NUCLEAR FIELD TREATMENT OF ^{209}Bi

P.F. Bortignon
The Niels Bohr Institute, University of Copenhagen,
Copenhagen, Denmark
Istituto di Fisica dell'Università, Padova, Italy*
Istituto Nazionale di Fisica Nucleare,
Sezione di Padova

From Prof. Broglia's talk |1| we learned how the graphical perturbation treatment of nuclear fields (particle-vibration theory) converges to the exact solution, thus correcting itself for violations of the Pauli principle and for the non-orthogonality of the elementary modes. The corresponding rules as well as all the interaction vertices required to obtain this convergence were presented.

In my contribution, I am going to discuss some results we obtained |2| by applying in a systematic way this nuclear field theoretical approach to the (2p-1h) type of states in ^{209}Bi. In this case the elementary modes of excitation (free fields) are surface and pairing vibrations (bosons) and single-particles (fermions). The unperturbed states in terms of the free fields are given by

$$|j_1^{-1}\otimes\lambda_1(^{210}\text{Po});IM> \quad (\lambda_1^\pi=0_1^+,2_1^+,4_1^+,6_1^+,8_1^+) \tag{1}$$

$$|j_2\otimes\lambda_1(^{208}\text{Pb});IM> \quad (\lambda_1^\pi=2^+,3^-) \tag{2}$$

where the states of ^{210}Po are the pairing vibrations and those of ^{208}Pb the surface vibrations. This is the basic set of states $\{\alpha_i\}$ which give rise to the model space. The single-hole and -particle states are:

---

* Permanent address.

$j_1^{-1} \equiv (1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 1h_{11/2}, 2d_{3/2}, 3s_{1/2})$ and $j_2 \equiv (1h_{9/2}, 2f_{7/2}, 1i_{13/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{11/2})$.

Their energies are taken from experiment as well as the energies of the bosons. All other states, e.g. 2p-1h, two phonons-1 particle etc., give rise to the complementary Hilbert space $\{a_i\}$ and appear only as intermediate states.

A multipole pairing and a multipole particle-hole force are used as residual interactions (for details see ref. |3|). According to the rules discussed in |1|, the coupling vertices are proportional to the model bare interaction coupling constant G_λ and k_λ and to the particle-vibration coupling strengths $\Lambda_n(\alpha\lambda)$, where λ is the multipolarity of the phonon, α the transfer quantum number and n indicates the root number. Their values are determined by the experimental energies $W_1(\alpha\lambda)$, through the RPA dispersion relation and the wave function normalization condition |3|. These coupling terms can act in all the allowed ways to generate the different perturbation theory graphs. They correspond to the expansion of the effective energy dependent residual interaction, (cfr. |2|, |3| and |4|)

$$h_{eff}(E) = h + h\frac{Q}{E-H_o}h_{eff} = h + h\frac{Q}{E-H_o}h + h\frac{Q}{E-H_o}h\frac{Q}{E-H_o}h + h\frac{Q}{E-H_o}h + \ldots \quad (3)$$

eliminating those diagrams which contain bubbles |1|. In (4) H_o is the part of the total hamiltonian diagonal in $|\alpha_i\rangle$ and $|a_i\rangle$, Q the projection operator in the complementary space and h includes all the coupling terms. Thus, the graphs displayed in fig. 1 correspond to the graphical expansion of h_{eff} up to 4. order in h. The analytic expression for the numerators are collected in appendix C of ref. |2|. The secular equation generated by the diagonalization of $h_{eff}(E)$ in the model space $\{\alpha_i\}$ must be solved in a self-

