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TREATMENT OF THE ANTISYMMETRIZATION EFFECTS IN NUCLEAR COLLISIONS BY PROJECTION OPERATOR TECHNIQUES.

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Abstract. The work consists of three parts. Part 1 formulates the Pauli projecting in terms of strict theory of many-body scattering. The concept of ghost states is inserted as a dynamic generalization of the states forbidden by the Pauli principle in terms of the resonating group method; the integral equations of many-body scattering are projected on the allowed-state space. In Part 2, Saito's model of orthogonality conditions (OCM) is generalized to a three-composite-particle system and particularly to an approximate inclusion of the Pauli principle in the theory of direct nuclear reactions. Part 3 presents the construction of the projection techniques and the derivation of the integral equations that make it possible to include not only the two-body but also three-body exchange effects. The relationship between the forbidden states and the convergence of the Born series for elastic and rearrangement scattering is briefly discussed. Consideration is also given to the methods for solving the derived equations.

1. The aim of the projection approach

Numerous microscopic calculations of the collision dynamics in the systems of two or several clusters have been carried out recently (mainly by H.Hackenbroich and his group /1/, Tang et al /2/, and some Japanese groups /3/). The relevant calculations of the spectra in many-cluster systems (such as ^{9}Be, ^{12}C, etc) /4/ should also be noted. The results of the calculations are clearly indicative of adequacy of the physical model (the cluster model with antisymmetrization and its mathematical realization through the resonanting group method (RGM) and the generator coordinate method (GCM)) applied to the treatment of such phenomena. At the same time, the RGM calculations are fairly tedious and, what is of greatest importance, such approach can hardly be applied to reveal the relationships between the basic physical factors affecting the results.

The above mentioned drawbacks become much more gravier when dealing with more complex systems composed of several fragments.In this case, even high-speed computer calculations consume tens of hours (!) of computer time and require a long series of programs/1,5/.

A successful attempt to overcome the above mentioned difficulties and to reveal the simple regularities in the problems of cluster-cluster scattering was Saito's model of orthogonality conditions/6/ and the optical model based on the microscopically substantiated local potentials /7,8/.It is assumed in these approaches that the main antisymmetrization effects can be reduced to exclusion of the forbidden states from the spectrum of the hamiltonian of the system or, in other words, to orthogonalization of the scattering wave functions (and the corresponding operators) to the forbidden states.It is also assumed that the exchange effects left after such exclusion, i.e.the allowed state exchanges, are weak and may be treated by the perturbation theory.This very attractive possibility of considerably simplifying the complex problem of many-body scattering has not been, however, realized as yet in practice mainly, as I think, because of two reasons, at first, in the overwhelming majority of real cases instead of strictly forbidden and allowed states it appears a lot of so called semiforbidden states which can give rise to such essential difficulties, as abnormal resonances etc./9/, and secondly,up to now there was no any appropriate orthogonality projecting technique applicable to consideration the off-shell effects in few-cluster systems For ex-ample, it can be shown/10/ that Saito's model (formulated in terms of the conventional projection technique) does not exclude all the poles corresponding to the forbidden states from Green function (GF) off mass shell but treats the poles as located at zero energy.

Recently, we have developed the projection operator techniques/10/ which are an alterhative to the Feshbach technique and can be readily used to solve a large class of the scattering theory problems and, in particular, to realize the above mentioned perturbation treatment of the exchange effects. Double application of the mentioned projection technique will be outlined in the present report. At first, we shall examine the Pauli projecting and the perturbation theory for residual exchange effects in two non-trivial and exactly solvable problems where the Pauli principle is of crucial importance, namely the quartet neutron-deutron scattering through the Pauli projection of the Faddeev equations (FE) and the quintet deuteron-deuteron scattering (through the Yakubovsky-Faddeev four-body equations). These exactly solvable problems will exemplify the exact meaning of the forbidden states in the many-body scattering theory (which mean the generalization of the eigenstates of Feshbach's antisimmetrization kernel), and such treatment will then make it possible to indicate the dubious aspects of the forbidden states in RGM. Besides that some relationships between the Pauli projecting and the Born series convergence in the general theory of many-body scattering will be discussed

The second application deals with the generalization of Saito's model of orthogonality condition to the three-cluster (and more complex) systems. I shall made an attempt here to show that the two-body Pauli projecting in the FE's is quite sufficient to include the two-body exchange effects in approximate manner, thereby avoiding the complicated three-body operators, which are usually inserted into the trial functions instead of total antisymmetrization over all the nucleons/11/ in the course of the variational calculations of three-cluster system. In the stead, the pure three-body projectors composed of three-body wave functions of the forbidden states will be inserted as the OCM-generalization to replace the three-body exchange effects. Such generalization can be readily carried out using our technique of orthogonal projecting. More than that, it will be shown that immediately integrable equations can obtained by increasing the dimensionality of the three-body projectors.

2. The ghost states and the Pauli projecting in the scattering theory.

The n-d and d-d scatterings will be examined here on the basis of strict integral equations from the viewpoint of the Pauli principle to clarify the general situation with the Pauli projecting in more complex systems where the rigorous many-body treatment is not yet practically realizable (though the formal equations for scattering of $\underset{\sim}{N}$ identical particles have been proposed/12/).The appoach as a whole will be based on the concept of ghost states which are of extreme importance to the said treatment.It should be noted that the very idea of non-physical states in the three-body scattering theory has been known since long ago (and was first inserted in the work by Lovelace in 1964/13/) but was never applied in practice.Consider first the low-energy n-d scattering in the quartet channel where the two-neutron interaction can be safely (due to the parallelizm of the neutron spins) and the rest two n-p interactions are the S-wave and triplet ones.Further, we shall call the states $\underline{\Psi}_a$ antisymmetric relative to the transposition P_{12} of two identical neutrons the physical states, and the states $\underline{\Psi}_s$ is symmetric relative to P_{12}-the ghost states.

