FIZIKA 9 (1977) Supplement 4, 497–512
(Proceedings of the Int. Symp. on Nuclear Collisions and
Their Microscopic Description, Bled, September 1977)

A MICROSCOPIC DESCRIPTION OF DEEP INELASTIC COLLISIONS[+]

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1. Introduction.

The purpose of this note is to give a short description of a model of heavy ion collisions in terms of excitation of surface vibrational modes. Complicated formulae and details of the work are outside of the scope of this presentation and can be found in Ref. 1.

A microscopic description of deep inelastic reactions aims to represent the observed features of these processes utilizing the nuclear structure information available for the colliding ions. One way to do this, for example, in the one exploited in the time-dependent Hartree-Fock calculations where the search for specific degrees of freedom is pushed all the way down to the independent-particle states. This is a costly process and the extent of work that can actually be done, though ever expanding, is severely limited to date.

In the study of systems with large number of particles, however, the concept of elementary modes of excitation proves to be extremely useful in reducing the complexities of the problem. The analysis of a reaction is thus greatly simplified by identifying the few collective degrees of freedom which may best represent the response of the system to a given external probe.

There is little doubt that among the different elementary modes of excitation the surface vibrational modes will play a role in the mechanism of excitation of of nuclei during heavy ion collisions. Their presence has been well established for quasi-elastic processes and it seems only natural to investigate further their role in deep inelastic reactions.

[+] The work reported here was performed in collaboration with R.A.Broglia, O.E.Civitarese, G.Pollarolo and A.Winther

2. Formalism.

The surface vibrational modes can be represented by the amplitudes $\alpha_{\lambda\mu}$ defining the deformation of the nuclear shapes through

$$R = R_0 \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\varphi)\right)$$

A simple picture is obtained by assuming that, to lowest order, the restoring forces in the $\alpha_{\lambda\mu}$ degree of freedom are proportional to the amplitudes. If C_λ is the corresponding constant and D_λ is the associated mass parameter we could represent the mode by the harmonic oscillator Hamiltonian

$$H_\lambda = \sum_\mu \left(\frac{|\pi_{\lambda\mu}|^2}{2D_\lambda} + C_\lambda \frac{|\alpha_{\lambda\mu}|^2}{2} \right)$$

where $\pi_{\lambda\mu}$ stands for the conjugate momentum to the variable $\alpha_{\lambda\mu}$.

The coupling to the relative motion of a collection of such modes is introduced by defining a nuclear interaction which depends on the distance between the surfaces of the reacting nuclei. To second order in the deformation parameters this will occur along the variable of relative motion $\vec{r}$. Thus, if

$$s = r - R_a(-\hat{r}, \alpha^{(a)}) - R_A(\hat{r}, \alpha^{(A)})$$

the surface-surface interaction is given by

$$U = U(s) = U(\vec{r}, \alpha^{(a)}, \alpha^{(A)})$$

where a, A label the reacting nuclei. The functional form of this interaction term was taken to be the proximity potential introduced by Swiatecki et al. (Ref. 2)

$$U(s) = 4\pi\gamma \bar{R} \phi(s)$$

γ being the surface tension and $\bar{R}$ being related to the principal radii of curvature along $\vec{r}$.

In addition to the nuclear interaction we have to include, of course, the electromagnetic interaction and the corresponding couplings for Coulomb-excitation. For these, only the monopole-multipole terms have been retained.

The general idea becomes clear when we write the Hamiltonian for the system. The expression is very simple indeed and only the intrinsic Hamiltonian merits a special section to elaborate on its construction.

$$H = \frac{1}{2\mu}\left(p_r^2 + \frac{p_\varphi^2}{r^2 \sin^2\theta} + \frac{p_\theta^2}{r^2}\right) + \frac{z_a z_A e^2}{r}$$

$$+ \sum_{\lambda\mu}\left\{\frac{|\pi^{(a)}_{\lambda\mu}|^2}{2D^{(a)}_\lambda} + C^{(a)}_\lambda \frac{|\alpha^{(a)}_{\lambda\mu}|^2}{2}\right\}$$

$$+ \sum_{\lambda\mu}\left\{\frac{|\pi^{(A)}_{\lambda\mu}|^2}{2D^{(A)}_\lambda} + C^{(A)}_\lambda \frac{|\alpha^{(A)}_{\lambda\mu}|^2}{2}\right\}$$

