THE INTERACTING BOSON MODEL

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Nuclei away from closed shells and regions of large deformations often appear to be characterized by a typical vibrational-like spectrum |1| with large splitting between members of the two-phonon $0^+, 2^+, 4^+$ triplet, relatively large quadrupole moment of the first excited 2_1^+ state, but rather small values for the matrix elements of the forbidden $\Delta n=2$ transitions. Typically, the BE2 ratio

$$R = \frac{B(E2; 2_2^+ \to 0_1^+)}{B(E2; 2_2^+ \to 2_1^+)} \tag{1}$$

ranges between 10^{-3} and 3×10^{-2}.

To accommodate these experimental results, we have recently proposed |2| a phenomenological model of vibrational nuclei described by the Hamiltonian

$$H = \varepsilon \sum_m b_m^\dagger b_m + \sum_L c_L [(b^\dagger b^\dagger)_L (bb)_L]_0 \tag{2}$$

and by the transition operator

$$T_m^2 = t(b_m^\dagger + (-)^m b_{-m}) + t'(b^\dagger b)_m^2 \tag{3}$$

where $b^\dagger$(b) are the creation (annihilation) operators for quadrupole phonons and the parentheses denote angular momentum couplings. In this model the large anharmonicity in the energies is caused by the phonon-phonon interaction described by the term c_L (L=0,2,4), the quadrupole moment of the first excited state arises from the second term in the transition operator, and the cross-over $\Delta n=2$ transitions are strictly forbidden. Our model can be obtained from previously discussed models by neglecting all anharmonic terms except the boson-boson interaction c_L. In particular it is identical to that of Das, Dreizler and Klein |3| if one drops the terms of d_I, c_{2I}, a_4 in their eqs. (8) and (9) and to that of Brink, Kerman and de Toledo Piza |4| if one assumes that the phonon changing terms are small from the outset and that therefore no unitary transformation is needed to bring the Hamiltonian to the form of eq. (2).

The main advantage of this model is that the Hamiltonian of eq. (2) is diagonal in the boson basis. Using standard group theoretical techniques |5| it is possible to obtain its eigenvalues in compact form. They are given by |2|

$$E(n,v,n_\Delta,L,M) = \varepsilon n + \alpha \frac{n(n-1)}{2} + \beta(n-v)(n+v+3) + \gamma\left[L(L+1)-6n\right] . \qquad (3)$$

The parameters α,β,γ (related to c_0, c_2 and c_4) describe the phonon-phonon interaction and n, v, n_Δ, L, M are the quantum numbers which completely label the representations of the five-dimensional rotation group O(5). As it can be seen from Eq. (3), the boson-boson interaction splits but does not admix the different representations of O(5) and thus it gives rise to a dynamical symmetry. The possible existence of this symmetry (also suggested in a slightly less general form by Ferreira, Alcarás and Navarro |6|) and its experimental verification is one of the main results of our approach. The observation of the cross-over transitions indicates that this dynamical symmetry is broken.

However, the breaking appears to be small, of the order of few percent, and it can therefore be treated in perturbation theory |7|.

In addition to the quadrupole mode, vibrational nuclei are expected to exhibit other excitation modes such as the octupole mode and two or more quasi-particle modes |8|. In view of the arguments above we now propose to treat these other modes in the same fashion as the quadrupole mode and thus we formulate a general interacting boson model by defining a Hamiltonian

$$H = \sum_{\ell m} \varepsilon_\ell b^\dagger_{\ell m} b_{\ell m} + \sum_{\substack{\lambda \\ \ell\ell'\ell''\ell'''}} c_{\lambda,\ell\ell'\ell''\ell'''} \left[(b^\dagger_{\ell'} b^\dagger_{\ell})^\lambda_\mu (b_{\ell''} b_{\ell'''})^\lambda_\mu \right]^0_0 \tag{4}$$

and transition operators

$$Q^\ell_m = q_\ell (b^\dagger_{\ell m} + (-)^{\ell-m} b_{\ell,-m}) + \sum_{\ell'\ell''} t_{\ell,\ell'\ell''} (b^\dagger_{\ell''} b_{\ell'})^\ell_m \tag{5}$$

where ℓ labels the different excitation modes. This Hamiltonian describes a system of interacting bosons and it is thus analogous to the shell-model Hamiltonian.