The FE's for the wave function describing the scattering of particle k(1 or 2) on the bound state of two other particles are

$$\begin{pmatrix} \underline{\Psi}^{(1)} \\ \underline{\Psi}^{(2)} \end{pmatrix} = \begin{pmatrix} \alpha_1 \underline{\varphi}^{(1)} \\ \alpha_2 \underline{\varphi}^{(2)} \end{pmatrix} + \begin{pmatrix} 0 & G_0 T_1 \\ G_0 T_2 & 0 \end{pmatrix} \begin{pmatrix} \underline{\Psi}^{(1)} \\ \underline{\Psi}^{(2)} \end{pmatrix} \tag{1}$$

where $\underline{\Psi}^{(1)}$ and $\underline{\Psi}^{(2)}$ are the Faddeev components of the total wave function $\underline{\Psi} = \underline{\Psi}^{(1)} + \underline{\Psi}^{(2)}$, $G_0=(E-H_0)^{-1}$-free GF; $T_i=V_i+V_iG_iV_i$ is the two-body t-matrix; $\underline{\varphi}^{(1)}=|\underline{\varphi}_{23}, \underset{\sim}{p}_1(0)\rangle$ (and, similarly, $\underline{\varphi}^{(2)}$) is the initial state wave function, where $\underline{\varphi}_{23}$-is the bound state of the system (2+3) which is here a deuteron with energy $\underline{\epsilon}_{23}$ so that $(p_1^{(0)})^2/2\mu + \underline{\epsilon}_{23}=E$; the factors $\underline{\alpha}_i = \underline{\delta}_{ik}$.Since the equation (1) is linear, any solution of eq.(1) may be expressed through the symmetric and antisymmetric solutions satisfying the equation

$$\underline{\Psi}_{s,a}^{(i)} = \underline{\varphi}^{(i)} \pm G_0 T_i P_{12} \underline{\Psi}_{s,a}^{(i)} \tag{2}$$

where

$$\underline{\Psi}_{s,a}^{(2)} = \pm P_{12} \underline{\Psi}_{s,a}^{(1)} \tag{3}$$

since $T_2=P_{12}T_1P_{12}$ due to the identity of particles 1 and 2.The sign +and - in eq.(3) corresponds to the subscripts $\underset{\sim}{s}$ and $\underset{\sim}{a}$ so that the Schrödinger total wave function is

$$\underline{\Psi}_{s,a} = (1 \pm P_{12})\, \underline{\Psi}^{(1)}_{s,a}$$

The matrix kernel of the FE

$$\hat{K}_2 = \begin{pmatrix} 0 & G_0 T_1 \\ G_0 T_2 & 0 \end{pmatrix}$$

has two sets of eigenvalues and, correspondingly, two sets of eigenfunctions of the $\underset{\sim}{s}$ and $\underset{\sim}{a}$ type, both types being contained in the relations of completeness and orthogonality/14/. Thus we have the eigenvalues equation;

$$\hat{K}_2 \hat{\underline{\chi}}_n^{s,a} = \pm \underline{\alpha}_n \hat{\underline{\chi}}_n^{s,a} \tag{4}$$

where the signs + and - are for the $\underset{\sim}{s}$ and $\underset{\sim}{a}$ respectively; $\hat{\underline{\chi}}_n^{s,a}$ is the two-line column.

Since, however, each kernel may be expanded in its system of eigenfunctions, the Faddeev kernel $\hat{K}_2$ is broken into two parts $\hat{K}_2^s$ and $\hat{K}_2^a$ either of them affects the spaces of the functions of its own symmetry.Then the total resolvent is also a sum of the resolvents of the two parts.It is known, however, that the bound states correspond to the eigenvalues $\underline{\alpha}_n=\pm1$, with $\underline{\alpha}_n=+1$ for the symmetric bound states (i.e.the ghosts) for which $P_{12}\underline{\Psi}_s=\underline{\Psi}_s$ and $\underline{\alpha}_n=-1$ for the true bound states, where $P_{12}\underline{\Psi}_a=-\underline{\Psi}_a$ and, hence, "-" should be used in eq.(2).Therefore, the ghost states correspond to the poles in the symmetrie part of the resolvent while the physical bound states relate to the poles in the antisymmetric part. Since these singularities belong to different spaces and the symmetric states (ghosts) can be explicitly distingnished from the antisymmetric states, the ghost states whould have seemed to fail to affect at all the scattering in the physical (antisymmetric) channel.

A bit more thorough examination of the scalar equation (2) shows, however, that this is not the fact.If, in fact, the symmetric channel comprises a bound state with energy E_0 (i.e. a ghost) then $\alpha_0(E_0)=+1$ and hence the Born series for the equation $\underline{\Psi}^{(1)}_s=\underline{\varphi}^{(1)}+G_0T_1P_{12}\underline{\Psi}^{(1)}_s$ will be divergent.On the other hand, it is obvious that the Born series for the antisymmetric solution i.e. that for the equation $\underline{\Psi}^{(1)}_a=\underline{\varphi}^{(1)}-G_0T_1P_{12}\underline{\Psi}^{(1)}_a$

will also be divergent since the kernel $-G_0 T_1 P_{12}$ has at least a higher-modulus eigenvalue $|\alpha_0| > 1$. In other words, the ghost will result in the Born series divergence for the physical channel too and exert, in this respect, the same effect on the scattering as the "physical" bound state. The situation outined above is identical with the potential scattering by a strong repulsive potential which is known to give rise to quite the same divergence of the Born series for the Lippmann-Schwinger equation as the attractive mirror potential with a great number bound states, since in both cases the kernel G_0V will be large and differ only by its sign. Specific calculations made in terms of the three-body model with separable Yamaguchi potential (fitted to the triplet data) show that the quartet triton (i.e. the spatially symmetrical state) has energy $E_0 \approx -5.5$MeV.

