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Dissipation and Fluctuations within the Linear Response Approach to Heavy Ion Collisions - a Critical Review of the Theory and its Application

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We describe the quasi-static approach to deep inelastic reactions and examine critically its assumptions. It is shown that the concept of a heat bath is not necessary for obtaining the final form of the Fokker-Planck equation. Response and correlation functions are calculated by assuming for the relevant squared matrix elements the form suggested by H.A.Weidenmüller and co-workers. It is seen that in this model the Einstein relation between the diffusion and friction coefficients is obtained if only very weak assumptions on the distribution of the intrinsic excitations are made. Possible implications arising from quantal fluctuations of the collective degrees are discussed. So far such quantum effects are neglected.

This paper contains: Introduction; Discussion of the basic ideas and assumptions; Definition of a perturbation approach; Dissipation within linear response theory (relaxation, friction and conservative forces, computation of response functions); Fluctuating forces (master equation, Markoff approximation, diffusion coefficients, choice of statistical weighting factors); Fokker-Planck equation (comments on neglection of quantal effects, equation for entire motion, equations for reduced densities); Cross sections and applications (formulas for differential cross sections, applications - computation of $d^2\sigma/d\Theta\, dE_{kin}$ for the 388 MeV Ar + Th system - computation of $d^2\sigma/d\Theta dX$ for the 365 MeV Cu + Au system); Summary and Discussion.

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I Introduction

Deeply inelastic reactions are usually described by following the time-development of a few degrees of freedom Q_μ (e.g. relative motion, rotation, mass asymmetry shape degrees etc.). Indeed, the experiments[1] show that the exit channel retains memory of the entrance channel. Therefore, despite the high intrinsic excitation not all degrees of freedom reach statistical equilibrium - as in a compound nuclear reaction.

The first theoretical task consists in finding the proper equations of motion for the Q_μ. As the next step, one has to understand how the coefficients depend on the microscopic structure of the nucleonic degrees.

To describe the energy loss many authors have used classical Newton equations with simple frictional forces[2)-7)]. For another treatment of dissipation we refer to the talk of C.H.Dasso at this conference[8)]. Nörenberg[9)] and Moretto[10)] have derived and used Master or Fokker-Planck equations to describe statistical fluctuations of quantities like mass asymmetry, intrinsic excitation energies etc. (For the work of Nörenberg and co-workers we refer also to the talk of G.Wolschin.[23)]) In this context the coefficients mentioned above are then often called transport coefficients.

It is worthwhile to recall that the way of posing the problem is by no means new to nuclear physics. Indeed, in the conventional collective model[11)] one is faced with practically the same problem. There the coefficients to be computed are inertia and stiffness (or the whole potential energy, for large scale motion). What is new is the need to understand the nature of dissipative and fluctuating forces - that is to say the nature of friction and diffusion coefficients. This problem was attacked in refs.12)-15) by applying the quasi static method known from statistical

mechanics. In this contribution we should like to review this approach as well as its application[16)-19)].

There exists another theory for the same problem developed by H.A.Weidenmüller and co-workers[20)-22)]. Similar to the approach just mentioned they too treat the dynamics along the trajectories as well as the statistical fluctuations on the same footing. Their starting point, however, is not the nuclear collective model but the random matrix theory for complex nuclear reactions. These authors have criticised strongly our use of the concept of a temperature. In this paper we shall, therefore, pay special attention to this problem. We shall also try to make clear that in our particular case using a temperature does not necessarily mean that nucleonic degrees must be embedded in a heat bath. In a simple model treated in the appendix we show that a possible uncertainty in the excitation energy has negligible influence on the value of the friction coefficient. The simple model consists in using within our theory the form for the average squared matrix element as defined and used in refs.20)-22).

II Discussion of the basic ideas and assumptions

The collective model obtains its great simplicity due to the requirement of the existence of slow modes Q_μ (the collective ones). They are assumed to define the shape (or a rotation angle) of the nuclear single particle field $V(x_i; Q_\mu)$. This picture works if the nucleonic degrees x_i are fast enough to follow instantaneously any change of the Q_μ. The Q_μ-dependent single particle potential then serves as the coupling between collective (Q_μ) and intrinsic (x_i) degrees from which the equation of motion is generated. This procedure is usually carried through by a perturbation approach. For this the Q-dependent (adiabatic) ground state of the system serves as the zero order approximation.

Clearly, for deep inelastic reactions the latter assumption has to be abandoned. The fact that the intrinsic degrees get

excited must be described in some way. It would be extremely helpful if that could be done by only one additional parameter. To achieve this the following fact can possibly be used. In the region of the expected excitation energy the level density is extremely high. Therefore, clearly, a statistical description is necessary and it is probably fair to assume a statistical equilibrium. We shall see later to what extend this is justified. As the additional parameter which characterizes the intrinsic system one may choose the mean excitation energy $\langle E \rangle$ or a temperature T. It is heartening to see that the simple model discussed in the appendix indeed shows that a parametrization of the intrinsic system by its mean energy is sufficient. We shall see that the influence of a possible fluctuation ΔE on the friction coefficient for instance is of second order in $\frac{\delta E}{\langle E \rangle}$ and thus negligible (see below).

The procedure developed above is the one of the so called quasi static approach to irreversible processes[28]: A change of the macroscopic coordinates Q_μ excites the intrinsic degrees. The latter always have time to relax to a thermal equlibrium described by a temperature T. For a large-scale motion of the Q_μ T must change in time. This picture has to be combined with the idea of the nuclear collective model that the coupling of the Q_μ with the x_i is given by the mean nuclear field $V(x_i, Q_\mu)$. Then we can derive the dynamical equations of motion for the Q_μ.

This procedure has several advantages, ones of principle as well as practical ones:

a) Due to the special choice of the coupling, the method takes into account essential properties of a nuclear system - in contrast to the piston model, for instance, or even to phenomenological hydrodynamical models.

b) The collective degrees appear explicitly in the Hamiltonian from the beginning. They need not be found by some projection or coarse graining method.

c) Due to the flexibility of the model the Q_μ can represent any slow collective degree.

So the main question arises: can this procedure be applied to heavy ion collisions? As we have seen, this is mainly a question of time scales. As a first hint we may use the following estimate. One of the fastest modes to be treated explicitly is certainly that for relative motion. For the reactions we are going to interpret the relative kinetic energy at the top of the Coulomb barrier is typically of the order of a few MeV per nucleon. This value is much smaller than the nucleons' fermi energy of about 35 MeV. So it is very likely that the main conditions may be fairly well fulfilled. Before we come to more details let us make more precise these main assumptions:

a) The reaction can be described in terms of a few collective or macroscopic degrees Q_μ.

b) The intrinsic degrees x_i relax to a statistical equlibrium in a time τ much shorter than a typical time for the collective motion τ_{coll}: $\tau \ll \tau_{coll}$

Later we shall assume also that

c) the statistical fluctuations ΔQ_μ are small. Then the concept of classical trajectories is meaningful.

This set of conditions is finally completed by neglecting any quantal fluctuations of the collective degrees. That is to say the equation of motion for the Q_μ shall not contain any quantum effects. (For more details we refer to section VI.1).

III Definition of a perturbation approach

In the following, in order to simplify the formulas, we shall restrict the discussion to the case of one collective degree Q. The Hamiltonian used in the collective model then has the form

$$\hat{\mathcal{H}}(\hat{x}_i, \hat{p}_i, Q) = \hat{H}(\hat{x}_i, \hat{p}_i) + \hat{V}(\hat{x}_i, Q) \tag{1}$$

and Q is treated as a classical time-dependent parameter. The first part of the Hamiltonian contains the nucleons' kinetic energy. It may further contain any interaction not represented by $V(x_i, Q)$. From (1) the equation of motion for Q(t) is then derived by the methods of the cranking model. It has been shown in ref. (13) how this derivation can be generalized so as to include dissipative processes. In a paper[12] previous to ref. 13) as well as in that,[14] in which the Fokker Planck equation was derived an auxiliary Hamiltonian $\hat{H}_{coll}$ was added:

$$\hat{\mathcal{H}} = \hat{H} + \hat{V} + \hat{H}_{coll} \tag{2}$$

It describes a bare collective motion:

$$\hat{H}_{coll} = \frac{\hat{p}^2}{2\mu} + \hat{U}(\hat{Q}) \tag{3}$$

The coordinate Q and its conjugate momentum are then treated as quantal operators. In the following discussion we shall follow this procedure and assume for $\hat{\mathcal{H}}$ the form

(2). In principle, the formal derivation presented below can be used also for a more general coupling V.

The method for obtaining an equation of motion for the subsystem of collective degrees is as follows: Start with an approximate equation for the whole system and average this equation with respect to the intrinsic degrees. For heavy ion reactions V certainly represents a strong interaction (for a Q which for instance describes relative motion V is responsible for the huge energy loss of some 100 MeV). So V cannot be treated in low order perturbation theory. If, nevertheless, we want to benefit from the simplicity of a perturbation approach we have to renormalize the interaction.

Suppose we want to follow the motion at a time t around an arbitrary but fixed time t_o:

$$t_o - \delta t < t < t_o + \delta t \tag{4}$$

At t_o the collective degree shall have the average value $Q_o = \langle Q \rangle_{t_o}$. We then rewrite $\hat{\mathcal{H}}$ as

$$\hat{\mathcal{H}} = \hat{H}_{int}(\hat{x}_i, \hat{p}_i; Q_o) + \delta\hat{V} + \hat{H}_{coll} \tag{5}$$

with

$$\delta\hat{V} = \hat{V}(\hat{x}_i, \hat{Q}) - \hat{V}(\hat{x}_i, Q_o) \tag{6}$$

and

$$\hat{H}_{int} = \hat{H}(\hat{x}_i, \hat{p}_i) + \hat{V}(\hat{x}_i; Q_0) \tag{7}$$

According to our assumption of small fluctuations of Q (see section II) δV is small whenever δt can be chosen sufficiently small: $\hat{Q}$ may then be estimated by its average value $\langle \hat{Q} \rangle_t$ which for small δt will be close to Q_o.

For the choice of the magnitude of δt we have to concider two limits. Certainly, δt must be chosen smaller than the typical time interval τ_{coll} after which the Q has changed considerably. On the other hand, we do not want to make it smaller than the intrinsic relaxation time τ. Otherwise we would loose important information about the intrinsic system. According to our assumption b) (see section II), τ and τ_{coll} are well separated. For δt we therefore find a value

$$\tau \lesssim \delta t \ll \tau_{coll} \tag{8}$$

which indeed guarantees that δV is small.