$$+ U\left(r - R_a - R_A - R_a \sum_{\lambda\mu} \alpha^{(a)}_{\lambda\mu} (-)^\lambda Y^*_{\lambda\mu}(\theta,\varphi) - R_a \sum_{\lambda\mu} \alpha^{(A)}_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\varphi)\right)$$

$$+ \sum_{\lambda\mu} \frac{3 z_a z_A e^2 R_c^{(a)\lambda}}{(2\lambda+1)} \frac{\alpha^{(a)}_{\lambda\mu} (-)^\lambda Y^*_{\lambda\mu}(\theta,\varphi)}{r^{\lambda+1}}$$

$$+ \sum_{\lambda\mu} \frac{3 z_a z_A e^2 R_c^{(A)\lambda}}{(2\lambda+1)} \frac{\alpha^{(A)}_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\varphi)}{r^{\lambda+1}}$$

It may be noted that if we expand the proximity potential in terms of α , the first order term can be combined with the Coulomb-excitation term to construct the familiar macroscopic form factor for inelastic excitation. Neglecting higher orders the resulting problem of excitation of the intrinsic degrees of freedom can be solved analytically in the context of quantum mechanics.

As long as we are interested in average quantities, the results of such treatment can be proven to be equal to the results of the corresponding classical calculation. In addition to being less restrictive in the size of the deformations, the classical calculation lends itself better to the introduction of the damping of the modes which, as we shall see later, becomes part of the story.

3. Intrinsic Hamiltonian. - In this project a considerable fraction of the effort is invested in identifying the relevant surface modes of the reacting ions which couple to the relative motion. During a collision particles are exchanged between projectile and target and the strict identity of these nuclei is lost. Thus, the response functions to be constructed are not meant to refer to any specific nucleus, but rather to describe the average distribution of strength for a collection of nuclei in the neighborhood of the original projectile and target.

The response functions are constructed with a combination of theoretical and experimental ingredients. The lowest-lying collective states for the different multipolarities are, when known, incorporated directly in the calculation. These states usually collect a small fraction of the total strength for the corresponding multipolarity as conveniently expressed by the energy-weighted sum rules. Information about the higher-energy range is usually scant and therefore the construction of the representative spectra is complemented with a theoretical calculation. This is done by diagonalizing a schematic multipole-multipole interaction containing both isoscalar and isovector terms in the framework of the random phase approximation. The coupling strengths are chosen so as to reproduce the experimental data for the lowest state of each spin and parity.

A couple of examples of the results obtained from this procedure are presented below for the quadrupole and octupole modes of ^{208}Pb

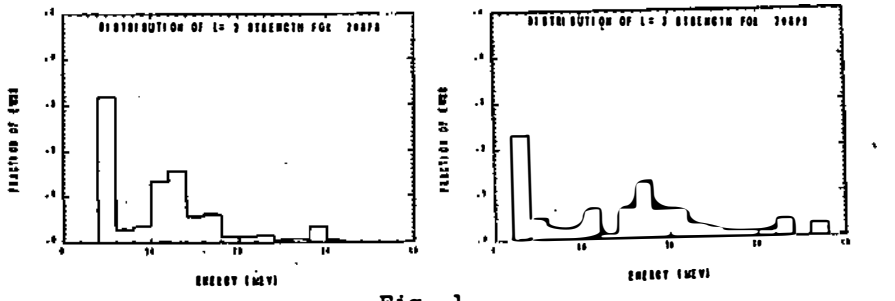

Fig. 1

In addition to the easily recognizable 2+(4.07 MeV) and 3-(2.62 MeV) states we can see in these plots the accummulation of strength around 12 MeV and 18 MeV which is identified with the so-called "giant" modes.