Now, let us construct an iteration series for equation (2) in a subspace orthogonal to the ghost state. With this purpose we shall briefly outline the orthogonality projection technique using orthogonalyzing pseudo-potentials (OPP)/10,15/.

Let the equation $\mathcal{L}\underline{\Psi}=0$ be necessary to solve with additional condition of solution orthogonality to some subspace $\{\underline{\varphi}_1, \underline{\varphi}_2, \ldots \underline{\varphi}_n\}$ of mutually orthogonal vectors $\underline{\varphi}_i$. This problem may be solved by the method of Lagrange multipliers following the frequently used procedure; it is more convenient, however, to use the pseudooperator $\widetilde{\mathcal{L}}_\lambda = \mathcal{L} + \underline{\lambda}\underline{\Gamma}$ where $\underline{\Gamma}$ is the orthoprojector onto the subspace $\{\underline{\varphi}\}$, i.e. $\Gamma = \sum_i |\underline{\varphi}_i\rangle\langle\underline{\varphi}_i|$; $\underline{\lambda}$ is the constant. Then, the equation $\widetilde{\mathcal{L}}_\lambda \widetilde{\underline{\Psi}} = 0$ should be solved already in the total subspace and, eventually, the limit $\underline{\lambda} \to \infty$ must be taken. It can be easily shown/10/ that in this case one obtains the solution $\widetilde{\underline{\Psi}}$ which is explicitly orthogonal to the subspace of the vectors comprised in $\underline{\Gamma}$. Such orthogonality projecting method is convenient to the scattering theory because it is not necessary first to construct the scattering basic in truncated space and, instead the operations in the complete space are quite possible thereby automatically giving the scattering operators in the orthogonal subspace in the limit $\lambda\to\infty$. Besides that, the addition $\lambda\Gamma$ is actually a separable potential resulting in simple algebraic manipulations.

Let as examine, as an example, free GF $G_0(E)=(E-H_0)^{-1}$ and use the above described technique to construct GF $\tilde{G}_0$ orthogonal to the subspace Γ. The ansatz described above gives immediately that

$$\tilde{G}_0(E) \underset{\lambda\to\infty}{=} (E-H_0-\lambda\Gamma)^{-1} = G_0 - G_0\Gamma(\Gamma G_0\Gamma)^{-1}\Gamma G_0 \qquad (5)$$

where the matrix $(\Gamma G_0\Gamma)^{-1}$ is inverse to the matrix $\langle\psi_i|G_0|\psi_j\rangle$. It can be easiby confirmed that $\Gamma\tilde{G}_0(E)=0$, i.e. the GF $\tilde{G}_0(E)$ exhibits the required orthogonality to the subspace Γ. We have successfully used such projection technique to rearrange the Born series of the scattering theory/10,15/ on the basis of the well known fact that the scattering functions for the Hermitian hamiltonian are orthogonal to the functions of the discrete spectrum for any particle number. In this case, the considerable decrease of the integral kernels will result in convergence of the Born series at all energies incluiding the lowest ones, and at any number off bound states in the system (for the interactions satisfying certain constraints, see in/10/). In particular such orthogonality projecting for FE's gives the equation whose iterations will be convergent even at the lowest energies. Instead of the operators $G_0T_i=G_iV_i$ (G_i is channel GF), the kernels of orthogonalized equations comprise the operators $\tilde{G}_iV_i=1-G_i\Gamma(\Gamma G_i\Gamma)^{-1}G_0T_i-(1-Q_i)G_0T_i$ where Q_i is a projection operator, though other then an Hermitian one.

In the case of interest to us, the projection should be onto the eigenstates of the three-body hamiltotian, i.e. onto the ghosts whose symmetry differs from the symmetry of the scattering states. The projecting technique for this case should be somewhat modified. Thus, in the examined case $\Gamma=|\Phi\rangle\langle\Phi|$ where Φ is the wave function of the ghost state (which is the quartet "triton" function for the n-d scattering), and then the projected FE is of the form,

$$\tilde{\Psi}_a^{(i)} = \tilde{\varphi}^{(i)} - \tilde{K}_i\tilde{\Psi}_a^{(i)} \qquad (6)$$

where $\tilde{K}_i = (1-Q_i)K_i = (1-Q_i)\, G_0 T_i P_{12}$

and $\tilde{\underline{\varphi}}^{(i)} = \underline{\varphi}^{(i)} - G_i \underline{\Gamma}(\Gamma G_i \Gamma)^{-1} \underline{\Gamma}\,\underline{\varphi}^{(i)} = (1-Q_i)\underline{\varphi}^{(i)}$

is the "plane" orthogonalized wave. It will be emphasized that the projecting does not change the total (Schrödinger) wave function

$$\tilde{\underline{\Psi}} = \underline{\Psi} - G\underline{\Gamma}(\Gamma G \Gamma)^{-1}\underline{\Gamma}\,\underline{\Psi} \equiv \underline{\Psi}$$

since, as it was said above, $\underline{\Gamma}\,\underline{\Psi} = 0$ for the Hermitian Hamiltonian.