We may hence think of δV as a small perturbation on the intrinsic system. It is important to remember that this is true only for δV and an intrinsic Hamiltonian which contains a large fraction of the total interaction $\hat{V}(\hat{x}_i, \hat{Q})$. It is only by this renormalization of $\hat{H}_{int}$ that we can benefit from the weak coupling limit. This has to be considered when we want to compare with inferences obtained in other

approaches like the one of refs 20)-22) where no such renormalization is performed.

For the further procedure $\delta\hat{V}$ is approximated by the first term of a Taylor expansion:

$$\delta\hat{V} = (\hat{Q}-Q_0)\frac{\partial\hat{V}(\hat{x}_i,Q_0)}{\partial Q_0} \equiv (\hat{Q}-Q_0)\,\hat{\bar{f}}(\hat{x}_i,Q_0) \qquad (9)$$

It is again stressed that neglecting higher terms, like for instance $\frac{1}{2}(\hat{Q}-Q_0)^2\frac{\partial^2 V}{\partial Q_0^2}$, is possible only if assumption (c) is fulfilled (and if quantal fluctuations are small, see below).

We still have to define the unperturbed state of the system. It is here where statistics enters: we may anticipate that during the reaction a situation is quickly reached where the level spectrum is very high. To follow then each individual excitation explicitly is hopeless. Therefore, we represent the intrinsic state by a statistical mixture. We even go a step further and assume that to order zero in $\delta\hat{V}$ the intrinsic density operator $\hat{\varrho}_{int}$ represents a statistical equilibrium.

In previous publications, we have chosen for $\hat{\varrho}_{int}$ the canonical form

$$\hat{\varrho}_{int}(Q_0,T_0) = \frac{1}{Z}\exp\left\{-\frac{\hat{H}_{int}(\hat{x}_i,\hat{p}_i;Q_0)}{T_0}\right\} \qquad (10)$$

Since H_{int} depends on Q_o in a parametric way, $\hat{\varrho}_{int}$ is also

a function of Q_o. The temperature T_o is that for the intrinsic system at time t_o. It is to be computed from the mean excitation energy

$$E(t_o) = tr\, \hat{\varrho}_{int}(Q_o, T_o)\, \hat{H}_{int}(\hat{x}_i, \hat{p}_j, Q_o) \qquad (11)$$

The latter is obtained by requiring conservation of energy on the average:

$$E_{tot}(t) = \langle \hat{H}_{int} \rangle_t + \langle \hat{H}_{coll} \rangle_t \equiv E(t) + E_{coll}(t) \qquad (12)$$

In (12) the weak coupling δV has been neglected for the energy balance.

The validity of using the temperature concept has been strongly questioned in refs. 20-22). It has been argued that the two reacting nuclei are not embedded in a heat bath but form an isolated system. One has to remember, however, that we are not interested in distributions of the total system but only in those for the subsystem of the collective degrees. Suppose for a moment <u>all</u> degrees had reached a statistical equilibrium and that the total energy can indeed be split up as in eq. (12). Then the temperature as defined by eq. (11) can indeed be used to find the distribution for the collective subsystem[24]. In our case the situation is different in that the collective degrees do <u>not</u> reach equilibrium. Thus to be sure we have to consider this problem a little more carefully. Therefore, we perform the formal derivation without specifying $\hat{\varrho}_{int}$.

We shall see that for our approach to be valid the special form (10) for $\hat{\varrho}_{int}$ is by no means necessary. Indeed, we could as well use a microcanonical distribution without changing the essentials of the derivation. In the latter case $\hat{\varrho}_{int}$ has a different form. The statistical weighting factors $\varrho(E_m)$ do not have as large a range as for the canonical distribution

$$\varrho(E_m)\Big|_{can} = \frac{1}{Z} \exp\left\{-\frac{E_m}{T}\right\} \tag{13}$$

but differ from zero only in a finite interval δE:

$$\varrho(E_m)\Big|_{microcan} = \begin{cases} \text{const} \neq 0 \text{ for } & E(t_0) - \delta E < E_m < E(t_0) \\ 0 \quad \text{elsewhere} & \end{cases} \tag{14}$$

(Here E_m are the eigenenergies of H_{int}. It is understood, of course, that $\varrho(E_m)$ does depend on Q_o and T_o(or $E(t_o)$)).

There is, however, one condition which the density operator must fulfill if it is to serve as a zero order approximation: The implicit time dependence via E(t) must be small. It must be such that during δt $\hat{\varrho}_{int}$ can be treated as a constant.

IV Dissipation within linear response theory

Formally, the perturbation approach is carried through by applying the theory of linear response. A response function

$\tilde{\chi}_{OF}$ measures a deviation, $\delta\langle\hat{O}\rangle$, of the mean value of an operator $\hat{O}$ from its unperturbed value, as induced by the interaction $(\hat{Q}-Q_o)\hat{F}$. In our case it measures the deviation of any intrinsic operator from its equilibrium value $\langle O\rangle_{eq} = \mathrm{tr}\,\hat{\varrho}_{int}\,\hat{O}$. It therefore also measures the relaxation of intrinsic excitations to equilibrium. We are mostly interested in the mean value of $\hat{F}$ itself:

$$\langle\hat{F}\rangle_t = \langle\hat{F}\rangle_{eq} - \int \tilde{\chi}(t-s)\left(\hat{Q}(s) - Q_o\right)ds \qquad (15)$$

Firstly, $-\langle\hat{F}\rangle_t$ defines the force exerted on the collective degree Q induced by V (as given by eq. (9)). Secondly, it is mainly this quantity $\hat{F}$ of the intrinsic system which will be affected by the time dependence of Q(s).

How, precisely, the form (14) is obtained is described extensively in refs. 12)-14). (For a general discussion of linear response theory we refer to an article by Kadanoff and Martin or to the book of Pines and Nozières[25]). Here we only repeat the definition of $\tilde{\chi}(t)$ as well as that of two other functions:

$$\tilde{\chi}(t) = 2i\,\Theta(t)\,\tilde{\chi}''(t) = \tilde{\chi}'(t) + i\,\tilde{\chi}''(t) \qquad (16)$$

for all t.

($\Theta(t)$ represents the step funtion). In (16) $\tilde{\chi}(t)$ is decomposed into the so-called reactive ($\tilde{\chi}'(t)$) and dissipative ($\tilde{\chi}''(t)$) parts. The latter is defined as:

$$\tilde{\chi}''(t-s) = \frac{1}{2}\left\langle \left[\hat{F}^{I}(t), \hat{F}^{I}(s) \right] \right\rangle = \frac{1}{2}\, tr\, \hat{\varrho}_{int} \left[\hat{F}^{I}(t), \hat{F}^{I}(s) \right] \qquad (17)$$

with $$\hat{F}^{I}(t) = e^{i\hat{H}_{int}t}\, \hat{F}\, e^{-i\hat{H}_{int}t}$$

The decompositon (16) leads to a separation of the Fourier transform of $\tilde{\chi}(t)$, viz $\chi(\omega)$, into its real and imaginary parts:

$$\chi(\omega) = \chi'(\omega) + i\chi''(\omega) \qquad (18)$$

(and hence $\chi'(\omega)$ is connected to $\chi''(\omega)$ by a Kramers-Kronig relation (see refs. 12) and 14))).

It is important to note that the properties of these functions described and used in refs. 12) and 14) also hold if $\hat{\varrho}_{int}$ represents a microcanonical destribution. All these properties are deduced from the fact that $\tilde{\chi}''(t)$ is an odd function. As can be seen from the definition (17) this is a property of the commutator only and not of $\hat{\varrho}_{int}$. (For a relation to correlation functions see below).

1) Relaxation

From the form (15) the meaning of a relaxation to equilibrium can be easily seen: Suppose for a moment the intrinsic system were excited by an impulse:

$$(\hat{Q}(s) - Q_0) \sim \delta(s) \qquad (19)$$

In this case $\langle \hat{F} \rangle_t$ is given by

$$\langle \hat{F} \rangle_t = \langle \hat{F} \rangle_{eq} - \tilde{\chi}(t) = \langle \hat{F} \rangle_{eq} - 2i\tilde{\chi}''(t) \qquad \text{for } t>0 \tag{20}$$

So we may say that the excitation decays to zero after a time τ if and only if the function $\tilde{\chi}''(t)$ approaches zero for $t \gg \tau$.

2) Friction and conservative forces

Let us now come back to the realistic case where Q(s) is a continuous function. The force as given by eq. (15) is non local in time. To achieve locality we may exploit the difference in the time scales for collective and intrinsic motions. If Q(s) is a slow function of s compared to $\tilde{\chi}(t-s)$ the integral in (15) may be evaluated by expanding Q(s) around s=t. Let us do this to second order:

$$\hat{Q}(t-s) - Q_0 \cong \hat{Q}(t) - Q_0 - \dot{\hat{Q}}(t)\, s + \ddot{\hat{Q}}(t)\, \frac{s^2}{2} \tag{21}$$

By inserting (21) into (15) (after changing the integration variables $(t-s) \to s$) one obtains:

$$\langle \hat{F} \rangle_t = \langle \hat{F} \rangle_{eq} - c(\hat{Q} - Q_0) + \gamma \dot{\hat{Q}}(t) + m \ddot{\hat{Q}}(t) \tag{22}$$

with

$$c(Q_0, T_0) = \int ds\ \tilde{\chi}(s) \qquad = \int ds\ \tilde{\chi}'(s) \tag{23}$$

$$\gamma(Q_0, T_0) = \int ds\ \tilde{\chi}(s)\, s \qquad = i\int ds\ \tilde{\chi}''(s)\, s \tag{24}$$

$$m(Q_0, T_0) = -\tfrac{1}{2}\int ds\ \tilde{\chi}(s)\, s^2 \qquad = -\tfrac{1}{2}\int ds\ \tilde{\chi}'(s)\, s \tag{25}$$

It should be noted that the time scale of the response functions needed to justify the form (22) can be expected to be even shorter than the relaxation time τ as defined above. In previous publications we used ratios of succeeding moments in time to define this scale. They appear quite natural for defining a smallness parameter by looking at expansion (22). Let us approximate the collective motion locally by a harmonic one with a frequency ω_{coll}. The the ratio of the fourth to the second term on the right hand side of (22) (both involve the same response function $\tilde{\chi}'(t)$) is given by $\omega_{coll}^2 \tau^2$ with

$$\tau^2 = \frac{\frac{1}{2}\int t^2 \tilde{\chi}'(t)\,dt}{\int \tilde{\chi}'(t)\,dt} \qquad (26)$$

This τ is identical to the decay time of the response function only if the latter does not show marked oscillations. Such oscillations are to be expected. Nevertheless, in the following the decay time is used as a measure of the intrinsic time, since it defines an upper limit.