Deferring a more detailed study of the properties of our model to a longer paper |7|, we quote here some results which can be inferred from eqs. (4) and (5). (i) The coupling of an excitation mode with angular momentum j to the states of the quadrupole phonon introduces in general many new states. However, only two sets of states appear to form stable "bands", a band being defined as a set of levels connected by strong E2 transitions. They correspond to the totally aligned, I=2n+j, and totally aligned minus one, I=2n+j-1, states where n is the number of quadrupole phonons. The energies of these states are given by simple expressions. As an example we consider the case of a single octupole boson, denoted by f, coupled to the quadrupole boson, denoted by d. The energies of the states belonging

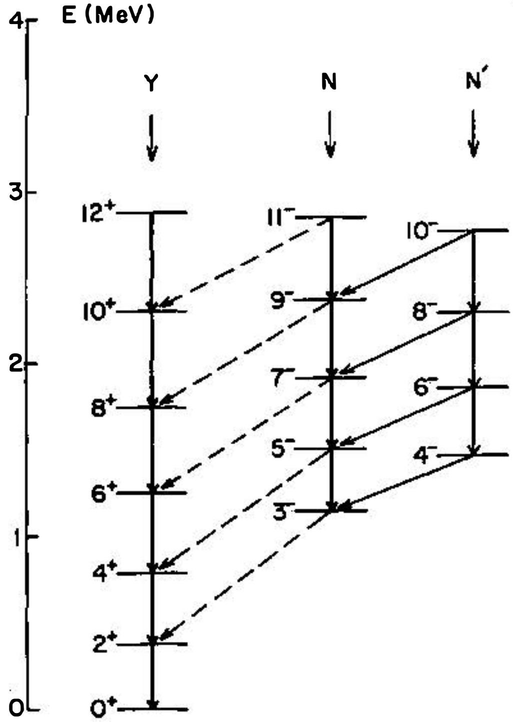

Fig.1. Schematic representation of the coupling of an octupole 3^- excitation mode to the quadrupole phonon. The two negative parity bands are called N and N´, respectively. The parameters used are ε_2=365 keV, $c_4^{(22)}$= =45 keV, ε_3=1123 keV, $c_5^{(23)}$=-10 keV, $\Delta_4^{(23)}$= =-40 keV. Broken lines represent E1 transitions, full lines represent E2 transitions. Only E1 transitions from the N to the Y band are shown.

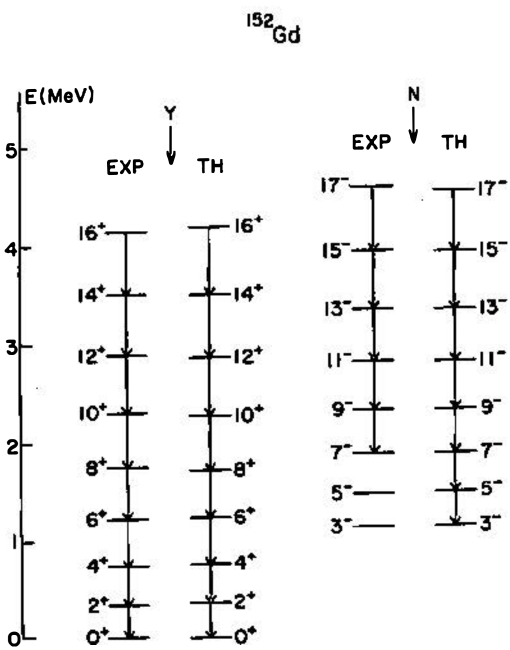

Fig.2. Comparison between experimental |9| and theoretical energies of the ground state (Y) and negative parity (N) band in ^{152}Gd. The parameters in the theoretical spectrum are the same as in fig. 1.