The iterations of the three-body equation (6) may be rigorously proved/15/ to converge from zero energy and, after that, the scattering amplitude will be found from the expresion,

$$F_a = -\frac{\mu}{2\pi}\langle \underline{\varphi}^{(i)} | G_i^{-1}(-K_i) | \tilde{\underline{\Psi}}_a^{(i)} \rangle + \frac{\mu}{2\pi}\langle \underline{\varphi}^{(i)} | \underline{\Gamma}(\Gamma G_i \Gamma)^{-1} \underline{\Gamma} | \underline{\varphi}^{(i)} \rangle \qquad (7)$$

It will be noted that the iterations begin from the zero-order term

$$\tilde{F}_0 = \frac{\mu}{2\pi}\langle \underline{\varphi}^{(i)} | \underline{\Gamma}(\Gamma G_i \Gamma)^{-1} \underline{\Gamma} | \underline{\varphi}^{(i)} \rangle \qquad (7a)$$

which does not contain the kernel K_i at all.

The subsequent iterations of (6) for the wave function give the corresponding series for the scattering amplitude F_a. Since the zero-order amplitude $\tilde{F}_0$ fails to contain the three-body rescattering processes (the kernel K_i is absent) and comprises only the inhomogeneous term of the equation, the amplitude $\tilde{F}_0$ essentially describes the scattering by orthogonality condition (to the ghost) and is the three-body generalization of the orthogonality scattering amplitude introduced by Shakin et al/16/ Fig.1 presents the convergence of the iteration series of eq.(7) for the n-d scattering, and Fig.2 shows the comparison between the zero-order approximation and the exact phase. In case of a single ghost state $\underline{\Phi}_G$ (as for the real n-d and d-d cases), the $\mathcal{I}$ amplitude of zero-order approximation (7a) becomes

$$\tilde{F}_0 = \frac{\mu}{2\pi}\,\frac{\langle \underline{\varphi}^{(i)} | \underline{\Phi}_G \rangle \langle \underline{\Phi}_G | \underline{\varphi}^{(i)} \rangle}{\langle \underline{\Phi}_G | G_i | \underline{\Phi}_G \rangle} \qquad (7b)$$

This amplitude gives a ~~very good description~~ of the quartet length of the n-d scattering and the low-energy S-phases (the difference from the exact results and from the experimental data does not exceed 7%). Thus, the simple orthogonalization of the plane wave to a ghost state permits the quartet low-energy n-d scattering to be explained.

It will be noted that even if the pole part of two-body GF in the deutron pole only is substituted in (7b), the approximation will remain of high quality, in particular the amplitude will be still unitary. In this case, $\tilde{F}_0$ will contain not the ghost function $\underline{\Phi}_G$ but its overlap with the deutron function, i.e. the formfactor $\langle \underline{\varphi}^{(i)} | \underline{\Phi}_G \rangle$ and we shall return to Saito's simplified model of orthogonality condition with $V_D=0$. In contrast to Saito's approach based on RGM, however, we always deal with explicitly defined one-particle states, to which the orthogonalization is to be made and which are merely the ghost-state formfactors $\langle \underline{\varphi}_d | \underline{\Phi}_G \rangle$. Now we shall examine at greater length the relation between the ghosts and the eigenstates of the antisymmetrization kernel $K(\underset{\sim}{R},\underset{\sim}{R}')$. These eigenstates are known to be determined /17/ by the equality

$$K|\underline{\chi}_n\rangle = \mu_n|\underline{\chi}_n\rangle \qquad (8)$$

In RGM, the states $\underline{\chi}_n$ for which $\mu_n=1$ and $\mu_n=0$ are called the forbidden and allowed states respectively. The intermediate case (it is this case that is actually realized in almost all instances) relates to the so called semiforbidden states giving rise to the major difficulties /9/. It will be noted that the eq. (8) corresponds to but a single degree of freedom (the functions $\underline{\chi}_n$ depend only on the variable of relative motion) and is not related to any Hamiltonian. In our approach, the ghost states are the exact eigenstates of the many-body Hamiltonian with a symmetry violating the Pauli principle. In accordance with this definition, the ghost states are defined (through their Faddeev expansion) by relation (4) with $\underline{\alpha}_n=+1$ for the antisymmetric channel. It can be seen that RGM (8) and the rigorous definition (4) are alike, but the exact definition involves the Faddeev kernel K_2 instead of the (pure structural) antisymmetrization kernel $K(R,R')$ and it is evident that such dynamic treatment fails to involve any semiforbidden states. The eigenstates $\underline{\chi}_n$ in FEq.(4) for which $\underline{\alpha}_n<1$ (for example, $\underline{\alpha}_n=0.9$) correspond not to the semiforbidden states, as in eq.(8), but to the "virtual" states of forbidden symmetry.For such a virtual state to become a bound state, it is sufficient to slightly increase the two-particle scattering amplitude (and not the potential as usual).

It may be supposed, therefore, that the forbidden states in RGM schould appear due to the limitation of the subspace by only the functions of the ground (or several discrete) states of clusters and, once limitation is removed (which essentially means that we quit RGM) such semiforbidden states become conventional virtual states (of forbidden symmetry)

Discussed briefly below will be the ghost states for the quintet channel of d-d scattering.

Four particles in the quintet channel all have parallel spins.The Hamiltonian comprises only four interactions (the interaction of two identical neutrons (their numbers are 2 and 4) and protons (their numbers are 1 and 3) is disregarded) and is invariant relative to the transpositions P_{13}, P_{24} and cyclic transposition $P_{1234}=C$.