The physical meaning of the different terms in eq. (22) is obvious: c, γ and m define stiffness, friction and inertial coefficients, respectively. Note that c and m[+] which represent conservative forces are defined by $\tilde{\chi}'(t)$, the reactive part of $\tilde{\chi}$, whereas the friction force is given by the dissipative part of the response function. We see that in the framework of linear response theory this separation is quite natural.

[+]The formula (25) for the inertia is a generalization of the cranking inertia to the case of finite temperature (see (12) and especially (13)).

Let us finish this subsection with the following two remarks. Firstly, due to the generality of the choice of the coupling $\hat{V}(\hat{X}_1,Q)$ the formula for the friction coefficients may be applied not only to heavy ion collisions but also to fission. Secondly, the one-body dissipation formula[27] appears to be a special case of (24). It is obtained if $V(X_1,Q)$ represents an infinite wall enclosing a big system of particles moving on classical trajectories[28].

3) Computation of response functions

Let us now briefly discuss the probem of how to compute the response functions. Simultaneously we can check to see whether these functions do indeed die out after some time τ. To this end we write $\tilde{\chi}''(t)$ in terms of the eigenstates $|m\rangle$ (with energies E_m) of the intrinsic Hamiltonian $\hat{H}_{int}$. Using the definition (17) we obtain

$$\tilde{\chi}''(t) = -i \sum_{n,m} \varrho(E_m) \left|\langle m|\hat{f}|n\rangle\right|^2 \sin(E_n - E_m)t \qquad (27)$$

The computation of the right hand side becomes very simple, if the double sum can be converted into a double integral:

$$\tilde{\chi}''(t) = -i \int_0^\infty \frac{dE}{D(E)} \varrho(E) \int_0^\infty \frac{dE'}{D(E')} \mathcal{F}(E,E') \sin(E'-E)t \qquad (28)$$

and if a simple ansatz can be made for the function $\mathcal{F}(E,E')$. Here this function represents an average of the squared matrix element $|\langle m|\hat{F}|n\rangle|^2$ computed for states with $E_m(E_n)$ lying in the neighbourhood of the energy $E(E')$.

In the step from (27) to (28) we have replaced the discrete level spectrum by a continuous one. This will be a fair approximation if the mean level spacing D is small and if we are only interested in knowing the function for times $t \ll \frac{t}{D}$ (this limit t/D is sometimes referred to as the Poincaré recurrence time). For the deeply inelastic reactions there exists very naturally such a finite observation time, namely that given by the duration of the reaction. For the latter one expects values of the order of $t/1\,MeV$ which, indeed, is much shorter than t/D.

In principle the function $\mathcal{F}(E,E')$ can be evaluated with the method of spectral distributions[26]. Up to now no detailed computation is available. Let us therefore use as a model ansatz the form suggested in refs. 20-22 (for the case of Q representing the relative distance between two heavy ions)

$$\mathcal{F}(E,E') = F_0^2 \, e^{-\frac{(E-E')^2}{2\Delta^2}} \sqrt{D(E)\,D(E')} \qquad (29)$$

The quantity Δ measures the width of the band of levels which are coupled by the operator $\hat{F}$. Since $\hat{F}$ ist a one body operator these levels will predominently lie within one major shell. It

can be expected, therefore, that Δ is of the order of 10 MeV or smaller. In refs. 20) -22) a value of 7 MeV is estimated. The last factor $\sqrt{D(E)D(E')}$ leads to a strong decrease of $\mathcal{F}$ with increasing E+E'. In this way one takes account of the fact that the complexity of the levels increases strongly with increasing energy (and thus their overlap decreases). To justify the special ansatz for this factor these authors argue that $\mathcal{F}\cdot(D(E)DE'))^{-\frac{1}{2}}$ can be viewed roughly as a spreading width and should not be strongly dependent on excitation energy.

In the appendix we describe extensively the implications of the ansatz (29) on the various response functions. Let us mention only one result: $\tilde{\chi}''(t)$ decreases with t as $\exp\left(-\frac{t^2\Delta^2}{2}\right)$. So in this model the relaxation time is determined by

$$\tau \cong \frac{1}{\Delta} \tag{30}$$

With the value estimated for Δ ($\approx$ 7 MeV) we thus obtain a τ of the order of 10^{-22}sec.

If the computation of $\tilde{\chi}''(t)$ discussed so far is to be more than an illustration or first orientation a microscopic computation of $\mathcal{F}$ is necessary (see above). It might, however, be possible to avoid the somewhat tedious method of spectral distributions (Note that the computation has to be performed for different Q values). Indeed (27) can be evaluated directly if it is possible to calculate the matrix elements

$\langle m|\hat{F}|n\rangle$ in a simple but realistic model for $\hat{H}_{int}$. The simplest possibility would be a deformed shell model without any residual interaction. Such a computation was performed in ref. (15) for a head on collision of ^{16}O on ^{238}U. The collective degrees chosen are two dimensional polar coordinates R and Θ. The result for γ''_{RR} is given in fig.1 (for more details we refer to ref 15)). Different from the case mentioned before, this function keeps oscillating also for $t > \tau \cong 0.3\ \hbar/MeV$, which time might be identified as its "decay"-time. These oscillations manifest themselves in that the first moment of this function, i.e. the friction coefficient, vanishes without further manipulation. The first modification one can imagine is to throw away contribution from times longer than the reaction time τ_{reac} - or, (what amounts to the same) to smear the levels with some uncertainty $\hbar/\tau_{reac}$ in the energy. Since τ_{reac} is not well defined the procedure is meaningful, only if the result is independent of the cut-off time τ_{cut}, if τ_{cut} is only of the order of τ_{reac}. Such a result is indeed obtained for the friction coefficient as shown in fig. 2.

We expect that the behaviour of the response function changes if the shell model computation is performed by including a residual two-body interaction. Very likely the oscillations for large times damp out more quickly since the interaction may make the level spectrum more complicated. Studies of such effect are in progress.

V Fluctuating forces

So far we have discussed only the conservative and dissipative forces which appear in the Newton equations for the mean values $\langle Q \rangle$. In addition there exist forces which create statistical fluctuations, sometimes called fluctuating forces. One way to take them into account is to look for the evolution of a density distribution d(t) in the phase space of the collective degree Q and its conjugate momentum P. The equation satisfied by d(t) is a Master or a Fokker-Planck equation. Below we briefly sketch its derivation, for details we refer to ref. 14).

1) Master equation

Let us still look for an equation which is valid in the time interval δt (see above), for which δV (see eq. (9)) is a small perturbation. One first solves the von Neumann equation for the total density operator $\hat{\varrho}_{tot}(t)$:

$$i \frac{\partial \hat{\varrho}_{tot}(t)}{\partial t} = - \left[\hat{\varrho}_{tot}(t), \hat{H}_{int} + \delta V + \hat{H}_{coll} \right]$$

to second order in δV. For the unperturbed density operator of the total system we use a factorized form:

$$\hat{\varrho}^{(0)}_{tot} = \hat{\varrho}_{int}(Q_0, T_0) \cdot \hat{d}(t_0) \qquad (31)$$

This is in accordance with our quasistatic approximation. To order zero in δV we do not allow for correlations between the collective and the intrinsic system other than those which arise from the readjustment of the intrinsic Hamiltonian to the mean value of Q. The solution to second order, $\hat{\varrho}^{(2)}_{tot}(t)$, will contain terms of lower order $\hat{\varrho}^{(i)}_{tot}(t)$ $(i<2)$. The same is true for the reduced density operator:

$$\hat{d}(t) = tr_{int}\, \hat{\varrho}_{tot}(t) \tag{32}$$

obtained after averaging over the intrinsic excitations. To get a closed equation+ for $d^{(2)}(t)$ (and thus for d(t)) one has to replace everywhere $d^{(i)}(t)$ $(i<2)$ by $d^{(2)}(t)$ such that the difference is of order three or higher in δV.

2) Markoff approximation

The equation we arrived at contains terms which involve integrals over remote times. In some of these terms the integrands are proportional to the response function. They have a form very similar to the one encountered before in

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+ We would like to mention at this point that this equation is identical to the one obtained from the Nakajima-Zwanzig projection technique by expanding there the integral kernel to second order in δV (cf. ref. 14).

eq. (15). There are, however, terms which involve - in the same way - another function, namely the correlation function:

$$\psi(t-s) = \frac{1}{2} \left\langle \left[\hat{F}^I(t) - \langle \hat{F} \rangle_{eq}, \hat{F}^I(s) - \langle \hat{F} \rangle_{eq} \right]_+ \right\rangle_{eq} \qquad (33)$$

It differs from $\tilde{\chi}''(t)$ in that the average is to be taken for the anticommutator. So if $\psi(t)$ is again expressed in terms of the intrinsic eigenstates we obtain (for $\langle \hat{F} \rangle_{eq} = 0$)

$$\psi(t) = -\sum_{n,m} \varrho(E_m) \left| \langle m | \hat{F} | n \rangle \right|^2 \cos(E_n - E_m)t \qquad (34)$$

We see that $\psi(t)$ has a structure very similar to that of $\tilde{\chi}''(t)$. Therefore $\psi(t)$ can be expected to change on a time scale τ_c very similar to τ. In the model mentioned before the decay time τ_c is completely identical to τ (see the appendix). Therefore also the integrals involving $\psi(t)$ can be estimated by a moment expansion. In this way the equation for $\hat{d}$ becomes local in time. It is here where the so-called Markoff approximation is made: at time t the density distribution (which describes completely the system of collective degrees) depends on t only and not on the history of the motion.

3) Diffusion coefficients

The moments of the correlation function define diffusion coefficients. In a phenomenological theory they might be treated as new parameters. However, it would be very helpful if a relation could be found to the moments of the response functions mentioned above. Such a relation is possible on very general grounds. Moments in t of the time dependent functions can be expressed by derivatives of their Fourier transforms, evaluated at $\omega = 0$. For instance, the friction coefficient can be written as

$$\gamma = \left.\frac{\partial \chi''}{\partial \omega}\right|_{\omega=0} \tag{35}$$

with

$$\chi''(\omega) = \pi \sum_{n,m} \varrho(E_m) |\langle m|\hat{f}|n\rangle|^2 \left(\delta(\omega - \Omega_{nm}) - \delta(\omega + \Omega_{nm}) \right) \tag{36}$$

being the Fourier transform of $\chi''(t)$.