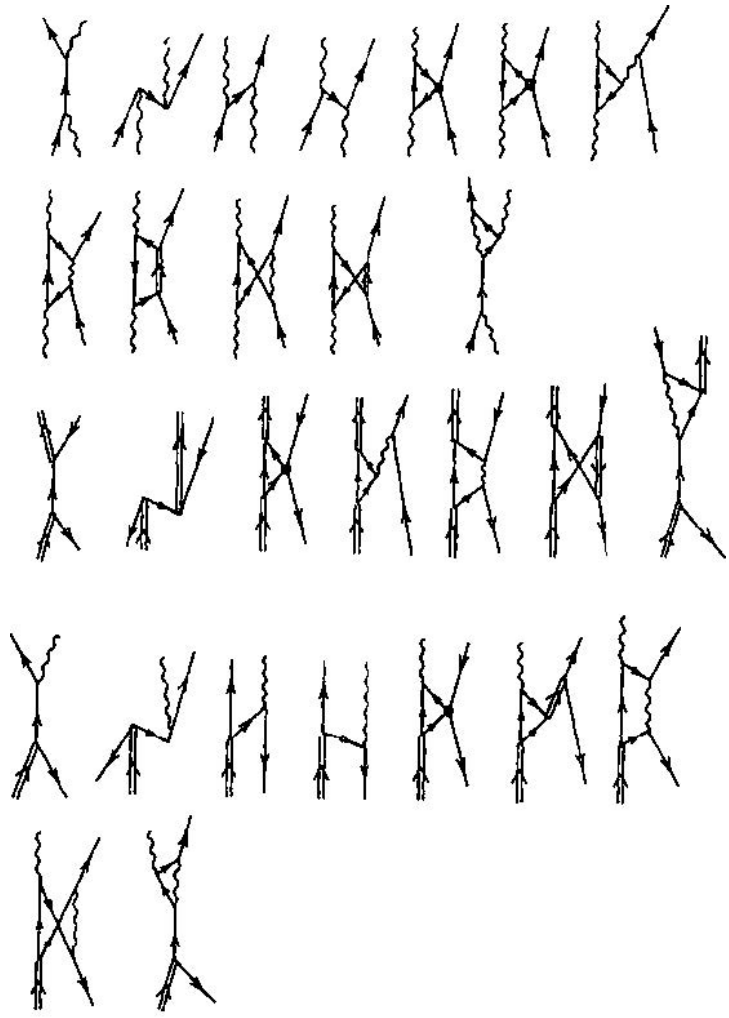

Fig.1. Contributions to the energy of the states up to 4. order in h. The wavy lines represent a surface vibration, while the double arrowed lines stand for a pairing vibration. The full dot and the full square correspond to the multipole particle-hole and to the multipole pairing model bare interactions.

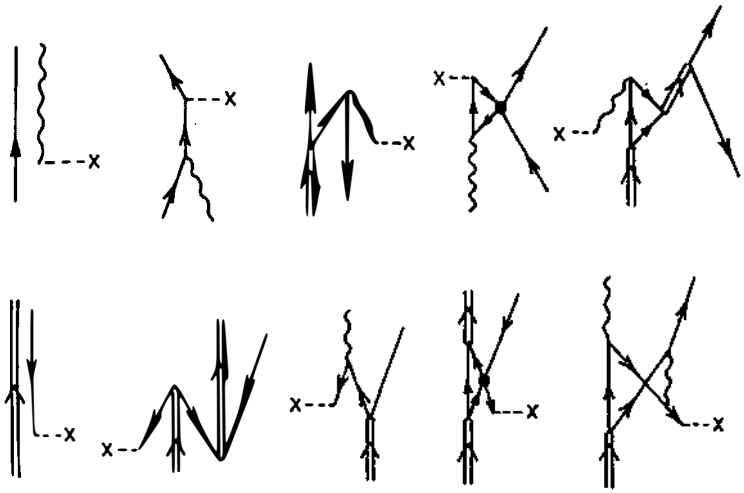

Fig.2. Examples of contributions to inelastic scattering and one-particle pick-up processes. The dotted lines represent the transition operator.

consistent way, because of the energy dependence of $h_{eff}(E)$, taking into account the finite overlap of $|\alpha_i\rangle$ and $|a_i\rangle$ (cfr. |2| and |4|).