The detailed examination in terms of four-body Yakubovski integral equations shows in this case that there is one physical channel (with taking into account of the initial channel symmetry) with antisymmetrical wave function and six the ghost channels (two symmetrical ones and four channels of the mixed symmetry).Such examination shows they that the projecting in the symmetric (ghost) channel results in a decrease of the norm of the kernel in the physical (antisymmetrical) channel too.

Consider further the most evident ghost channel, namely the symmetric channel.The ground state in this channel is the deep-bound compact spatially symmetric state resembling the α-particle.The existence of an excited ghost state in the channel is also possible.If, now, the four-body equations are projected onto the ghost states (it will be emphasized that this means projecting onto the eigenfunctions of other kernels), the zero-order approximation amplitude $\widetilde{F}_0$ will be generated, as in all other cases, by the inhomogeneous term of the equations.In case of a single ghost state, we get the evident generalization of eq.(7b)

$$\tilde{F}_{o\,dd} = \frac{\mu_d}{2\pi} \cdot \frac{\langle \underline{\varphi}_{dd} | \underline{\Phi}_G \rangle \langle \underline{\Phi}_G | \underline{\varphi}_{dd} \rangle}{\langle \underline{\Phi}_G | G_{12,34} | \underline{\Phi}_G \rangle} \qquad (9)$$

where $\underline{\varphi}_{dd} = \underline{\varphi}_d(12) \cdot \underline{\varphi}_d(34) \cdot \underline{\delta}(p-p_o)$; $G_{12,34} \equiv (E - H_o - V_{12} - V_{34})^{-1}$ is the GF of the d-d channel.Similarly to the case of the n-d scattering the formula (9) may be further simplified by using only the pole parts $G_{12,34}$ (without continuum), with the deuteron wave functions $\underline{\varphi}_d(12)$ and $\underline{\varphi}_d(34)$ being the residua.Here again we get a simplified OCM where, as earlier, the forbidden state function is represented not by the eigenfunction of the antisymmetrization kernel (as in OCM) but by the formfactor $\langle \underline{\varphi}_d(12) \cdot \underline{\varphi}_d(34) | \underline{\Phi}_G(1234) \rangle$ of the ghost state function, which is the eigenfunction of the total four-body Hamiltonian with forbidden symmetry.It will also be noted that in both cases (i.e.when the use is made of either the complete GF or its pole parts) the amplitude $\tilde{F}_o$ is unitary.There is every reason to assume that the orthogonality scattering amplitude (9) (with a symmetric ghost) should be as good approximation of the low-energy d-d scattering as it is for the three-nucleon case.It is of importance to emphasize that the formula (9) can be explicitly generalized for scattering of any composite particles and, at the same time, if the near-threshold bound or resonance states are absent from, while the Pauli principle is dominant in, the examined channels, the amplitude (9) will adequately describe the low-energy scattering.Such examples are quite numerous in the region of light-cluster scattering.For example, the n-^{4}He and ^{3}He(^{3}H)-^{4}He systems in the S-channel, the n-^{3}He(^{3}H) system in the channels with ST=01, 10, 11, etc.will be noted.Summarizing in brief the discussion of the ghost projecting, it may be indicated that the derived conclusions can probably help in overcoming the difficulties met with the projecting onto the partly forbidden states in RGM (and OCM), which will be always the fact when the studied clusters can be described by oscillator functions of different radii (for example, in the α-O^{16} case)or when the non-oscillator functions of clusters are used (for example, in case of deuteron scattering by any nucleus).In such cases (and, preferably, in all other cases) it is desirable to use not the eigenfunctions of the antisymmetrization kernel $K(R,R')$ but the formfactors $\langle \underline{\varphi}_A \hat{A} \underline{\varphi}_B \underline{\delta}(p-p_0) | \underline{\Phi}_G \rangle$ constructed on the basis of eigenfunctions of the N-body Hamiltonian $\underline{\Phi}_G$ with a symmetry forbidden by the Pauli principle. It may be expected bearing in mind of the usual high binding energy of such ghost states that the functions may be found using many approximate methods (for example, through the shell model, the hyperspherical expansion, etc).

3. Integral equations for three composite particles; Generalization of OCM

Discussed in this Section will be the second topic which is closely associated with the problems examined above and where the replacement of antisymmetrization by the orthogonality projection may prove to be highly effective. Namely, the scattering theory for three composite nuclear particles (clusters) will be discussed on the basis of the developed method of orthogonality projection. In other words, an attempt will be made to generalize OCM for the case of three clusters.

The aim of the examination is to formulate a theory permitting mainly the description of the continuum in a system of three composite particles and also the discrete and quasidiscrete spectra of three-cluster nuclei though these last problems may be treated successfully by RGM or GCM. In particular the proposed approach permits the formulation of rigorous integral theory of direct nuclear reactions in terms of three-body model (e.g. the direct reactions with deutrons and ^6Li and ^7Li ions) /18/. In this case we shall extend the conventional formalisms based on DWBA and attempt to formulate rigorous equations (of course, in terms of the assumed model) which may subsequently be used to derive various approximations.

Recently with the same aim in view, Schmid/19/ suggested a RGM generalization for three-cluster systems. His approach, however, involves very tedious exchange potentials and seems at present to be hardly tractable for the three-body continuum.