A small dose of imagination allows us to replace the sharp histograms of figure 1 by the following curves

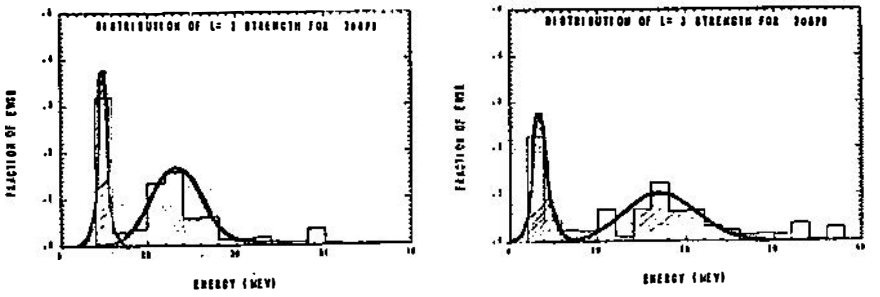

Fig. 2

in which the main features of the distribution are retained by adscribing to each concentration of strength a centroid, an area and a representative width. These three quantities are readily related to the mass, restoring force and damping term of the classical vibration which couples to the relative motion.

In fact, the classical equation of motion for the amplitude $\alpha_{\lambda\mu}$ of such a surface mode looks as follows

$$\ddot{\alpha}_{\lambda\mu} + \left(\frac{\gamma_\lambda}{D_\lambda}\right)\dot{\alpha}_{\lambda\mu} + \left(\frac{C_\lambda}{D_\lambda}\right)\alpha_{\lambda\mu} = \left(\frac{1}{D_\lambda}\right) f_{\lambda\mu}.$$

where $f_{\lambda\mu}$ stands for the driving force. The quantities in parenthesis are respectively determined by width, average frequency and accumulated strength.

It may be noted that the substitution of a collection of modes by a single damped mode which accumulates their strength is mathematically correct if the distribution of energy-weighted sum rule follows a Lorentzian curve.

There are two points in connection with the strength distributions which need to be stressed, not the least because they are easily mistaken.

A typical collision time for a reaction may be estimated, for example, by dividing the sum of the nuclear radii by the relative velocity at the barrier. Expressed in units of $\hbar$ one gets

$$\frac{\tau_{coll}}{\hbar} \cong 1 \text{ MeV}^{-1}$$

If this was also a typical "interaction" time, one would conclude that only vibrational modes of up to a few MeV could possibly be excited in the process. The argument, well known from the case of Coulomb excitation, follows from the fact that in a forced oscillator the high frequency of a mode literally averages out the effect of a slowly varying external interaction.

The nuclear form-factors, however, are sharply defined functions which change rapidly over the range of a nuclear diffusivity (~ 0.5 fm). A characteristic interaction time τ_{char} thus turns out to be an order of magnitude shorter than τ_{coll}, namely

$$\frac{\tau_{char}}{\hbar} \cong 0.1 \text{ MeV}^{-1}$$

The giant resonances can then be easily excited and results indicate that indeed they are.

The other question that arises in connection with the spectrum is concerning its stability. In deep inelastic processes large amounts of energy are transferred into the intrinsic degrees of freedom and one can be inclined to think that as the temperature increases the nuclear response function will no longer resemble the initial one (probably looking more like the uncorrelated particle-hole distribution). This is not in fact the case. Calculations of the response function were carried out in the context of the temperature-dependent RPA and the results indicate that the basic features of the zero-temperature distribution are retained for even quite high

excitation energies.

See, for example, the following comparison of the octupole response function for ^{208}Pb at T = 0 MeV and T = 2 MeV

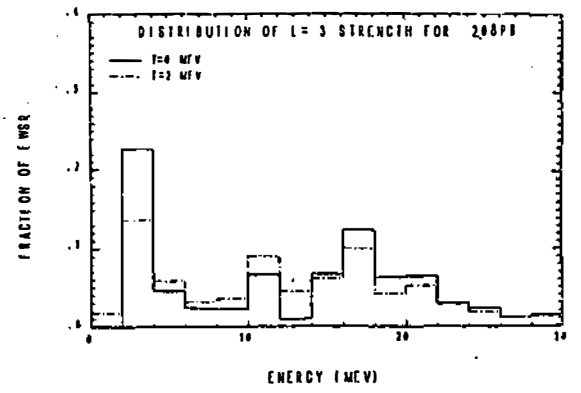

Fig. 3

Notice that in ^{208}Pb a temperature of 2 MeV corresponds to a thermal excitation of about 100 MeV

