APPLICATION OF THE MARUMORI BOSE-EXPANSION TO TRANSITIONAL NUCLEI

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The idea of using weakly coupled quadrupole bosons to describe the low-lying states of even-even transitional nuclei has met with considerable success |1|. In many cases it has been possible to add anharmonic corrections to a harmonic collective Hamiltonian in such a way that the number of low-lying states, their spins, energies and E2 transition rates agree surprisingly well with experimental spectra.

In fig. 1 we give five examples where coefficients of a fourth-order power series in terms of quadrupole Bose-operators

$$\begin{aligned} H_{coll} = \tfrac{1}{2}\, h_{11} [B^+B]_0 + h_{20} [B^+B^+]_0 + h_{21} [[B^+B^+]_2 B]_0 \, + \\ + \, h_{30} [[B^+B^+]_2 B^+]_0 + \tfrac{1}{2} \sum_{J=0,2,4} h_{22}^{(J)} [[B^+B^+]_J [BB]_J]_0 \, + \\ + \, h_{31} [B^+B^+]_0 [B^+B]_0 + h_{40} [B^+B^+]_0 [B^+B^+]_0 + \text{h.c.} \qquad (1) \end{aligned}$$

have been adjusted to fit the eigenvalues of H_{coll} to the known experimental energies. (The brackets $[\ ]_J$ indicate angular momentum coupling). The $h_{20} [B^+B^+]_0$ terms can always be chosen to be zero and the corresponding coefficients are given in table 1. The fact that such a correspondence can easily be established indicates that in these nuclei the collective phonon states should be well separated from particle degrees of freedom.

In such cases the boson expansion technique seems to be the proper means to obtain H_{coll} from a microscopic

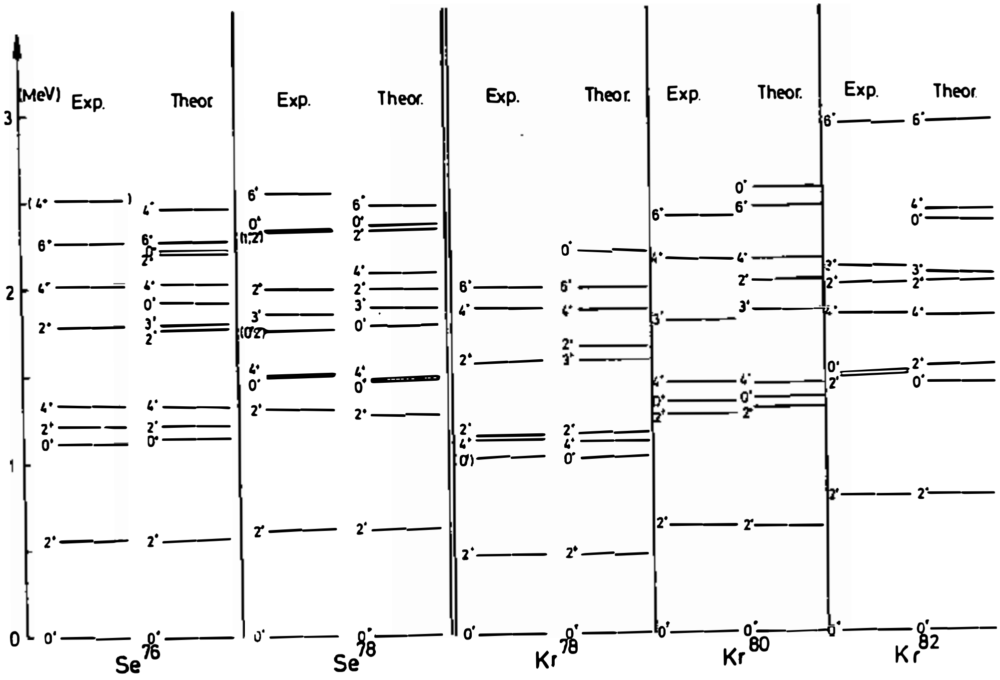

Fig.1. Low-lying spectra of five transitional nuclei as compared with the eigenvalues of H_{coll} from eq. (1) with coefficients given in table 1.