At first, we shall formulate the integral equations where the Pauli principle is included only through the two-body forbidden states and, correspondingly, through two-body projecting. After that, the formalism will be generalized for the case of three-body exchange effects, i.e. the triple exchanges due to the operators P_{ijk}, where i, j, k all relate to different clusters. Either the OCM-interaction, i.e. the t-matrix given by the solution of Saito's model

$$(H_0+V_D)\chi = E\chi;\quad \langle \chi_i|\chi\rangle = 0;\quad i=1,\ldots n. \tag{10}$$

or the microscopically substantiated cluster-cluster optical potentials with the forbidden states proposed by our group/7/ and by Buck et al/8/ will be used as the input cluster-cluster or nucleon-cluster interaction In terms of this model, the forbidden states are, in contrast to OCM, the eigenatates of the Hermitian potential and, therefore, the function of scattering on the mass shell are automatically orthogonal to the functions of the forbidden states. However, a contribution from the forbidden states appears to be outside the mass shell and, therefore, the scattering operators in such optical model should be so modified that the amplitudes in the three-cluster system would not contain the contributions from the virtual transitions into such forbidden states. It is evident that the construction of correct scattering operators in OCM as described by eq.(10) is sufficient for the corresponding operators for the above optical model to be obtained merely as a special case of the OCM operators.

The orthogonality projecting technique discribed above can be easily used to find the expression for the wave function of scattering in OCM:

$$\tilde{\chi}_E = \chi_E - G\Gamma(\Gamma G\Gamma)^{-1}\Gamma\chi_E \tag{11}$$

where χ_E is the solution for the equation $(H_0+V_D)\chi_E=E\chi_E$, and similarly for GF,

$$\tilde{G} = G - G\Gamma(\Gamma G\Gamma)^{-1}\Gamma G \tag{12}$$

It is obvious that $\Gamma\tilde{\chi}_E=0$ and $\Gamma\tilde{G}=0$; i.e. they satisfy OCM.

Consider now the three-body problem. The two-body interactions in the pair ij (and the corresponding projectors) will be denoted with subscrript K (i,j,k=1,2,3 and their cyclic transpositions).The action of the operator Γ_k on the three-body wave function is defined as

$$\Gamma_k \Psi = \sum \varphi_k(r_k) \int \varphi_k(r_k') \Psi(r_k', \rho_k)\, dr_k'$$

Then, the projected and non-projected two-body GF's in the three-body space $\tilde{G}_k(E)$ and $G_k(E)$ are introduced which satisfy the conventional resolvent equations $\tilde{G}_k=G_o+G_o\tilde{V}_k\tilde{G}_k$ whence $\tilde{G}_k=(1-G_o\tilde{V}_k)^{-1}G_o$ where the orthogonalizing pseudopotential $\tilde{V}_k=V_k+\lambda\Gamma_k$.Now, for the total projected GF which satisfies the usual eq. $\tilde{G}=G_o+G_o\hat{V}\tilde{G}$ (where $\hat{V}$ is the total pseudopotential) we introduce the Faddeev decomposition on the channel GF $\tilde{G}^{(i)}=G_o\tilde{V}_i\tilde{G}$ and, after simple manipulations, we shall get the modified set of the Faddeev equations

$$\tilde{G}^{(i)} = \tilde{G}_i \tilde{V}_i \left(G_o + \tilde{G}^{(j)} + \tilde{G}^{(k)}\right) \tag{12}$$

It can be easily seen that in the limit $\lambda\to\infty$ the constant λ is rigorously excluded from the obtained equations.In fact,

$$\lim_{\lambda\to\infty} (\tilde{G}_i\tilde{V}_i) = \tilde{G}_i V_i - G_i\Gamma_i(\Gamma_i G_i \Gamma_i)^{-1}\Gamma_i \tag{13}$$

and

$$\Gamma_i\tilde{G}_i\tilde{V}_i = -\Gamma_i .$$

Further, acting the projector Γ_i on eq.(12), we find

$$\Gamma_i\left(G_o + \tilde{G}^{(i)} + \tilde{G}^{(j)} + \tilde{G}^{(k)}\right) \equiv \Gamma_i\tilde{G} = 0$$

which means that the total projected GF is orthogonal (for the limit $\lambda\to\infty$) to all forbidden states contained in all Γ_i (i=1,2,3).It will be emphasized that the individual Faddeev components $\tilde{G}^{(i)}$ are not orthogonal to Γ_i, i.e. $\Gamma_i\tilde{G}^{(i)}\neq 0$ and, hence, the channel GF comprise the contribution from the forbidden states but the total GF is free of such contribution.In contrast to the variational calculations of three-cluster systems/11/ where the complicated three-body projectors are used to take into account the Pauli principle, our approach does not necessitates the construction of such three-body projectors and only two-body projecting is sufficient.

The found equations for GF $\tilde{G}$ can also be directly used to find the three-body equations for the full wave function whose Faddeev components are defined as $\tilde{\Psi}^{(i)}=\lim_{\varepsilon\to 0} i\varepsilon\tilde{G}\Phi$.After that, we get the equations for the scattering of cluster 1 by the bound state of clusters 2 and 3

$\underline{\tilde{\varphi}}(23)$ (corresponds to the, ~~for~~ pseudopotential $\tilde{V}_{23}$)

$$\underline{\tilde{\Psi}}_1 = \underline{\tilde{\Psi}}_1^{(1)} + \underline{\tilde{\Psi}}_1^{(2)} + \underline{\tilde{\Psi}}_1^{(3)}$$

$$\underline{\tilde{\Psi}}_1^{(i)} = \underline{\tilde{\varphi}}(23)\underline{\delta}(q_i - q_{10}) + \tilde{G}_i \tilde{V}_i \left(\underline{\tilde{\Psi}}_1^{(j)} + \underline{\tilde{\Psi}}_1^{(k)}\right) \tag{14}$$