According to the remarks made above it is obvious that the Fourier transform of $\psi(t)$, which is denoted by S, is obtained from (36) by replacing $\delta(\omega + \Omega_{nm})$ by $-\delta(\omega + \Omega_{nm})$:

$$S(\omega) = \pi \sum_{n,m} \varrho(E_m) |\langle m|\hat{f}|n\rangle|^2 \left(\delta(\omega - \Omega_{nm}) + \delta(\omega + \Omega_{nm}) \right) \tag{37}$$

The first diffusion coefficient, which we denote by Δ, can therefore be expressed as

$$\Delta = \frac{1}{2}\int_{-\infty}^{+\infty} \psi(t)\,dt = \frac{1}{2}\,S(0) \tag{38}$$

As may be suspected from the similarity of the forms of (36) and (37) that there exists a relation between $\chi''(\omega)$ and $S(\omega)$. It is the so-called fluctuation dissipation theorem:

$$\chi''(\omega) = \operatorname{tgh}\frac{\beta\omega}{2}\,S(\omega) \tag{39}$$

By differentiating with respect to ω we find immediately that the diffusion coefficient is proportional to the friction coefficient

$$\Delta = \frac{\gamma}{\beta} \tag{40}$$

For a canonical ensemble β is identical to the temperature T and (40)' is called the Einstein relation. As the reader may have noticed, this is the first (and only) place in the derivation where the concept of a temperature is important. Let us therefore discuss now the

4) Choice of the statistical weighting factors.

Usually, the relation (39) is proved[25] for the case that the $\varrho(E_m)$ are the canonical weighting factors. As mentioned before, in previous publications we have used always such an ensemble. However, the relation (39) remains true also for other distributions. This is clearly seen

from the discussion of the model prescribed above and in the appendix. The only condition one needs is that the function

$$\beta(E) = -\frac{\partial \ln D(E)}{\partial E} \tag{41}$$

varies slowly in the region where the excitation probability $W(E)$ is different from zero. The function $\beta(E)$ can then be approximated by the constant value

$$\beta \equiv \beta(E = \langle E \rangle) \tag{42}$$

which defines the quantity appearing in (39). From the definition of $\beta(E)$ as given by (41) it is seen that β is identical to the inverse temperature. This particular result has its origin in the special choice for $\mathcal{F}(E,E')$ - viz that it is proportional to $\sqrt{D(E')}$. The reason for this choice is discussed above.

The error resulting from the use of the approximation (42) is estimated in the appendix and found to be of second order in $\frac{\Delta E}{\langle E \rangle}$, with ΔE being the width of $W(E)$. Even for a canonical distribution applied to a typical nuclear system, this width ΔE is small enough for the error to be less than ten per cent. This result is worthwhile to be looked at also from a general point of view. It says that the distribution of the collective degrees can be found once we define the

intrinsic system on its mean. We mentioned above the same feature for the case of all degress being in a statistical equilibrium. Now we see that it is true also for the case where the subsystem of the collective degrees is away from equilibrium.

We may finally ask whether the relation (39) between $\chi''(\omega)$ and $S(\omega)$ can be used even for a more general function $\mathcal{F}(E,E')$ and a distribution different from the canonical one. The answer is yes, as can be seen in the appendix. The only condition on $\mathcal{F}$ we need is that the difference E-E' is bound to values small in comparison to the mean excitation energy $\langle E \rangle$. (This should be a weak condition if one considers that $\hat{F}$ is a one body operator). The definition of $\beta(E)$ then changes to

$$\beta(E) = 2\left[\frac{\partial}{\partial \Omega} \frac{\mathcal{F}(E, E+\Omega)}{D(E, E+\Omega)}\right]_{\Omega=0} \tag{43}$$

but β is still given by (42).We may thus draw the following conclusions:

a) The form for the (Einstein) relation (40) is not restricted to the canonical distribution. We recall an important implication of this form: for a finite β there is no friction without diffusion and vice versa.

b) For the model based on the averaged squared matrix element as suggested by Weidenmüller's group, β is identical

to the inverse temperature no matter how $\varrho(E)$ is chosen.

c) So far there is no special reason to believe in a function $\mathcal{F}(E,E')$ which would change the definition of β drastically as compared to eq. (41) (with (42.)). So for practical applications we may continue to work with β defined as for the canonical ensemble. This point certainly deserves more studies.

VI Fokker-Planck equation

To obtain the final form of the equation for the density we make two further assumptions. They are made in order to simplify the structure of this equation.

Firstly we shall neglect all forms which contain moments of $\psi(t)$ higher than order zero as well as those of $\tilde{\chi}(t)$ higher than order one. It can be shown that both restrictions are consistent with each other if the temperature is not too small. (In ref. 14) an estimate of $\omega \ll 3.5\ T$ was given).

The equation for the density operator d(t) valid in the interval δt, then reads (for the sake of simplicity we assume here and in eq.(45): $Q_o = \langle \hat{F} \rangle_{eq} = 0$; see the discussion on page 467 of ref.14)):

$$i\frac{\partial \hat{d}(t)}{\partial t} = -\left[\hat{d}(t), \hat{H}_{coll}\right] - c\left[\frac{\hat{d}(t)\hat{Q}+\hat{Q}\hat{d}(t)}{2}, \hat{Q}\right] \qquad (44)$$

$$-\left[\left(\gamma\frac{\hat{d}(t)\hat{P}+\hat{P}\hat{d}(t)}{2\mu} + i\Delta\left[\hat{d}(t), \hat{Q}\right]\right), \hat{Q}\right]$$

This equation still contains quantal effects. As the second approximation we neglect all quantum effects and replace eq.(44) by its classical analogue, which turns out to be the following Fokker-Planck equation[14] (FPE):

$$\frac{\partial d(t)}{\partial t} = -\frac{P}{\mu}\frac{\partial d(t)}{\partial Q} - K\frac{\partial d(t)}{\partial P} + cQ\frac{\partial d(t)}{\partial P} \qquad (45)$$

$$+\gamma\frac{\partial}{\partial P}\frac{P}{\mu}d(t) + \Delta\frac{\partial^2 d(t)}{\partial P^2}$$

1) Comments on neglection of quantal effects

A few comments should be made on this last step. Quantal effects which are caused by the bare collective Hamiltonian $\hat{H}_{coll}$ are probably small. This should be the case at least when Q represents the relative motion. For typical reactions the de Broglie wave length is less than 0.1 fm (see ref. 30)) and thus much shorter than typical

variations of U(Q) with Q. Furthermore, quantal effects in eq. (44) which arise from the friction and diffusion terms may also be expected to be small. Remember that we have assumed in some way a high temperature ($\omega_{coll} \ll 3.5\ T$). We know from equilibrium statistical mechanics that this is exactly the regime where quantal effects are small.

It is certainly true that this problem of quantal fluctuations deserves a more detailed study, especially after the findings of refs.21) and 22). These authors find strong quantal off-shell effects due to the strong interaction V. We believe that in our approach the strong interaction of V is to a large extent taken into account by our renormalization of $\hat{H}_{int}$ along the trajectory. However, the influence of the second term, $\frac{1}{2}(Q-Q_o)^2 \frac{\partial^2 V}{\partial Q_o^2}$, of the expansion of δV (see eq.(9)) still has to be studied (by treating δV to the next order). It is mostly from this term that non trivial quantal effects may arise.

Before closing this section we should like to refer to a recent paper by K. Dietrich and Ch. Leclerq-Willain, which also discusses implications of quantal effects.

2) Equation for entire motion

The FPE eq.(45) still is able only to describe the motion in the interval δt. Remember that the coefficients c, γ and D still contain the parameters Q_o and T_o (or t_o). An equation

for any t is obtained if Q_o and T_o can be replaced by the values of $\langle \dot{Q} \rangle$ and T at time t . But this replacement is again in accordance with our assumption that the collective degrees change slowly. For $\langle Q \rangle$ close to Q_o the equation is changed to an order higher than that considered if Q_o is replaced by $\langle \dot{Q} \rangle$ in all transport coefficients (Note that the latter are of second order in δV). Since all these coefficients are deduced from response functions we may phrase the condition in terms of two inequalities[12]:

$$\frac{1}{\tilde{\chi}} \frac{\partial \tilde{\chi}}{\partial Q} \frac{dQ}{dt} \delta t \ll 1 \qquad (46)$$

$$\frac{1}{\tilde{\chi}} \frac{\partial \tilde{\chi}}{\partial T} \frac{dT}{dt} \delta t \ll 1 \qquad (47)$$

Both compare the implicit time dependence of $\tilde{\chi}$ via $\langle \dot{Q} \rangle$ and T to its explicit time dependence which is defined by the characteristic value $\tau \leq \delta t$ (see eq. (8)). It should be noted, however, that the actual conditions we need are weaker. This is because the transport coefficients are determined by particular properties of $\tilde{\chi}$ only, namely its first few moments.

Before we write down the form of the F.P.E. actually used in the application a further manipulation should be mentioned. We use again assumption c) (section II) of

small fluctuations in the collective degrees and approximate the bare collective force by

$$K(Q) = -\frac{\partial U}{\partial Q} \cong K(Q_c) + \frac{\partial K(Q_c)}{\partial Q_c}(Q - Q_c) \tag{48}$$

Then the FPE reads

$$\frac{\partial d(t)}{\partial t} = -\frac{P}{\mu}\frac{\partial d(t)}{\partial Q} - \left(K(Q_c) - C^{eff}(Q_c)(Q-Q_c)\right)\frac{\partial d(t)}{\partial P} \tag{49}$$

$$+ \gamma(Q_c)\frac{\partial}{\partial P}\left(\frac{P}{\mu} + T(Q_c)\frac{\partial}{\partial P}\right) d(t)$$

with

$$C^{eff}(Q_c) = C(Q_c) - \frac{\partial K(Q_c)}{\partial Q_c} \tag{50}$$

The quantity Q_c introduced defines the mean value of Q:

$$Q_c(t) = \int dQ\, dP\; d(t)\, Q \tag{51}$$

We also use the mean value of P

$$P_c(t) = \int dQ\, dP\; d(t)\, P \tag{52}$$

The equations for these quantities are easily derived by differentiating with respect to t and applying the FPE (49) to the right hand side. We obtain:

$$\frac{dQ_c}{dt} = \frac{P_c}{\mu}. \tag{53}$$

$$\frac{dP_c}{dt} = K(Q_c) - \gamma(Q_c)\frac{P_c}{\mu} \tag{54}$$

It can be seen that these equations form a closed set and that they are identical to Newton's equations with frictional forces. This is to say the mean values of the density destribution follow the classical trajectories. Of course, this feature is due to our approximation for δV being linear in Q and/or to the assumption of small fluctuations. The latter can be measured by the second moments of d(t):

$$\omega(t) = \frac{1}{2}\int dQ\,dP\;\; d(t)\,(P - P_c(t))^2 \tag{55}$$

$$\psi(t) = \frac{1}{2}\int dQ\,dP\;\; d(t)\,(Q - Q_c(t))(P - P_c(t)) \tag{56}$$

$$\chi(t) = \frac{1}{2}\int dQ\,dP\;\; d(t)\;\,(Q - Q_c(t))^2 \tag{57}$$

They, too, fulfill a set of first order differential equations. It is probably not necessary to write down these equations explicitly (see refs.14),17) and 19)), but suffice it to mention a few properties: This set is not closed but coupled to the equations for the first moments, which is a consequence of the dependence of the transport coefficients on Q_c. The equations are inhomogeneous, the inhomogeneity term being given by the diffusion coefficient $D = \gamma T$. This implies that ω, χ, ψ reach finite values even if they are taken to be zero before the interaction (limit of Newtonian mechanics).

