ON THE VALIDITY OF THE QUASI-BOSON APPROXIMATION

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In the last years, the foundations of the RPA have been subjected to careful scrutiny and doubts about its consistency have arisen |1|. Most of the corresponding theoretical work, however, has been devoted to the particle-hole version of the RPA. It is our purpose here to discuss the consistency of this approximation, i.e., the validity of the quasi-boson approach, as applied to the description of vibrational states in superconducting nuclei. In these cases, the method employed is referred to as the Quasi-particle RPA (QRPA).

QRPA descriptions of collective states are found in the literature in connection, mostly, with two kinds of nuclei, i.e., single-closed shell (S.C.S.) ones (see for example ref. |2|) and deformed heavy nuclei (rare earth, transuranic) (see for example ref. |3|).

These two types of nuclide present us with widely different peculiarities. In the latter we find a stable deformed shape, highly collective character of the vibrations,

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great single-particle level density and a large number of both protons and neutrons outside closed shells. In the S.C.S. case, we have just a few particles outside closed shells, in relatively widely spaced single-particle levels. The equilibrium shape is spherical and the collective character of the vibrations around it is weak. Since the conditions for the two kinds of nuclei look quite antithetical, one cannot a priori expect similar results concerning the consistency of the theoretical approach.

The QRPA eigenvalue problem for an excited state of energy ω is given by the familiar equation |3|

$$\begin{pmatrix} P & Q \\ -Q & -P \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix} , \qquad (1)$$

while the QRPA correlated ground state is given by operating on the Hartree-Fock-Bogoliubov one with an exponential operator,

$$|c.g.s.\rangle = \exp\{-\frac{1}{2}\sum_{\substack{nL\\ ik\ell m}} C^{L}_{ik\ell m}\,|B^{+L}_{ik}\,B^{+L}_{\ell m}|^{0}_{0}\}\,|HFB\rangle , \qquad (2)$$

where L denotes the set of quantum numbers characterizing the excited state of energy ω, n distinguishes among different states of equal L and the B´s are two quasi-particle (q.p.) operators $b^{+}_{i}\, b^{+}_{k}$ coupled to L. The C´s are the so-called correlation coefficients, which in matrix notation relate the X and Y amplitudes of (1) in the following way

$$Y = C\,X \qquad (3)$$

The critical assumption made in deriving (1) can be stated as

$$<\mathrm{c.g.s.}|[B^{+}_{ik}, B_{\ell m}]|\mathrm{c.g.s.}> \simeq <\mathrm{HFB}|[B^{+}_{ik}, B_{\ell m}]|\mathrm{HFB}> \quad , \tag{4}$$

This is the so-called quasi-boson approach. First order corrections to (4) are linear in the q.p. density matrix. |5|.

$$\rho_{ij} = <\mathrm{c.g.s.}|b^{+}_{j}\ b^{+}_{i}|\mathrm{c.g.s.}> \quad . \tag{5}$$

Evaluation of ρ allows one for a test of the consistency of the QRPA, since one assumes (4) to hold in order to derive (1), solving it one is able, after inversion, to obtain C from (3). The correlation coefficients permit one, afterwards, to build up explicitly the c.g.s. (2), and, in consequence to extract ρ, which should be small if the original assumption is valid.

We have followed this procedure in order to evaluate the diagonal elements of the q.p. density matrix in the case of a) Lead S.C.S. isotopes and b) quadrupole vibrations in rare earth nuclei. In the first case, the largest diagonal elements range from 0.026 in ^{206}Pb to 0.358 in ^{198}Pb. Table 1 illustrates the situation for the rare earth nuclei.

TABLE 1

^{152}Sm	0.067	^{158}Gd	0.048	^{162}Dy	0.047
^{154}Sm	0.068	^{160}Gd	0.077	^{164}Dy	0.064
^{166}Er	0.083	^{174}Yb	0.085	^{176}Hf	0.093
^{168}Er	0.089	^{176}Yb	0.085	^{178}Hf	0.076
		^{180}W	0.095	^{184}Os	0.097
		^{182}W	0.081	^{186}Os	0.086

All the calculations were performed with a Surface Delta Interaction as the residual force. The corresponding numerical details are given in ref. |6|.

Inspection of table 1 allows one to conclude that the validity of the quasi-boson approach holds.

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