It can be easily seen that $\underline{\Gamma}_i \underline{\tilde{\Psi}}_1 = 0$, i.e. the solution of the eq.(14) is orthogonal to all $\underline{\Gamma}_i$. In the case where the mass of one of the particles (say, the third) may be considered to be infinitely great, i.e. $m_3 = \infty$ (which is a good approximation for the deuton-unduced reactions on medium and heavy nuclei, the Faddeev reduction can be conveniently made in somewhat other way by decomposing the total wave function (and, correspondingly, all the three-body operators) into the sum of two parts in accordance with the decomposition of the Hamiltonian $H = H_0 + V_3 + V_{12}$, with $V_3 = V_{13} + V_{23}$. Now, the Pauli principle can be included on the nucleon-nucleus variables, i.e. r_{13} and r_{23}. Applying a procedure similar to that described above, we obtain the projected set of two equations

$$\tilde{G}^{(12)} = G_{12} V_{12}\left(G_0 + \tilde{G}^{(3)}\right)$$

$$\tilde{G}^{(3)} = \tilde{G}_3 \tilde{V}_3 \left(G_0 + \tilde{G}^{(12)}\right) \tag{15}$$

where $\tilde{G}_3 = G_3 - G_3 \underline{\Gamma}(\underline{\Gamma} \underline{G_3} \underline{\Gamma})^{-1} \underline{\Gamma} G_3$ and $\underline{\Gamma}$ is the total three-body projector; $G_3(E) = g_{13} * g_{23} = -\frac{1}{2\pi i}\int g_{13}(\underline{\epsilon}) g_{23}(E - \underline{\epsilon}) d\underline{\epsilon}$ is the convolution of two one-particle GF's, and similarly $\tilde{G}_3 = \tilde{g}_{13} * \tilde{g}_{23}$. Similarly to the above, it can be easily shown that $\underline{\Gamma}_{13}\tilde{G} = 0$; $\underline{\Gamma}_{23}\tilde{G} = 0$ at $\underline{\lambda} \to \infty$ i.e. total GF found from the solution of eq.(15) is strictly orthogonal to the forbidden states in the nucleon-nucleus system.

Two equations (15) can be easily used to derive a single equation for the three-body wave function

$$\underline{\chi}^{(12)} = V_{12} \underline{\Phi} + T_{12} \Delta \tilde{G}_3 \underline{\chi}^{(12)} \tag{16}$$

where $\underline{\chi}^{(12)} \equiv V_{12} \underline{\tilde{\Psi}}$, T_{12} is the t-matrix of the n-p scattering; $\Delta\tilde{G}_3 = \tilde{G}_3 - G_0$; $\underline{\Phi} = \underline{\varphi_{12}}(K_{12})\underline{\delta}(p_3 - P_3)$ is the wave function of the initial state. The equation (16) may be a convenient starting point to examine the direct nuclear reactions with deutons (stripping, pick-up, disintegration of deutons in the field of nucleus, etc) where the Pauli principle can be approximately included by orthogonality projecting (generalization see-below) while the three-body continuum and the coupling channels are accurately included.A model of a similar type was recently used Austern /20/ to describe the deuton-induced nuclear reactions.The Schrödinger formalism used by him (instead of the Faddeev technique) prevents however, the promotion to the range higher than the deuton disintegration threshold from being correctly applied.

Now let us attempt to generalize the equations derived to taking into account of triple exchange forces.As it was shown above, the inclusion of the Pauli principle at the two-body level in a system of three composite particles is generally insufficient.If the three-body version of RGM is used, the effective Hamiltonian will comprise three-body nonlocal exchange terms/19/.Assume, following the OCM ideology, that the basic importance of such terms may be included by orthogonalizing to three-body functions of the forbidden states, or more strictly (bearing in mind Section 2) to the three-body formfactors of the form $|\Phi_\beta\rangle = \langle\varphi_{\alpha_1}\varphi_{\alpha_2}\varphi_{\alpha_3}|\Psi_B\rangle$ where Ψ_B is the wave function of a bound state with forbidden symmetry of the N-body Hamiltonian and φ_{α_i} is the internal wave function of the i-th cluster.The formfactors found in terms of the shell model (consideration will be given only to the configurations forbidden in the conventional shell model) are probably a good approximation for $|\Phi_\beta\rangle$ and for the corresponding two-body formfactor.The set of three-body integral equations which include the three-particle Pauli projecting is derived as in the preceding case.In the most general case when there are both two-body and three-body projecting the set of equations becomes

$$\tilde{G}^{(i)} = \left\{ \hat{G}_i V_i - G_i \Gamma_i (\Gamma_i G_i \Gamma_i)^{-1} \Gamma_i + \tilde{G}_i \Gamma (\Gamma \tilde{G}_i \Gamma)^{-1} \Gamma G_i \Gamma_i (\Gamma_i G_i \Gamma_i)^{-1} \Gamma_i \right\} \times \times \left\{ \tilde{G}_0 + \tilde{G}^{(j)} + \tilde{G}^{(k)} \right\} \qquad (17)$$

where $\hat{G}_i \underset{\lambda,\mu\to\infty}{=} (E - H_0 - V_i - \lambda\Gamma_i - \mu\Gamma)^{-1} = (1 - \tilde{G}_0 \tilde{V}_i)^{-1} \tilde{G}_0$;