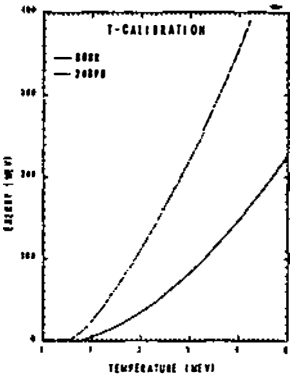

Fig. 4

The intrinsic Hamiltonians for projectile and target are constructed from the information contained in fig. 1-3 and the corresponding ones for the other multipolarities. In principle the search should be extended to $\lambda \sim 2A^{1/3} \lesssim 8$ for most nuclei. Earlier calculations including such high multipolarities have shown that besides increasing the number of channels to uncomfortable levels, modes with $\lambda \geq 6$ have little effect in the results obtained. Thus as a matter of convenience these modes have been excluded from the calculations reported here. It is possible, however, that for some matter of detail their presence may be of consequence.

4. Mass transfer. - Clearly one cannot be quite satisfied with a formalism which does not include the transfer of particles between projectile and target. In the present framework, a consistent treatment would require to identify the relevant collective degrees of freedom associated with particle-transfer. This perhaps could be done for the pairing modes and some effort has been invested in this direction. More generally, for the treatment of single-nucleon transfers we should revert to the semi-classical equations (Ref. 3) which indeed allow the handling of inelastic and transfer degrees of freedom on similar footing.

For the time being, however, none of these ideas have been fully implemented. To fill the vacuum and mostly to check how other current ideas would fit in the dynamics of this model we have incorporated the treatment of mass diffusion by means of the Fokker-Planck equation. This was suggested during a prolonged visit to Copenhagen by L.Moretto and his help in this aspect of the program has been most welcome.

We introduce an asymmetry potential

$$V(h,g) = (\text{liq.drop})_1 + (\text{liq.drop})_2 + V_{coul}(1,2) + V_{prox}(1,2) + E_{rot}(1,2)$$

which essentially represents the way in which the total mass and charge, Coulomb, nuclear and rotational energy can be split in two fragments 1,2 as a function of the mass and charge

asymmetry parameters

$$h = (A_2 - A_1) \qquad g = (Z_2 - Z_1)$$

If the relaxation of the charge asymmetry degree of freedom takes place faster than the mass drift it is possible to introduce a function g(h) defined by

$$\left.\frac{\partial V(h,g)}{\partial g}\right|_h = 0$$

which leaves the mass asymmetry parameter h as the only relevant quantity. Assuming that the resulting potential V(h) can be approximated by

$$V(h) \sim \frac{Ch^2}{2}$$

the mass distribution will follow a Gaussian whose centroid and second-moment are given by integrating

$$h(t) = h_o \exp\left(-\frac{Ct}{\varkappa}\right)$$

$$\sigma^2(t) = \frac{T}{C}\left(1 - \exp\left(-\frac{2Ct}{\varkappa}\right)\right)$$

respectively.

In this equations the temperature T is assumed to result from the distribution of the excitation energy over the whole system. The constant $\varkappa$ is the so-called frictional coefficient in the mass asymmetry degree of freedom and is a quantity strongly dependent on the geometry and dynamics of the reaction (for details see ref. 4).

The scheme is rather crude and actually at odds with the fact that there is not enough time to reach a thermal equilibrium between the two fragments. In consequence, the result reported here on the mass transfer should only be taken as temptative.

5. Applications. - We would like to illustrate the results that may be obtained from this model by showing an example of a reaction between quite heavy ions, namely $^{86}Kr + ^{208}Pb$ and another one involving a relatively lighter

nucleus, $^{40}Ca + {}^{40}Ca$.

For heavy systems the main objective is to get an adequate description (i.e. cross sections, angular distributions, etc.) of the deep inelastic events, which build up almost all of the reaction cross section. On the other hand, reactions for lighter ions are dominated by the formation of a compound nucleus. Thus, the description of the variation of fusion cross section with energy provides an important test of the ability of the model to describe the entire range of heavy ion reactions.

A considerable amount of information is generated during an actual calculation. Keeping explicit track of hundred of channels provides not only the average excitation energy and angular momenta associated with each of the fragments, but also detailed information on their distribution into the different modes, nuclear shapes, temperatures, etc.

One gets, however, a quick feeling of the results from the classical deflection function and final-energy curve shown below for the Kr + Pb reaction and for three different incident energies.