After solving for the first and second moments we have important information on the density distribution. As a matter of fact, for a FPE with the particular form (49), this is all the information we need to define the solution d(t) completely. Due to the fact that the equation is linear in Q and P this function d(t) is a Gaussian.

3) Equations for reduced densities

In applications to heavy ion collisions very often not the full equation (49) is used but equations for reduced densities $\sigma(P,t)$ or $\sigma(Q,t)$. Let us briefly discuss how they can be obtained from eq. (49). This is especially simple for the case where we want to average (integrate) over the coordinate Q. Let us do that for the case that P represents an angular momentum and Q an angle. We may then put K(Q) and c equal to zero (assuming that there exists no conservative force to rotate the system). If we integrate over Q we obtain

$$\frac{\partial \sigma(P,t)}{\partial t} = \gamma \frac{\partial}{\partial P}\left(\frac{P}{\mu} + T\frac{\partial}{\partial P}\right)\sigma(P,t) \qquad (58)$$

without making any approximation. An equation of that type is used by Nörenberg[9] to describe the probability for intrinsic angular momentum. The friction and diffusion coefficients are even put equal to constants.

To obtain a useful equation for $\sigma(Q,t)$ is not as simple and not possible without further assumptions. A simple integration over P would lead to the continuity equation only[32]. According to Kramers[33] (see also ref. 32)) one has to integrate over the path $Q + \frac{P}{\gamma} = Q_0$ and assume[14]

$$\frac{1}{K}\frac{dK}{dQ}\sqrt{\mu T}\,\frac{1}{\gamma} \ll 1 \tag{59}$$

$$\frac{1}{d}\frac{\partial d}{\partial Q}\sqrt{\mu T}\,\frac{1}{\gamma} \ll 1 \tag{60}$$

Then one obtains:

$$\frac{\partial \sigma(Q,t)}{\partial t} = -\frac{\partial}{\partial Q}\left(\frac{K(Q_c) - c(Q_c)(Q-Q_c)}{\gamma(Q_c)} - \frac{T(Q_c)}{\gamma(Q_c)}\frac{\partial}{\partial Q}\right)\sigma(Q,t) \tag{61}$$

That this equation does not fully describe the dynamics can easily be seen by looking at the equation for the mean value $Q_c(t)$.[17]

$$\gamma(Q_c)\,\frac{dQ_c}{dt} = K(Q_c) \tag{62}$$

It can be deduced from eq. (61) in the same way as eqs. (53) and (54) from eq. (49). We see that eq. (62) results from the full Newton equation (54) only in the case of overdamped motion: the acceleration term is neglected. For the sake of completeness we also write down the eq. for the second moment $\chi(t)$.[17]

$$\frac{d\chi(t)}{dt} = -2\,\frac{C(Q_c)}{\gamma(Q_c)}\,\chi(t) + \frac{T(Q_c)}{\gamma(Q_c)} \qquad (63)$$

An equation of the type (61) was used by Nörenberg[9] and Ngô[34] for the mass asymmetry degree. All the coefficients were taken to be time independent and C put equal to zero. In this case the solution of eqs. (62) and (63) become trivial: they are linear in t.

VII Cross sections and applications

1) Formulas for differential cross sections

The solution d(t) of the FPE can be used to obtain cross sections which contain the effects of statistical fluctuations. We can do that for any quantity which can be expressed as a function of the slow degrees we are able to take into account in our model. Imagine our set of collective degrees consists of two polar coordinates R and Θ for relative motion and a few other degress Q_n, like mass asymmetry, shape degrees and rotation angles of the fragments. The corresponding momenta shall be denoted by P_R, P_Θ and P_{Q_n}. Suppose we want to obtain, for instance, the differential cross section $\frac{d\sigma}{d\Omega}$ for the angular distribution. We

obtain that by integrating first over all coordinates (and momenta) but Θ :

$$W(\theta,t) = \int dR \prod_n dQ_n \int dP_R\, dP_\theta \prod_n dP_{Q_n}\, \mathcal{A}(R,\theta,Q_n;P_R,P_\theta,P_{Q_n};t) \quad (61)$$

This quantity W defines the probability for finding the system in the range $d\theta$ of the angle θ - provided it has started at $t \to -\infty$ from a given initial configuration. This configuration is described by means of an impact parameter b (Note that the widths of d are taken to be zero before the interaction (see above)). To obtain $\frac{d\sigma}{d\theta}$ we therefore have to integrate over $2\pi\, bdb$, and let t go to $+\infty$:

$$\frac{d\sigma}{d\theta} = \int 2\pi b\, db\; W(\theta, t \to \infty) \quad (65)$$

If we are interested in a multi-differential cross section we proceed in the same way. The only difference is that the coordinate or momentum for which the probability is to be computed has to be left out from the integrations on the right hand sides of eq. (64) . We do not want to give further details or other examples but refer to ref. 17).

The integrations needed to obtain the probability W can be performed analytically. Remember that d is a simple function, namely a multidimentional Gaussian. In the case of eq. (64) we obtain

$$W(\theta,t) = \frac{1}{\sqrt{4\pi\,\chi^{\theta\theta}(t)}}\,\exp\left\{-\frac{(\theta-\theta_c(t))^2}{4\,\chi^{\theta\theta}(t)}\right\} \qquad (66)$$

where $\chi^{\theta\theta}$ is the second moment in θ as defined by eq. (57) for $Q = \theta$.

An interesting feature is that for finite $\chi^{\theta\theta}(t\to\infty)$ the cross-section $\frac{d\sigma}{d\Omega}$ obtained from eq. (65) by inserting the form (66) remains finite even at the classical rainbow angle. If we may recall, the purely classical cross section (based on Newtonian mechanics only) diverges at this rainbow angle. This classical result can be obtained from eq. (65) (together with eq. (66)) as the limiting value for $\chi^{\theta\theta}\to 0$. Indeed, we get

$$\left.\frac{d\sigma}{d\theta}\right|_{class} = \lim_{\chi^{\theta\theta}\to 0} \int 2\pi b\,db\; W(\theta,t\to\infty) =$$

$$= \int 2\pi b\,db\;\delta(\theta-\theta_c(b)) = \sum_n 2\pi b_n \frac{1}{\left|\frac{d\theta_c}{db_n}\right|} \qquad (67)$$

(For examples we refer to a previous paper[16]).

2) Applications

The simplicity of the final version (49) of the FPE makes it very easy to combine this equation with phenomenological models for trajectories[2)-7)]. This is true because the diffusion coefficient (tensor) can be approximated by the friction coefficient (tensor) times the temperature of the intrinsic system (see above). So once we know the forces appearing in the Newton equations we can compute the statistical fluctuations. For these forces up to now there indeed exist only phenomenological ansatzes. A fully microscopic computation of, for instance, the friction force form factor f(R) is not available yet.

In refs. 16)-19) we have applied the FPE to several such phenomenological models. Here I would like to discuss briefly only the computation of the cross sections $\frac{d^2\sigma}{d\theta\, E_{kin}}$ and $\frac{d^2\sigma}{d\theta dx}$. ($E_{kin}$ = kinetic energy for relative motion, x = mass asymmetry).

a) Computation of $\frac{d^2\sigma}{d\theta\, dE_{kin}}$ for the 388 MeV Ar + Th system

For this calculation[19)] we used the friction force as suggested and applied in ref. 7) to the same system. Although there are objections to some of its assumptions we have chosen that particular phenomenological model, mainly because of its simplicity in comparison with other models (for a detailed discussion we refer to ref. 19)).

Fig. 3 shows the cross section as a function of the scattering angle and the kinetic energy in the center of mass system. The dashed line represents the result of the trajectory computation. The full lines show the influence of the statistical fluctuations in energy and angle.It is to be noted that for the deep-inelastic branch for given impact parameter the uncertainty in the angle ($\sqrt{\overline{\theta-\theta_c}}$) amounts to an order of 10°. Without taking into account these fluctuations this pattern could hardly be obtained. As one may see it looks very similar to the experimental curve which can be found in ref. 29). This is not the case, of course, for the quasi elastic part for which our theory cannot be applied.

a) Computation of $\frac{d^2\sigma}{d\theta dx}$ for the 365 MeV Cu + Au system

The computation was performed in the following way. For the relative motion (described in polar coordinates R and θ) the forces were taken to be those of ref. 2). For the mass asymmetry degree x it was assumed that:

i) it is statistically uncorrelated to the relative motion, i.e. the density distribution for the total system was assumed to factorize in the following way:

$$d(R,\theta,x;P_R,P_\theta,P_x,t) = d_1(R,\theta;P_R,P_\theta,t)\cdot d_2(x,P_x,t) \qquad (68)$$

ii) it is overdamped, i.e. the equation for $d_2(x,t)$ has the form (61).

Both assumptions do not mean that there is no dynamical coupling between the x-mode and the relative motion. Such a coupling comes in via the Newton equations for the mean values. The assumptions were made for simplifying reasons. They imply that the only force which has to be parametrized anew is the friction force for the x-degree. Two possible choices have been used. The results are shown in figure 4a and b. For fig. 4a the friction coefficient γ_{xx} was chosen constant[17] whereas for fig 4b γ_{xx} was taken to be proportional to the formfactor $f(R)$ of the radial friction force[18]. By comparing these figures we get an idea of the importance of the dynamical coupling of the x-mode to the relative motion (Note, however, that this coupling is not only given by $f(R)$ but also by the conservative forces). The experimental curve is shown in fig. 4c (for details see ref. 17)). It is seen that essential properties of this pattern are reproduced by the theoretical curves. The agreement is certainly better for fig. 4b. It should be strengthened, however, that the models are too crude for a hope for better quantitative agreement.

VIII Summary and Discussion

We described how it is possible, to derive a FPE within a weak coupling approach. For this it is necessary to renormalize the Hamiltonian for the intrinsic system along the trajectories of the collective motion. We showed that the transport coefficients are determined by specifying the mean of the excitation energy only and not its width. At the same time we showed that the use of a temperature in the FPE does not imply the assumption of heat bath (canonical ensemble). This result was obtained by considering carefully the particular situation of interest, namely a small system like a nucleus with the collective degrees away from equilibrium.