TABLE 1

The ratios of the coefficients $r_{ij} = h_{ij}/h_{11}$ from the Hamiltonian of eq. (1) fitted to the experimental spectra.

	r_{21}	r_{30}	$r_{22}^{(0)}$	$r_{22}^{(2)}$	$r_{22}^{(4)}$	r_{31}	r_{40}
Se^{76}	0.278	0.062	0.192	0.028	0.375	0.389	0.070
Se^{78}	0.212	0.071	0.277	0.189	0.526	0.605	0.115
Kr^{78}	0.380	0.126	0.309	-0.032	0.328	0.380	0.095
Kr^{80}	0.263	0.088	0.344	-0.035	0.296	0.375	0.096
Kr^{82}	0.156	0.052	0.230	-0.067	0.232	0.363	0.170

fermion Hamiltonian. This method is especially suited to describe groups of excited states related to certain well-separated degrees of freedom and the expansion will converge the better the more collective, i.e. bosonlike these degrees of freedom are.

For the expansion we use the prescription suggested by Marumori et al. |2| which to our knowledge has not yet been applied to transitional nuclei. The separation of the nuclear degrees of freedom is achieved in the space of two quasi-particle states |3| (Tamm-Dancoff approximation) which means that the B^+B-part of the Bose-Hamiltonian is diagonalized. To derive the anharmonicities for the collective branch, we select the creation operator for the collective $2^+(T=0)$ mode:

$$A^+ = \sum c_{\alpha\beta} b^+_\alpha b^+_\beta$$

Here the b^+_α are quasi-particle operators ($\alpha \equiv n_\alpha \ell_\alpha j_\alpha m_\alpha t_{3\alpha}$) and the ($c_{\alpha\beta}$) represent the lowest lying $2^+(T=0)$ eigenvector of the Tamm-Dancoff equation. From this we construct the normalized 2,4,6-quasiparticle states

$$|2\rangle = \frac{1}{c_2} A^+ |0\rangle$$

$$|4\rangle_J = \frac{1}{c_{4(J)}} [A^+ A^+]_J |0\rangle$$

$$|6\rangle_J = \frac{1}{c_{6(J)}} [A^+ A^+ A^+]_J |0\rangle$$

where $|0\rangle$ is the quasiparticle vacuum (BCS-ground state) and the $c_{i(J)}$ are normalization constants. If we denote the $(b^+)^i (b)^j$ parts of the microscopic Hamiltonian as H_{ij}, the Marumori expansion leads to the following expressions for the coefficients h_{ij}:

$$h_{11} = \langle 2|H_{11}|2\rangle + \langle 2|H_{22}|2\rangle$$

$$h_{20} = {}_0\langle 4|H_{40}|0\rangle/\sqrt{2!} \qquad (2)$$

$$h_{21} = {}_2\langle 4|H_{31}|2\rangle/\sqrt{2!}$$

$$h_{31} = ({}_2\langle 6|H_{40}|2\rangle / \sqrt{14\cdot 3!} - h_{20})\sqrt{5}$$

$$h_{22}^{(J)} = ({}_J\langle 4|H_{11}|4\rangle_J + {}_J\langle 4|H_{22}|4\rangle_J - \sqrt{2J+1}/\sqrt{5}\cdot h_{11})/2!$$

$$h_{30} = h_{40} = 0$$

The last equality is due to our choice of the operator A^+ containing only two quasiparticle creation operators (TD approx.). By unitary transformations

$$O^+ = xB^+ + yB \qquad \text{with } x^2-y^2 = 1$$

the $[O^+O^+]_0$-terms can always be made to disappear, thereby creating nonvanishing h_{30} and h_{40} terms. The resulting coefficients h_{ij} can then be directly compared with the anharmonicities obtained from a fit to the experimental spectrum.

For the actual calculation we have used the surface delta interaction |4|

$$V = -4\pi(\chi_s P_s + \chi_t P_t)\delta(r_1-R)\delta(r_2-R)\delta(\theta_{12})/r_1 r_2$$

with χ_s and χ_t as parameters for singlet and triplet force strengths, respectively, P_s, P_t being the corresponding projectors. This force was used as residual interaction between particles whose single-particle energies were based on the work of Gustavsson |5|.