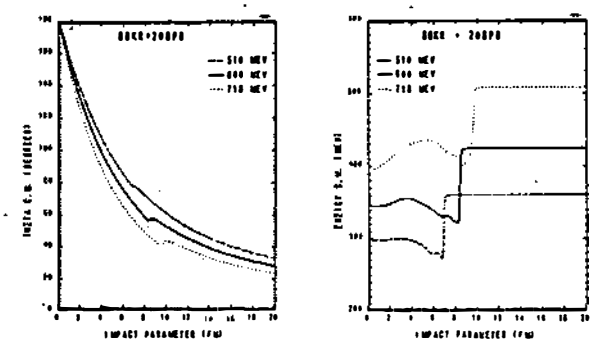

Fig. 5

The energy-loss curve enables us to identify the range of impparameters associated with the elastic, quasi-elastic and deep inelastic events and determine the corresponding cross sections. The classical deflection function allows us to construct the angular distributions for the deep-inelastic processes and they are shown in Fig. 6. The agreement with the experimental data is quite good (Ref. 5)

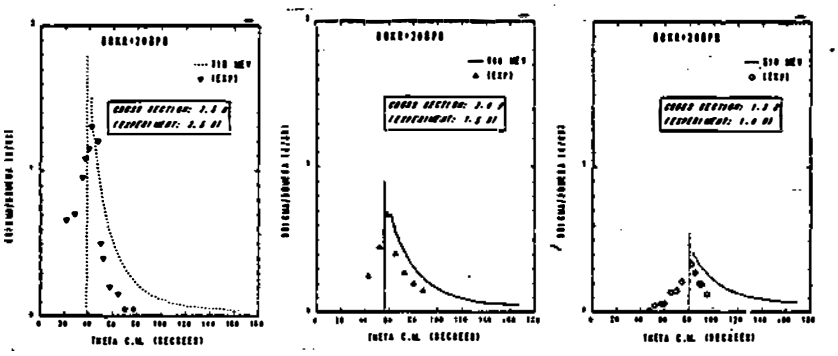

Fig. 6

In this picture the concentration of cross section at an angle slightly ahead of the grazing peak is an almost inescapable consequence of the shape of the elastic deflection function. In fact, Coulomb-dominated for small impact parameters, the elastic deflection function would practically imply a similar angular distributions for the inner branch of the deflection function (i.e. impact parameters which are smaller than the one corresponding to the nuclear rainbow).

The distribution of mass for the events with an energy loss of more than 20 MeV as predicted by the Fokker-Planck equation is shown in Fig. 7

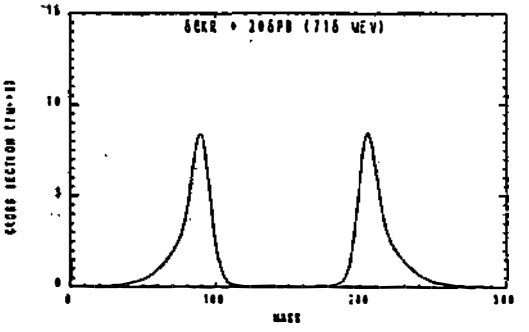

Fig. 7

So far no attempt has been made to feed-back this information into the equations of motion which define the classical averages. It is likely that when this step is taken the mass transfer will favor higher transfers of angular momenta from the relative motion into the nuclei. The maximum angular momentum absorbed by ^{208}Pb in this example is about 36 units of $\hbar$.

The deflection function and energy loss for the reaction $^{40}Ca + {}^{40}Ca$ are shown below for six bombarding energies