The application of the theory within simple models led to a qualitative agreement between computed and experimental cross sections. We may thus ask the question: is the approach realistic enough such that the agreement could be made more quantitative? The answer at this stage is that it is probably not. All the assumptions made in the theory are unlikely to be fulfilled - at least not for the entire duration of the reaction:

1) We certainly did not take into account all slow degrees. Shape degrees, for instance, are certainly important in the last stage of the reaction (see ref.6)).

2) The statistical assumption about high excitation energy (high temperature) are not valid at the beginning of the reaction.

3) The estimated relaxation time of $\tau \cong 10^{-22}$ sec is very likely not small enough so that at the beginning of the reaction a clean separation of the fast from the slow modes is possible (for a

discussion of this point see also ref.15)).

In which way could the theory be improved?

i) As the most important problem appears to be the treatment of the initial stage of the reaction. It is very unlikely that for this stage one can use any kind of statistical assumptions.

ii) The statistical theory presented above could be improved by not assuming a high temperature limit. We should also look more carefully at the problem of quantal fluctuations in Q. If they lead to a sizeable uncertainty ΔQ δV has to be treated to higher order (see section VI.1). It should be mentioned in this context that the model computations showed the statistical fluctuations ΔQ to be fairly small.

iii) In the practical applications done so far[16-18] the temperature T(t) was estimated by assuming $\ln D = 2\sqrt{a E(t)}$ and taking for the level density parameter a the usual ansatz: $a = \frac{A}{10}$. This estimate, of course, might be <u>too crude:</u> It does not accou for the special situation of a composite system whose level stru ture will be quite different from that of spherical nuclei. We may also put it differently: In this way we do not account for the necessity of renormalizing H_{int} along the trajectory. We would do better justice to this particular problem by computing the temperature from the relation (11) (cf. also ref.14)).

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Appendix

Response and correlation functions in a simple model

In this appendix we study a simple model in order to exhibit important properties of response and correlation functions used in the text. Special emphasis is paid to the question of the importance of the intrinsic energy distribution. We shall see that only very weak conditions on this distribution are needed.

All the functions we are interested in have the form

$$\underline{\underline{\Xi}}(x) = \sum_{nm} \varrho(E_m)\, |\langle m|\hat{F}|n\rangle|^2\, \xi(x, E_n - E_m) \tag{A1}$$

with x representing either a time t or a frequency ω . This is easily seen by using the following relation (cf eqs. (27), (34), (36), (37)):

$$\xi = \begin{cases} -\frac{1}{2}\left(e^{i\Omega_{nm}t} + e^{-i\Omega_{nm}t}\right) \\ -\frac{1}{2}\left(e^{i\Omega_{nm}t} - e^{-i\Omega_{nm}t}\right) \\ \pi\left(\delta(\omega-\Omega_{nm}) - \delta(\omega+\Omega_{nm})\right) \\ \pi\left(\delta(\omega-\Omega_{nm}) + \delta(\omega+\Omega_{nm})\right) \end{cases} \longleftrightarrow \begin{cases} \tilde{\chi}''(t) \\ \psi(t) \\ \chi''(\omega) \\ S(\omega) \end{cases} = \underline{\underline{\Xi}} \tag{A2}$$

Our first assumption is that the level spectrum is almost continuous, with an average level spacing D(E). Then the double sum in eq. (A1) can be replaced by a double integral:

$$\underline{\underline{\Xi}}(x) = \int_0^\infty \frac{dE}{D(E)}\, \varrho(E) \int_0^\infty \frac{dE'}{D(E')}\, \mathcal{F}(E, E')\, \xi(x, E'-E) \tag{A3}$$

Here the function $\mathcal{F}(E_m E_n)$ of the energies E_m and E_n represents the squared matrix element $|\langle m|\hat{F}|n\rangle|^2$. In principle, this function can be obtained by the method of spectral distributions [26]. For our purpose it is enough to use some simple approximation,

for instance the one suggested in refs. (20-22). We therefore write:

$$\mathcal{F}(E,E') = F_0^2 \, e^{-\frac{(E-E')^2}{2\Delta^2}} \sqrt{D(E)\,D(E')} \tag{A4}$$

A possible Q-dependence is contained entirely in the strength factor F_0^2 (compare section IV.3). For present purposes such a factor is completely unimportant. Therefore, we put F_0^2 equal to unity in the following. By inserting the ansatz (A4) into (A3) we obtain after a trivial change of integration variables:

$$\Xi(x) = \int_0^\infty dE \, W(E) \, \mathcal{F}_\xi(x,E) \tag{A5}$$

Here, $\mathcal{F}_\xi(x,E)$ is defined as

$$\mathcal{F}_\xi(x,E) = \int_{-E}^{\infty} d\Omega \sqrt{\frac{D(E)}{D(E+\Omega)}} \, e^{-\frac{\Omega^2}{2\Delta^2}} \, \xi(x,\Omega) \tag{A6}$$

and

$$W(E) = \frac{\varrho(E)}{D(E)} \tag{A7}$$

defines the probability function for the intrinsic excitation energy E.

To compute the integral $\mathcal{F}_\xi(x,E)$ we make use of the following general property of a statistical equilibrium: No matter how the statistical weighting factor $\varrho(E)$ is chosen, W(E) will be different from zero essentially only in some finite interval ΔE around the mean value $\langle E \rangle$. Possible choices for $\varrho(E)$ are:

$$\varrho(E) = \frac{e^{-\frac{E}{T}}}{Z} \tag{A8}$$

or

$$\varrho(E) = \begin{cases} \text{constant} & \text{for } E_0 - \delta E < E < E_0 \\ 0 & \text{elsewhere.} \end{cases} \tag{A9}$$

They correspond to the canonical or microcanonical ensemble,

respectively. (Note that because D(E) is a strongly decreasing function of E, for the microcanonical distribution ΔE is smaller than δE. Also, usually, $\Delta E/_{\text{microc.}}$ is smaller than $\Delta E/_{\text{can.}}$).

The integrand in (A6) is bound to small values of Ω by the Gaussian factor $e^{-\frac{\Omega^2}{2\Delta^2}}$. The width Δ is expected to be smaller than 10 MeV. (The authors of refs.[20)22)] estimate $\Delta \simeq 7$ MeV; compare section IV.3). On the other hand, the mean excitation energy $\langle E \rangle$ is much bigger:

$$\Delta \ll \langle E \rangle \tag{A10}$$

This is certainly true except for the initial stage of the reaction. . The following computation is made assuming the inequality in eq.(A10). Then Ω is much smaller than E and thus the following approximation can be used:[+]

$$D(E+\Omega) \simeq D(E) \exp\left(\Omega \frac{\partial \ln D}{\partial E} + \frac{\Omega^2}{2} \frac{\partial^2 \ln D}{\partial E^2} \right) \tag{A11}$$

Moreover, the lower limit in the integral of (A6) can be replaced by $-\infty$. Then $\sigma_\xi(x,E)$ reads

$$\sigma_\xi(x,E) = \int_{-\infty}^{+\infty} d\Omega \exp\left(-\frac{\Omega^2}{2(I(E))^2} + \frac{\Omega}{2} \beta(E) \right) \xi(x,\Omega) \tag{A12}$$

with

$$\beta(E) = -\frac{\partial \ln D(E)}{\partial E} \tag{A13}$$

and

$$\frac{1}{I^2} = \frac{1}{\Delta^2} + \frac{1}{2} \frac{\partial^2 \ln D}{\partial E^2} \tag{A14}$$

With the usual ansatz for the level density, $D(E) \sim e^{-2\sqrt{aE}}$, β and I can be estimated to be

$$\beta(E) = \sqrt{\frac{a}{E}} \tag{A15}$$

[+] Such logarithmic expansions are often used in thermodynamic computations, since D depends strongly on E.

$$\frac{1}{I^2} = \frac{1}{\Delta^2} + \frac{1}{4}\frac{\sqrt{\Omega}}{E^{3/2}} \qquad \text{(A16)}$$

By inserting a $\cong \frac{A}{10}\left[\frac{1}{MeV}\right]$ we see that for a typical situation (number of particles $A \gtrsim 100$, $E \gtrsim 100$ MeV) the second term on the right hand side of eq. (16) is negligibly small. Therefore, I is given by

$$I \cong \Delta \qquad \text{(A17)}$$

and thus independent of E (if a small E dependence of Δ is neglected).

In the model described above, let us now test two assumptions which are essential for obtaining the final form of the Fokker Planck equation. They concern the use of a temperature and the condition that the response as well as the correlation functions ($\chi''(t)$ and $\psi(t)$) die out after relaxation times τ and τ_c. To begin with the temperature: essentially it enters (see section V) only via the

a) fluctuation-dissipation theorem.

This reads:

$$\chi''(\omega) = \tanh\frac{\beta\omega}{2}\, S(\omega) \qquad \text{(A18)}$$

or, in the notation used above,

$$\int_0^\infty dE\, W(E)\, \mathcal{F}_{\chi''}(\omega, E) = \tanh\frac{\beta\omega}{2} \int_0^\infty dE\, N(E)\, \mathcal{F}_S(\omega, E) \qquad \text{(A19)}$$

The integrals $\mathcal{F}_{\chi''}$ and $\mathcal{F}_S$ are given by

$$\mathcal{F}_S = \pi \int d\Omega\, \exp\left(-\frac{\Omega^2}{2I^2} + \frac{\Omega}{2}\beta(E)\right)\left[\delta(\omega-\Omega) + \delta(\omega+\Omega)\right] = 2\pi\, e^{-\frac{\omega^2}{2\Delta^2}} \cosh\frac{\omega}{2}\beta(E) \qquad \text{(A20)}$$

$$\mathcal{F}_{\chi''} = \pi \int d\Omega\, \exp\left(-\frac{\Omega^2}{2I^2} + \frac{\Omega}{2}\beta(E)\right)\left[\delta(\omega-\Omega) - \delta(\omega+\Omega)\right] = 2\pi\, e^{-\frac{\omega^2}{2\Delta^2}} \sinh\frac{\omega}{2}\beta(E) \qquad \text{(A21)}$$

From these expressions it is clearly seen that a sufficient condition for (A19) (and thus (A18)) to hold is:

$$\beta(E) \cong \beta(E=\langle E\rangle) \equiv \beta \tag{A22}$$

That is to say, the function $\beta(E)$ must be sufficiently smooth in the interval ΔE around $\langle E\rangle$ such that it can be approximated by a constant. Had we used for $\varrho(E)$ the canonical distribution (A8), $\beta(\langle E\rangle)$ would indeed be identical to the inverse temperature (see textbooks on statistical mechanics, especially ref. 24). Different to the usual derivation of the fluctuation dissipation theorem[25] we have used explicitly a special form for the matrix element of the operator $\hat{f}$. The advantage is that our procedure also holds true for other choices of $\varrho(E)$, for instance, the microcanonical distribution (A9). In this case (A22) serves as the definition of the quantity β appearing in (A18).