It can be seen from table 1 that the differences in the coefficients h_{ij} for the considered nuclei generally are not very large, which means that the spectra depend sensitively on the precise values of these coefficients. Therefore we cannot expect that microscopic evaluations of the anharmonicities could lead to perfect agreement with the low-lying spectrum of a given nucleus. It further turned out during this investigation that small changes in the values of the single-particle energies affect the expansion coefficients as strongly as changes

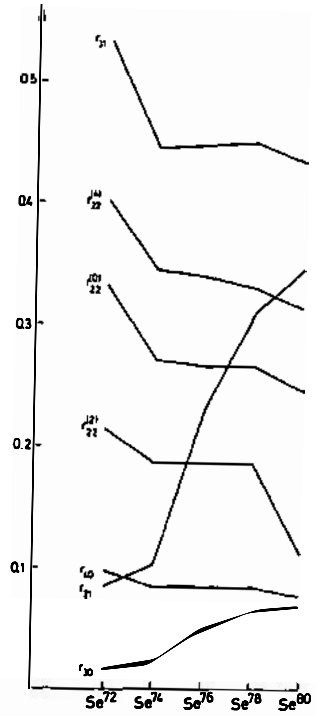

Fig.2. The ratios of the Bose-expansion coefficients $r_{ij} = h_{ij}/h_{11}$ as functions of neutron number for the Se-isotopes. (Gustavsson single-particle energies, $\chi_s = 17/A$, $\chi_s/\chi_t = 1$).

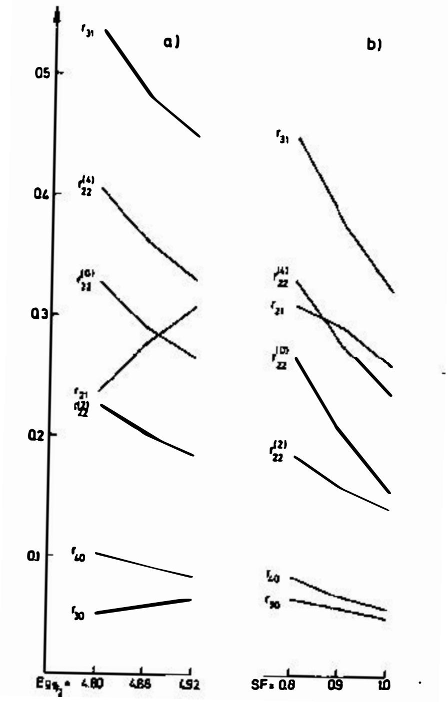

Fig.3. The ratios of the Bose expansion coefficients $r_{ij} = h_{ij}/h_{11}$ as functions of a) the $g_{9/2}$ neutron-single particle energy and b) a common scaling factor for the single-particle energies.

TABLE 2

Comparison for the nucleus Se^{76} between the coefficients from table 1 and the results of eqs. (2) evaluated with slightly adjusted single-particle energies.

	r_{21}	r_{30}	$r_{22}^{(0)}$	$r_{22}^{(2)}$	$r_{22}^{(4)}$	r_{31}	r_{40}
Se^{76}	0.278	0.062	0.192	0.028	0.375	0.389	0.070
Se^{76}	0.272	0.060	0.228	0.165	0.360	0.397	0.073

in proton or neutron number when going from one nucleus to the other (see figs. 2 and 3). Nevertheless we can see from comparing the numbers in fig. 2 with table 1 that the calculated values of the expansion coefficients lie well within the range of parameters that are needed to fit the experimental spectra. By slightly changing the single-particle energies it is not too difficult to obtain a set of expansion coefficients from the microscopic calculation which agree closely with the values of table 1. As an example we present such a comparison for ^{76}Se in table 2.

We would like to conclude that the Bose expansion method leads to a consistent qualitative picture for the low-lying states of transitional nuclei even if the details of the underlying single-particle structure are not sufficiently well known to allow for definite quantitative statements.

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