motion. Has such an energy-dependence been observed?

M.A. Nagarajan: No! To my knowledge, the type of energy dependence you refer to has not been observed.

G. Delić: Did you make any comparison of your calculations for ^{11}B (^{16}O, ^{15}N) ^{12}C with those which I made (see Phys. Rev.Lett.36 (1976) 569)?

M.A. Nagarajan: I am sorry. I was not aware of your paper.