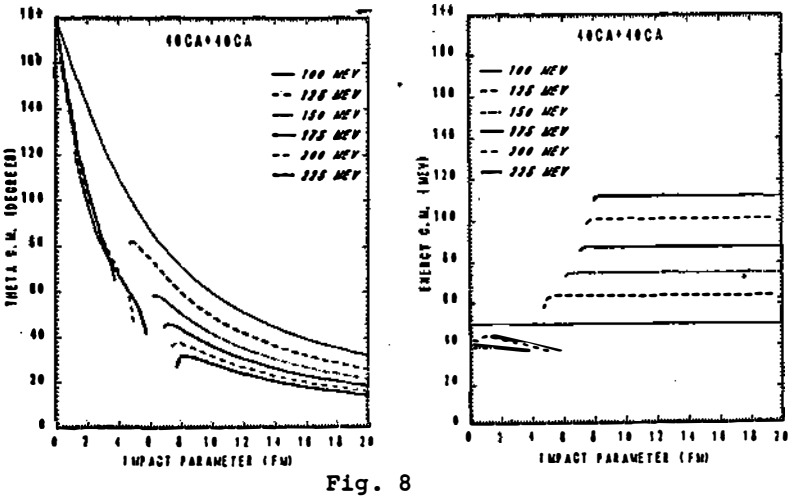

Fig. 8

Whenever these curves are interrupt, it means that there is no emerging trajectory for the corresponding impact parameters.

As soon as the energy in the center of mass goes above the Coulomb barrier this "capture" occurs for all impact parameters below the one corresponding to grazing. Under these circumstances, the fusion cross section increases with energy following essentially the well established geometrical law

$$\sigma_{reac} = \pi R^2_{reac}(1 - \frac{E_B}{E})$$

Once a certain threshold of energy is reached (~ 1.5 times the Coulomb barrier in this example) dynamical difficulties develop which inhibit the formation of the compound nucleus at low impact parameters. The corresponding fusion cross section deviates from the geometrical value so that the plot of fusion cross section energy eventually looks as shown in Fig. 9 (Ref. 6)

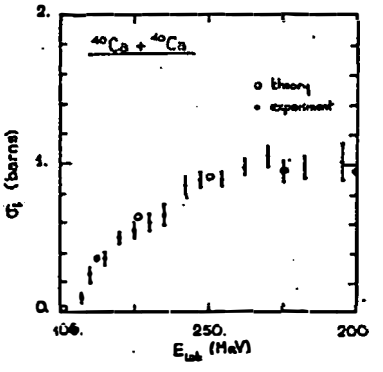

Fig. 9

where we see that, again, agreement with the experiment is good.

Just for amusement and without elaboration we can see in Fig. 10 and in a series of frames how the collision of Ca + Ca would look in the center of mass for one of the deep inelastic events. The trajectory corresponds to an impact parameter of 3 fm at a laboratory energy of 175 MeV.

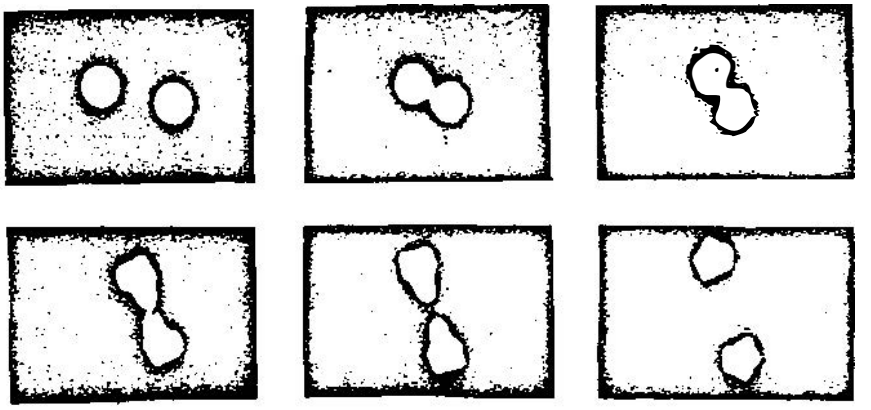

Fig. 10

Uncertainties in the energies of the giant modes arising from the use of different single-particle bases and/or coupling strengths may be in the order of one to three MeV. More troublesome is the question of the widths. In fact, the range in which the strength associated with a giant resonance may be spread over in the RPA calculation is only a part of the problem. The actual width of such collective states is also due to the coupling of the modes to a background of two particle-two hole states and other higher-order excitations. Ignoring these effects, the RPA distribution tends often to underestimate the spread at least for cases where there is experimental evidence. Work is in progress in Copenhagen which eventually may lead to theoretical estimations of the widths of the giant modes. So far, however we have relied on figures provided by the strength distribution and experiments. Reassuringly, changing widths by up to factors of two and shifting centroids by a few MeV does not affect the qualitative nature of the results presented here.

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