$\Gamma = |\Phi_\beta\rangle\langle\Phi_\beta|$ -is the three-body projection operator and $\tilde{G}_0 \underset{\mu\to\infty}{=} (E - H_0 - \mu\Gamma)^{-1}$. The indices i,j,k=1,2,3 and their cyclic permutations.This is our final set of equations.It can be easily found after the action of projectors Γ_1 and Γ on the i-th equation of (17) that $\Gamma_1 \tilde{G} = 0$ and $\Gamma \tilde{G}^{(1)} = 0$ and hence $\Gamma \tilde{G} = 0$.This means that the solution of the derived equations (17) is orthogonal bo h to the two-body and three-body forbidden states, i.e.just what was necessary.It should be noted if we want to include the three-body projecting only, it is necessary to substitute $\bar{G}_1$ instead of $\hat{\tilde{G}}_1$ and to neglect the all terms with $\underline{\Gamma}_1$ in the equations.Similarly if we want to limit ourselves to the two-body projecting only it is necessary to substitute $\tilde{G}_1$ instead of $\hat{G}_1$ and to neglect the all terms with $\underline{\Gamma}$.

It seems expedient in conclusion to return again to the relationship between the forbidden (or ghost) states and the convergence of the Born series as it was discussed above.It was already noted in Section 2 that the ghost states give rise to the divergence of Born series for the integral equations (Faddeev and Yakubovski) of many-body scattering just as the physical bound states do.Therefore, the projected equations (17) have smaller-norm kernels that the corresponding non-projected equations which facilitates the iterative solution of these equations.However, it is not sufficient that all the forbidden states should be projected out to obtain the convergent series.It appears necessary to include also all the allowed bound states.We have rigorously shown recently/15/ that, in case of the conventional three-body scattering, for the Born series to be convergent at low energies it is sufficient to project the Faddeev equations onto the entire discrete spectrum (i.e.the ghost and physical bound states).It is very probably (though has not strictly proved) that this theorem is also valid in our case of three composite particles.If this is so, the equations (17) need not being necessarily changed to obtain the convergent Born series for them.It is sufficient to supplement the three-body projectors . $\underline{\Gamma}$ by (apart from the forbidden-state formfactors) the contribution from all, or (if the total energy is not too low) the deep-lying physical bound states.Then, the system (17) may be directly iterated.

Thus, the above described orthogonality projecting technique not only makes it possible to derive the necessary equations for composite particles but also gives a direct and convenient method of their solution.

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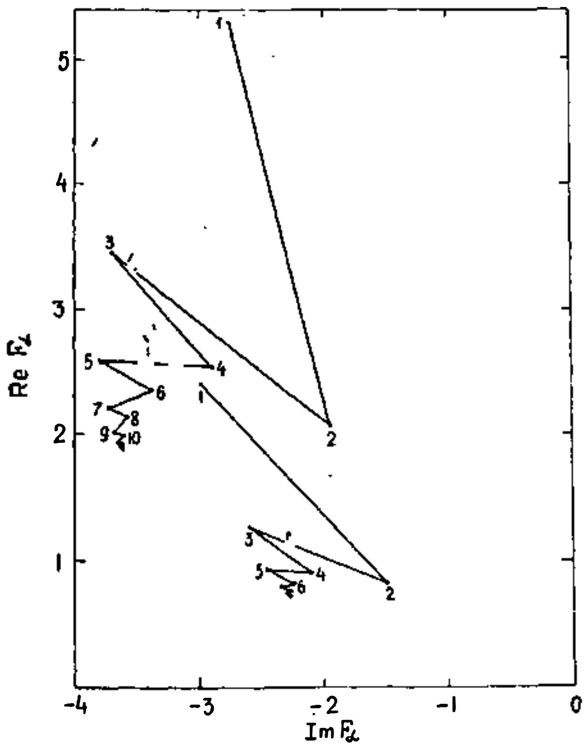

Fig. 1

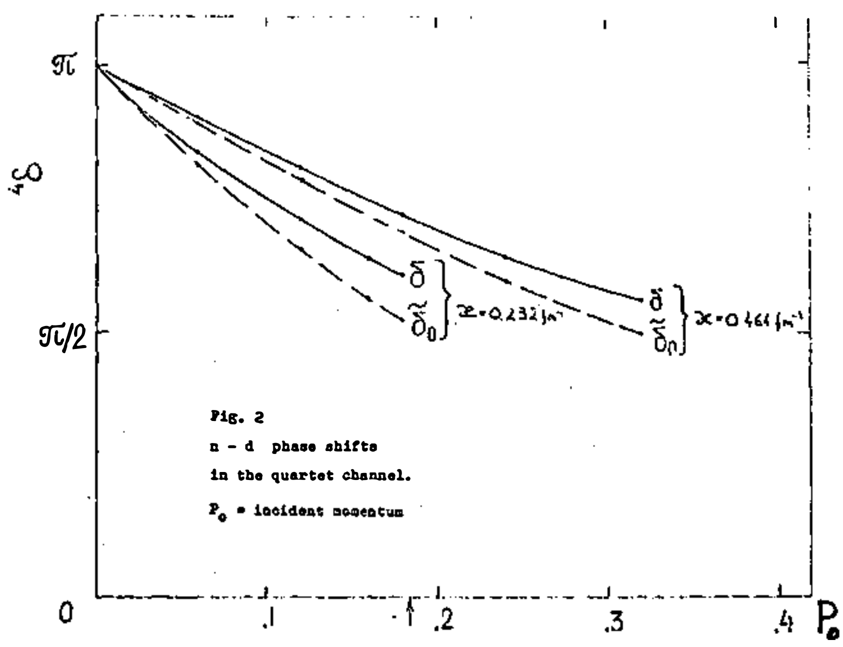

Fig. 2
n - d phase shifts
in the quartet channel.
P_0 = incident momentum