to the totally aligned band are given by

$$E|d^n(J_1=2n)f;I=2n+3| = E|d^n(J_1=2n)|+\varepsilon_3+nc_5^{(23)} \qquad (6)$$

and those of the states belonging to the totally aligned minus one band by

$$E|d^n(J_1=2n)f;I=2n+2| = E|d^n(J_1=2n)|+\varepsilon_3+nc_5^{(23)}+ + \frac{2n+3}{5}\Delta_4^{(23)} . \qquad (7)$$

In these expressions the first term on the right-hand side represents the energy of the ground state band (y-band in ref. |2|) and $c_5^{(23)}$ and $\Delta_4^{(23)}$ are two new parameters related to the boson-boson interaction. The corresponding spectrum is shown in fig. 1. (ii) The two bands which arise from the coupling of the octupole to the quadrupole mode can de-excite either by E2 transitions, first term in eq. (5), or by E1 transitions, second term in eq. (5) with $\ell''=2$ and $\ell'=3$. For the totally aligned band the branching ratio E1/E2 is given by

$$\frac{B|E1;(I=2n+3)^- \to (I=2n+2)^+|}{B|E2;(I=2n+3)^- \to (I=2n+1)^-|} = \frac{n+1}{n}\, c \qquad (8)$$

where c is a constant for each nucleus. There is compounding evidence that these bands are observed in nuclei. A typical application of eq. (6) is shown in fig. 2. The nucleus to which it refers, ^{152}Gd, is not completely vibrational but its ground state band can still be described by the boson formula

$$E|d^n(J_1=2n)| = \varepsilon_2 n + c_4^{(22)}\frac{n(n-1)}{2} . \qquad (9)$$

Similar relations apply to the coupling of a two

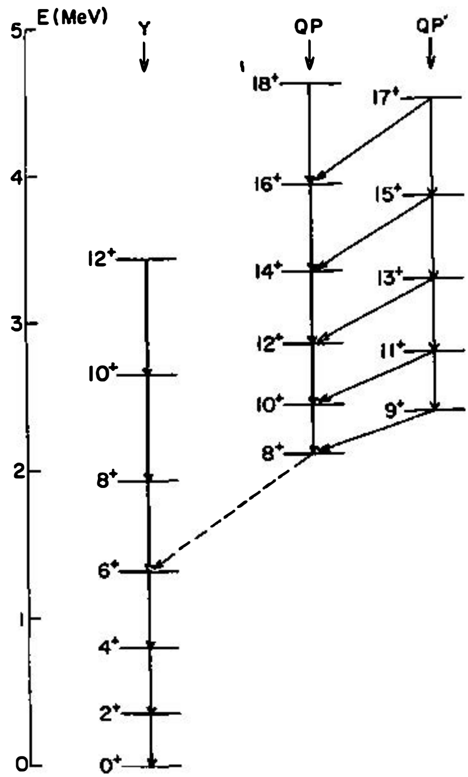

Fig.3. Schematic representation of the coupling of a high-spin two quasi-particle state (8^+) to the quadrupole phonon. The two new bands are called QP and QP′, respectively. E2 transitions within bands are represented by full lines. The full E2 transition de-exciting the lowest member of the QP band is indicated by a broken line.

quasi-particle state in even-even nuclei or to the coupling of a single quasi-particle state in odd-A nuclei. The expected behavior for the coupling of a two quasi-particle state with spin 8^+ is shown in fig. 3. It is worth mentioning that in the case in which the two quasi-particle state occurs at an energy below the corresponding state of the same spin in the ground state band, it will cause a dramatic backbending effect. However, since the states on top of the two quasi-particle state are built on the same basic excitation mode, the expected behavior of the band built on such a state will very closely resemble that of the corresponding ground state band. A full detailed description of these properties will be given elsewhere |7|.

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