It is obvious that a similar procedure can be performed for a function $\mathcal{F}(E,E')$ more general than that given by eq. (A4). In this case we would perform the logarithmic expansion for $f(E,\Omega) \equiv \mathcal{F}(E,E+\Omega)/D(E+\Omega)$ instead of $D(E+\Omega)$:

$$f(E,\Omega) = f(E)\, \exp\left(-\frac{\Omega^2}{2I^2} + \beta\frac{\Omega}{2}\right) \tag{A23}$$

where β and I now are defined as

$$\beta(E) = 2\,\frac{\partial \ln f(E,\Omega)}{\partial \Omega}\bigg|_{\Omega=0} \tag{A24}$$

$$I^{-2} = -\,\frac{\partial^2 \ln f(E,\Omega)}{\partial \Omega^2}\bigg|_{\Omega=0} \tag{A25}$$

(The new factor f(E) will not invalidate the arguments used above to find eq. (A19) true).

Let us finally estimate the error which results from the restriction (A22). It is not necessary to do this for the whole range of frequencies ω since we are mainly interested in the low frequency domain (see sections IV + V). So let us estimate the

error for the friction coefficient γ [+]:

$$\gamma = \frac{\partial \chi''}{\partial \omega}\Big|_{\omega=0} = 2\pi \int_0^\infty dE\, W(E) \left[\frac{\partial}{\partial \omega} e^{-\frac{\omega^2}{2\Delta^2}} \sinh \frac{\omega}{2} \beta(E)\right]_{\omega=0}$$

$$= \pi \int_0^\infty dE\, W(E)\, \beta(E) \qquad \text{(A26)}$$

To evaluate the last integral we expand $\beta(E)$ around $E=\langle E\rangle$. It is enough to use for $\beta(E)$ the estimate (A15)

$$\beta(E) = \sqrt{\frac{a}{E}} \cong \beta\left(1 + \frac{1}{2}\frac{E-\langle E\rangle}{\langle E\rangle} + \frac{3}{8}\frac{(E-\langle E\rangle)^2}{\langle E\rangle}\right) \qquad \text{(A27)}$$

and to assume $\Delta E \ll E$. Then we see immediately that the relative deviation of the friction coefficients evaluated with and without assuming (A22) is of second order in $\Delta E/\langle E\rangle$:

$$\frac{\delta\gamma}{\gamma} \approx \frac{3}{8}\frac{(\Delta E)^2}{\langle E\rangle^2} \qquad \text{(A28)}$$

The linear term vanishes according to the definition of the mean value

$$\langle E\rangle = \int_0^\infty dE\, W(E)\, E \qquad \text{(A29)}$$

In previous papers[12)-18)] we have always suggested the use of the canonical distribution to parametrize the equilibrium of the internal degrees. On the other hand the two reacting nuclei form an isolated system. Thus the incertainty in the excitation energy might be expected to be smaller than ΔE as given by the canonical distribution:

$$(\Delta E)^{-2} = \frac{\partial^2 \ln D(E)}{\partial E^2}\Big|_{E=\langle E\rangle} = \frac{1}{2}\frac{1}{\langle E\rangle}\sqrt{\frac{a}{\langle E\rangle}} = \frac{1}{2\langle E\rangle}\cdot\frac{1}{T} \qquad \text{(A30)}$$

(see for instance ref. 24). Thus we see that the use of a

[+] Note that according to formula (A20) the diffusion coefficient (which is given by $\frac{1}{2}S(0)$) appears to be independent of $\beta(E)$.

canonical distribution produces an error of the order of a few percent only - contrary to the suspicion expressed in refs.2o)-22).

Let us now come to the problem of estimating

b) relaxation times.

They are defined as the times τ (and τ_c) after which the functions $\tilde{\chi}''(t)$ (and $\psi(t)$) become negligibly small. As can be seen from eqs.(A2) and (A3) they are given by linear combinations of the functions

$$\tilde{\chi}^{(\pm)}(t) = \int_0^\infty dE\, W(E) \int_{-\infty}^{+\infty} d\Omega\, \exp\left(-\frac{\Omega^2}{2\Delta^2} + \frac{\Omega}{2}\beta(E) \pm i\Omega t\right) \qquad \text{(A31)}$$

(Here, we have used again (A5) together with the approximate form (12) for $\tilde{\chi}_s(t,E)$). The trivial integration over $d\Omega$ leads to

$$\tilde{\chi}^{(\pm)}(t) = \sqrt{2\pi}\,\Delta\, e^{-\frac{t^2\Delta^2}{2}} \int_0^\infty dE\, W(E)\, \exp\left(\frac{1}{2}\left(\pm\frac{\beta\Delta}{2}\right)^2 \pm it\beta(E)\frac{\Delta^2}{2}\right) \qquad \text{(A32)}$$

As a result we thus obtain in this model that the relaxation times τ and τ_c are equal and determined by $\frac{1}{\Delta}$:

$$\tau = \tau_c \cong \frac{1}{\Delta} \qquad \text{(A33)}$$

Note again that this conclusion is reached without specifying the statistical density function $\varrho(E)$.

References

1) We refer the reader to the following review papers where references to the experimental work can be found:
J.Galin,European Conference on Nucl.Physics with Heavy Ions, Caen (1976),J.Phys.C5 37(1976)83
M.Lefort,Rep.Prog.Phys.39(1976)129
L.G.Moretto and R.Schmitt,European Conference on Nucl.Physics with Heavy Ions, Caen(1976),J.Phys.C5 37(1976)1o9
A.Gobbi,invited paper presented at the Fall Creek Falls Meeting on Heavy Ion Collisions,Pikeville,Tennessee,June 1977
J.R.Huizenga,W.U.Schröder,J.R.Birkelund and W.Wilcke,ibidem. see also ref. 30)

2) D.H.E.Gross and H.Kalinowski,Phys.Lett.48B(1974)3o2
J.N.De,D.H.E.Gross and H.Kalinowski,Z.Phys.A277(1976)385

3) J.P.Bondorf,M.I.Sobel and D.Sperber,Phys.Lett.Rep.15C(1975)83
J.P.Bondorf,J.R.Huizenga,M.I.Sobel, and D.Sperber,Phys.Rev.C11 (1975)1265

4) A.J.Sierk and J.R.Nix,Proc.of the 3rd IAEA Symposium on the Physics and Chemistry of Fission, Rochester vol.II(1973)273 (IAEA Vienna)
J.R.Nix and A.J.Sierk,Phys.Script.1oA(1974)4o1

5) C.F.Tsang,Phys.Script.1oA(1974)9o
F.Beck,Phys.Lett.62B(1976)385

6) H.H.Deubler and K.Dietrich,Phys.Lett.56B(1975)241
H.H.Deubler and K.Dietrich,Nucl.Phys.A277(1977)493

7) K.Siwek-Wilczynska and J.Wilczynski,Nucl.Phys.A264(1976)115

8) C.H.Dasso,invited paper at this conference

9) W.Nörenberg,Phys.Lett.53B(1974)289
W.Nörenberg,Z.Phys.;274(1975)241 and A276(1976)84
S.Ayik,B.Schürmann and W.Nörenberg,Z.Phys.A277(1976)299 and Z.

Phys.A279(1976)145

W.Nörenberg,European Conference on Nuclear Physics with Heavy Ions, Caen(1976)C5 141

1o) L.G.Moretto and J.S.Sventek,Phys.Lett.58B(1975)26

J.S.Sventek and L.G.Moretto,Lawrence Berkeley Report LBL-5o12 (1976)

11) A.Bohr and B.R.Mottelson,Nucl.Structure vol.II W.A.Benjamin, INC(1975)

12) H.Hofmann and P.J.Siemens,Nucl.Phys.A257(1976)165

H.Hofmann and P.J.Siemens,Phys.Lett.58B(1975)417

H.Hofmann,XIIIth Int.Winter Meeting on Nucl.Phys.,Bormio(Italy) 1975

13) H.Hofmann,Phys.Lett.61B(1976)243

14) H.Hofmann and P.J.Siemens,Nucl.Phys.A275(1977)464

15) P.J.Johansen,P.J.Siemens,A.S.Jensen and H.Hofmann,Nucl.Phys.in press

16) H.Hofmann and C.Ngô,Phys.Lett.65B(1976)97

17) C.Ngô and H.Hofmann,Z.Phys.A282(1977)83

18) H.Hofmann and C.Ngô, invited paper presented at the Fall Creek Falls Meeting on Heavy Ion Collisions, Pikeville, Tennessee, June 1977, by C.Ngô

19) M.Berlanger, P.Grangé,H.Hofmann,C.Ngô, and J.Richert,Z.f.Phys. in press.

2o) D.Agassi,C.M.Ko, and H.A.Weidenmüller,preprint MPI-H(1976)-V25

21) C.M.Ko,D.Agassi, and H.A.Weidenmüller,preprint MPI-H(1977)-V5

22) H. A.Weidenmüller,invited paper presented at the Fall Creek Falls Meeting on Heavy Ion Collisions,Pikeville,Tennessee, June,1977

23) G.Wolschin, invited paper at this conference

24) W.Brenig, Statistische Theorie der Wärme. Berlin-Heidelberg - New York: Springer 1975

L.D.Landau and E.M.Lifschitz, Statistical Mechanics,
London. Pergamon press 1951
F.Reif, Fundamentals of Statistical and Thermal Physics.
New York : Mc Graw Hill Company, 1965

25) L.P.Kadanoff and P.C.Martin, Ann.of Phys. 24 (1963) 419
D.Pines and P.Nozières, The Theory of Quantum Liquids,
Vol. 1. New York : Benjamin 1966

26) J.B.Frenchand K.F.Ratcliff, Phys.Rev.C3 (1971) 94;
F.S.Chang, J.B. French and T.H.Thio, Ann.of Phys. 66 (1971) 137

27) W.J.Swiatecki, Lawrence Berkeley Laboratory Report:
LBL-4296 (1975);
see also: D. H.E. Gross, Nucl.Phys. A240 (1975) 472

28) S.E.Koonin, R.L.Hatch and J.Randrup, Nucl.Phys. A277 (1977) 509;
S.E.Koonin and J.Randrup, The Niels Bohr Institute preprint (1977)

29) J.Wilczynski, Phys.Lett. 473 (1973) 484

30) W.U.Schröder and J.R.Huizenga, to be published in Ann.Rev.
Nucl.Science, Vol. 27

31) K.Dietrich and C.Leclercq-Willain, Lawrence Berkeley Laboratory
Report. LBL-5815 (1977);
see also: K.Dietrich contribution to this conference

32) R.Becker, Theorie der Wärme. Berlin-Göttingen-Heidelberg-
New York: Springer 1964

33) H.A.Kramers, Physica VII (1940) 284

34) C.Ngô, J. Péter, B.Tamain, M.Berlanger and F.Hanappe,
Nucl.Phys. A267 (1976) 181.