The matrix element $\langle n|T|n'\rangle$ of any transition operator can be expressed as a perturbative expansion in terms of h, which can also be worked out graphically. This expansion defines a generalized effective operator. The diagrams of fig. 2 provide examples.

We calculated |2| energies, single-particle transfer spectroscopic factors and inelastic scattering cross sections associated with the members of the septuplet $(1h_{9/2} \otimes 3^-(^{208}Pb))$ up to second order in the interaction vertices. Also, the second $3/2^+$ state, strongly excited in the $^{210}Po(t,\alpha)$ reaction at E_x=2.95 MeV, is studied.

I am going to talk only about the two $3/2^+$ states,

TABLE 1

Energies, electromagnetic transition probabilities and one-particle pick-up spectroscopic factors for the two $3/2^+$ states. See refs. |5| an |7| for the experimental values.

E_n (MeV)		$\frac{d\sigma(g.s.(^{209}Bi)\to J(^{209}Bi))^*}{d\sigma(g.s.(^{208}Pb)\to 3^-(^{208}Pb))}$		S(t,α)	
Theory	Exp.	Theory	Exp.	Theory	Exp.
2.577	2.48	3.4	4.2±0.3	1.81	1.8
2.922	2.95	1.6	1.1	2.14	2.2

*Ratio of inelastic scattering cross sections (%).

which provide a typical example to the coupling between surface and pairing vibrations, as first pointed out in refs. |3| and |5|. The problem is solved by expanding $h_{eff}(E)$ and the effective transition operators up to 4. order in the interactions, including the coupling terms proportional to k_λ and G_λ (four point vertices). The model space is spanned by the two basic states

$$|1\rangle \equiv |1h_{9/2}\otimes 3^-(^{208}Pb);3/2^+\rangle \qquad E^{(o)} = 2.615 \text{ MeV} . \quad (4)$$

$$|2\rangle \equiv |2d_{3/2}\otimes g.s.(^{210}Po);3/2^+\rangle \qquad E^{(o)} = 2.733^* \text{ MeV} . \quad (5)$$

The predictions in comparison with the experimental data are collected in table 1.

*The Coulomb correction $\Delta E_C = -670$ keV, |2|, |5| to the unperturbed energy has been taken into account.

The two basic states start 118 keV apart and they mix strongly through the couplings of the type depicted in the graphs of the last two rows in fig. 1. Because of the energy dependence of $h_{eff}(E)$, there is one matrix for each final state and there are thus two off-diagonal matrix elements.

Both are $M \equiv \langle 1|h_{eff}(E)|2\rangle \gtrsim 200$ keV. A typical value for the energy denominators is $\Delta E \gtrsim 1.5$ MeV. Thus $M/\Delta E \gtrsim 0.1$, ensuring the applicability of the perturbation techniques to the problem. In ref. |6| the matrix element M is estimated by counting only the third mixing diagram of fig. 1 and the result is M = 300 keV.

The contributions of the different diagrams to the energies, inelastic scattering and pick-up reaction are given in detail in ref. |2|.

REFERENCES

|1| D.R. Bes and R.A. Broglia, invited talk to this Conference;
D.R. Bes, R.A. Broglia, G.G. Dussel, R. Liotta and B. Mottelson, Phys. Lett. 52B (1974) 253.

|2| P.F. Bortignon, R.A. Broglia, R. Liotta, D.R. Bes, V. Paar, to be published.

|3| D.R. Bes and R.A. Broglia, Phys. Rev. C3 (1971) 2349.

|4| G.G. Dussel and R. Liotta, Phys. Lett. 37B (1971) 477.

|5| P. Barnes, E. Romberg, C. Ellegaard, R. Casten, O. Hansen, J. Mulligan, R.A. Broglia and R. Liotta, Nucl. Phys. A195 (1972) 146.

|6| I. Hamamoto, invited talk to this Conference; Phys. Rev. 10C (1974) 63 and references therein.

|7| J. Ungrin, R.M. Diamond, P.O. Tjøm and B. Elbek, Mat. Fys. Medd. Dan. Ved. Selsk. 38 (1971) No. 8.