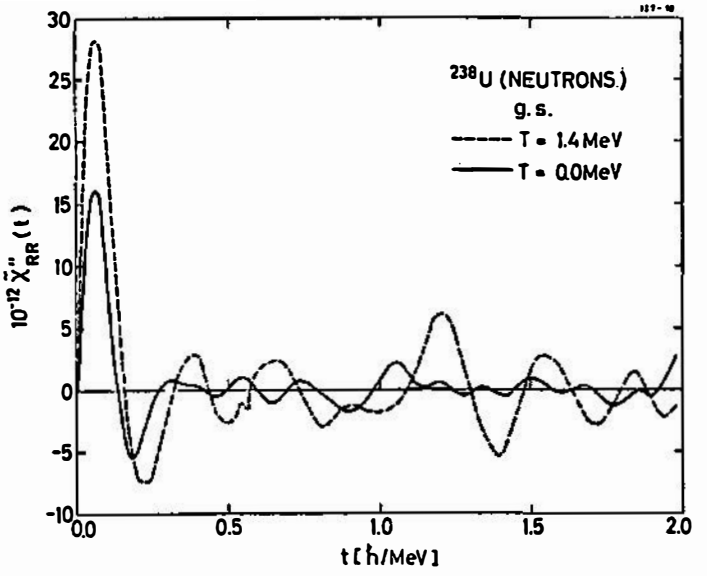


fig. 1 : Response function for the case of relative motion (see section IV.3).

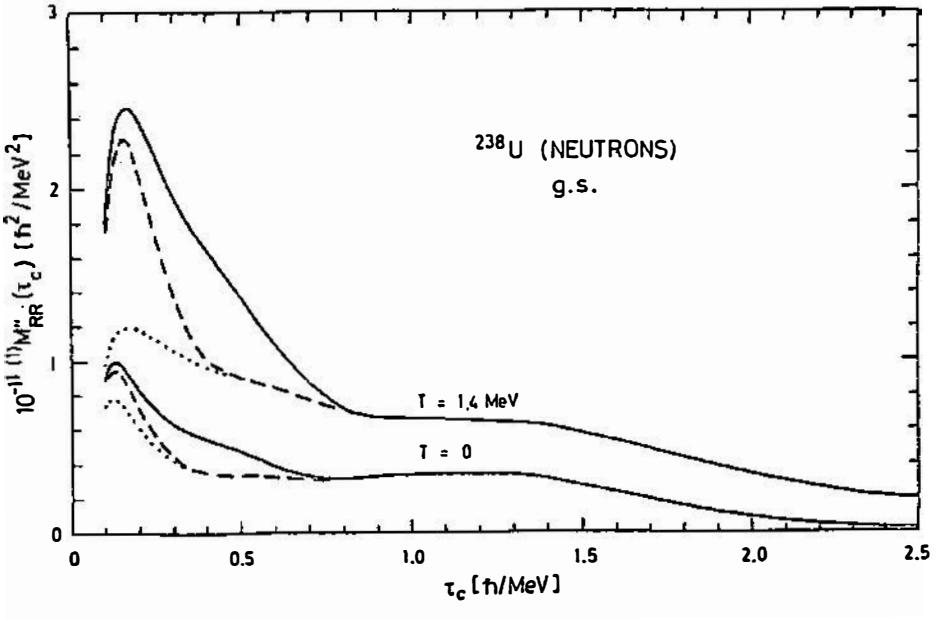


fig. 2 : First moment of the response function of fig. 1 (see section IV.3).

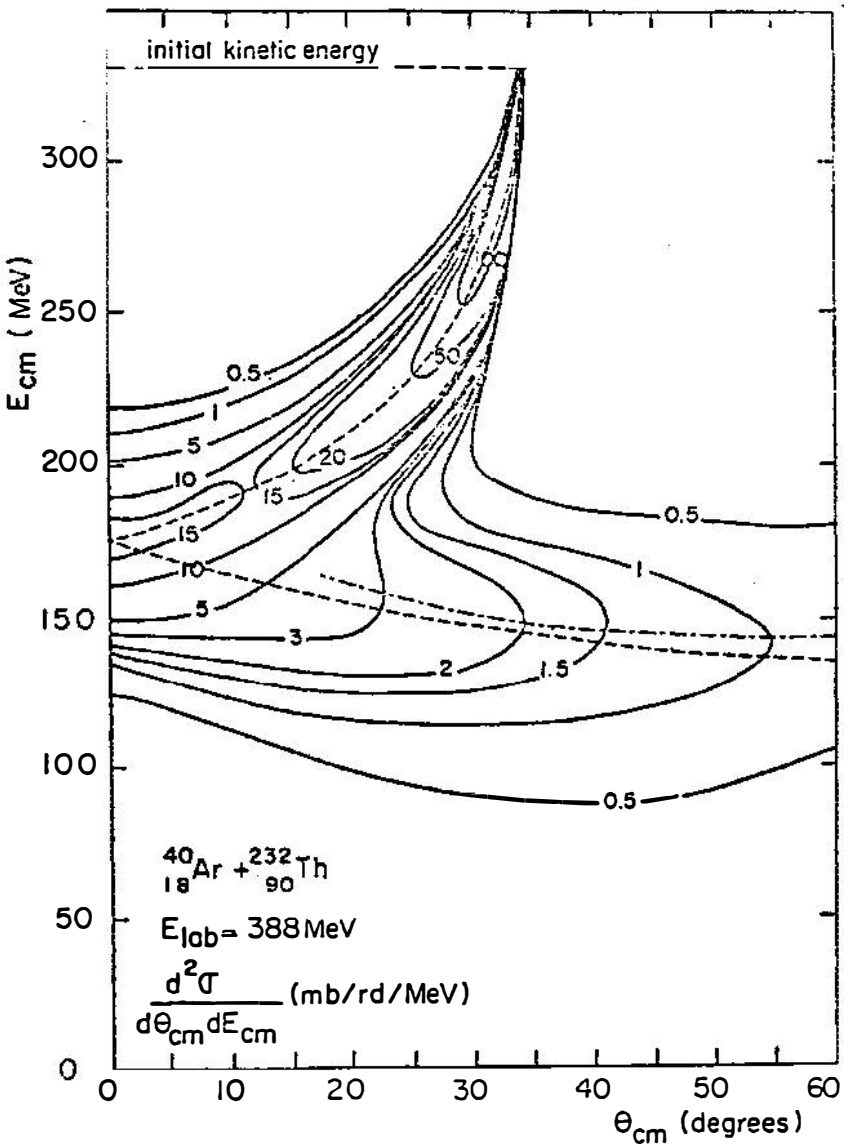
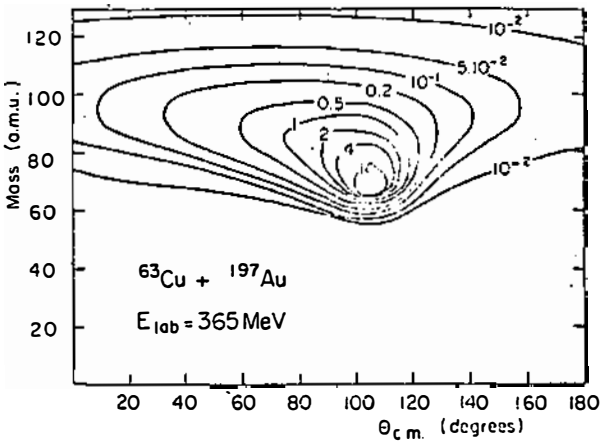
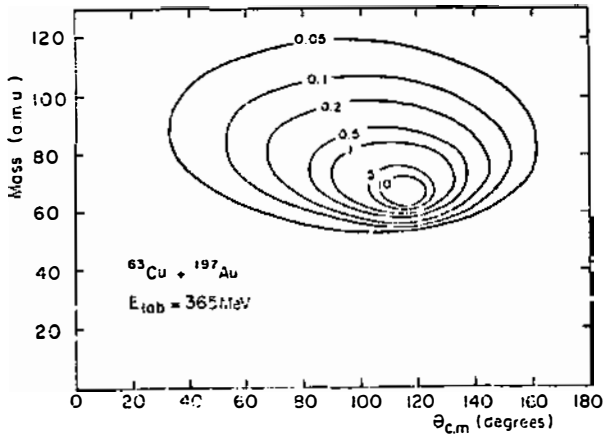


fig. 3 Computed cross section for angle and kinetic energy of the reaction $^{40}Ar + ^{232}Th$ at E_{lab} = 388 MeV.





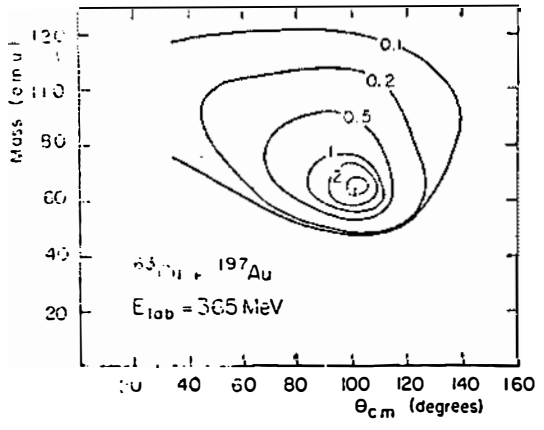


Fig. 4.

Cross section for angle and mass asymmetry of the reaction $^{63}Cu + ^{197}Au$ (compare the text).

a) Theoretical curve with γ_{xx} = constant

b) Theoretical curve with $\gamma_{xx} \sim f(R)$

c) Experimental curve.

DISCUSSION

R.W. Hasse: I wonder whether your formalism works for phase oscillations, too. At some point you employed an expansion in terms of time derivatives of Q, i.e., Q, $\dot{Q}$, and $\ddot{Q}$, and truncated them. By comparing with the Newtonian equation of motion, you identified the mass and friction coefficients. If oscillations are concerned, however, $\dot{Q} \propto \ddot{Q} \propto \dddot{Q}$ and so on, so that the coefficient of $\dddot{Q}$ is by no means vanishing. The above mentioned identification should therefore not work.

H. Hofmann: It works perfertly if the conditions mentioned at the beginning are fulfilled. The smallness parameter for this expansion may be chosen to be $\omega_{\omega R}^2 \varphi^2$ with φ^2 defined by the ratio of the second moment of the response function to the zeroth one, for instance.