A MICROSCOPICAL DESCRIPTION OF THE COUPLING BETWEEN DIFFERENT DEGREES OF FREEDOM

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Recently, experimental evidence has been found about some levels of negative parity whose electromagnetic properties point to a pronounced collective character and which could not be described in the framework of the so-called random phase approximation (R.P.A.). Indeed, the energies of the above mentioned levels suggest that they are two-phonon states but, on the other hand, the R.P.A. predicts that the multiphonon states are degenerated. It is then necessary to correct the harmonic picture given by R.P.A., by switching on the coupling between different modes.

In this way the observed experimental [1] levels 1^-, 3^{-1}, 5^- in ^{114}Cd and ^{150}Sm and 1^-, 3^{-1}, 5^-, 4^- in ^{148}Sm as well as the corresponding ratios $B(E3;3^{-1}\rightarrow 0^+)/B(E3;3^-\rightarrow 0^+)$ have been interpreted in the framework of the phenomenological model [2] as member of the quadrupole-octupole quintuplet. The γ decay which follows the decay of the 60 d and 15 m isomers of ^{124}Sb indicates [3] that the second 3^- level of ^{124}Te is a member of a quintuplet. Also, it has been shown by using the coupled channel theory, that the collective levels 1^-, 3^{-1}, 5^- in ^{110}Pd excited by inelastic proton scattering might result from the coupling of one octupole and one quadrupole vibrations [4]. Recently, in neutron capture experiment [5] the sequence of levels 5^-, 4^-, 3^-, 2^- have been populated in ^{148}Sm by the subsequent primary γ decay of the neutron resonances 3^+ and 4^+.

On the other hand, since the collective and degenerate - R.P.A. - quintuplet occurs in the region of two quasiparticle states, the interference of collective and noncollective degrees of freedom ought to be considered.

In previous papers |6-8| we attempted to give a microscopic interpretation for the levels of the kind mentioned above by considering their energies as eigenvalues of a boson expanded Hamiltonian. The Hamiltonian we used is the usual shell model Hamiltonian plus pairing, quadrupole-quadrupole and octupole-octupole effective interaction. This Hamiltonian is written in the quasiparticle representation.

Following the Belyaev-Zelevinsky procedure |9| we expressed the operators $A_{nq}^{(\pm)}$ (ab) (two-quasiparticle operator) and $B_{nq}^{(\pm)}$ (ab) (one quasiparticle-one quasihole) as series of R.P.A. quadrupole and octupole phonon operators of the collective ($\overset{o}{C}_n$, n=2,3) as well as of the noncollective type ($\overset{o}{C}_n(i)$, n=2,3; i=1,2,...p_n). These series have been determined so that the $A_n^{(\pm)}$, $B_n^{(\pm)}$ commutators algebra be consistently reproduced. This condition was fulfilled in the first order ignoring the recoupling terms.

In order to improve the convergence of the B-Z expansion we changed the R.P.A. collective phonons by a linear and canonical transformation chosen in such a way that the ground state energy be minimum.

$$\overset{o}{C}{}^{+}_{nq} = a_{n1}C^{+}_{nq} + a_{n2}(-1)^{q}C_{n-q}, \qquad n = 2,3$$

It was proved in ref. |7| that our Hamiltonian expressed in terms of "correlated" phonons is the same as that would have been obtained if the expansion from the beginning was performed with respect to the phonons $\{C_n\}_{n=2,3}$, $\{\overset{o}{C}_n(i)\}_{\substack{n=2,3\\ i=1,2,\dots p_n}}$. The Hamiltonian thus modified has a smaller convergence parameter than the initial one and consequently the convergence of the B-Z expansion is improved. Also, it has been shown that in the new representation the "dangerous" graphs cancel and the coefficients of the anharmonic terms decrease. This fact yields a reasonable magnitude for the dimension of phonon space at which the stability of observables of interest, is realized.

Now, the Hamiltonian can be written as

$$H = H^{\mathrm{coll}} + H^{\mathrm{ncoll}} + H^{\mathrm{coup}}$$

where H^{coll} and H^{ncoll} contain only collective and non-collective phonons, respectively, while H^{coup} contains only terms involving both the collective and noncollective phonons. The explicit form of H^{coll} is:

$$H^{\mathrm{coll}} = U + H_2 + H_3 + H_{23}$$

where

$$\begin{aligned} H_2 = \{ & \tfrac{1}{2}\,\omega_2 \sum C^+_{2q} C_{2q} + A_{30}(C^+_2 C^+_2 C^+_2)_0 + A_{21}(C^+_2 C^+_2 C_2)_0 + \\ & + A_{40}(C^+_2 C^+_2)_0 (C^+_2 C^+_2)_0 + A_{31}(C^+_2 C^+_2)_0 (C^+_2 C_2)_0 + \\ & + A_{22}(C^+_2 C^+_2)_0 (C_2 C_2)_0 + M_2 N_2 (N_2 - 1) + P_2 (J^2_2 - 6N_2) \} + \mathrm{h.a.} \end{aligned}$$

$$\begin{aligned} H_3 = \{ & \tfrac{1}{2}\,\omega_3 \sum C^+_{3q} C_{3q} + B_{40}[(C^+_3 C^+_3)_2 (C^+_3 C^+_3)_2]_0 + \\ & + B_{31}[(C^+_3 C^+_3)_2 (C^+_3 C_3)_2]_0 + B^0_{22}(C^+_3 C^+_3)_0 (C_3 C_3)_0 + \\ & + B^2_{22}\ [(C^+_3 C^+_3)_2 (C_3 C_3)_2]_0 + M_3 N_3 (N_3 - 1) + P_3 (J^2_3 - 12N_3) \} + \\ & + \mathrm{h.a.} \end{aligned}$$

$$H_{23} = \{ D^{10}_{20}[C^+_2 (C^+_3 C^+_3)_2]_0 + D^{10}_{11}[C^+_2 (C^+_3 C_3)_2]_0 +$$

$$+ D^{01}_{20}\left[(C^{+}_{3}C^{+}_{3})_{2}C_{2}\right]_{0} + \sum_{J=0}^{4}\Big[{}^{(J)}D^{20}_{20}\left[(C^{+}_{2}C^{+}_{2})_{J}(C^{+}_{3}C^{+}_{3})_{J}\right]_{0} +$$

$$+ {}^{(J)}D^{20}_{11}\left[(C^{+}_{2}C^{+}_{2})_{J}(C^{+}_{3}C_{3})_{J}\right]_{0} + {}^{(J)}D^{11}_{20}\left[(C^{+}_{2}C_{2})_{J}(C^{+}_{3}C^{+}_{3})_{J}\right]_{0} +$$

$$+ {}^{(J)}D^{11}_{11}\left[(C^{+}_{2}C_{2})_{J}(C^{+}_{3}C_{3})_{J}\right]_{0} + D^{20}_{02}\left[(C^{+}_{2}C^{+}_{2})_{J}(C_{3}C_{3})_{J}\right]_{0}\Big]\} +$$

$$+ \text{h.a.}$$

Here, the coefficients A, B, D, M, P are functions of the matrix elements of multipole operators, the U, V coefficients as well as of the amplitudes and the energies of RPA phonons. Their expressions are given in refs. [7,10].

Further approximations regard only the terms H^{ncoll} and H^{coup}. We retain only the RPA harmonic terms from H^{ncoll} and the third order terms of H^{coup} including one-noncollective phonon. The results are denoted by H^{ncoll}_{RPA}, $\tilde{H}^{coup}$, $\tilde{H}(\equiv H^{coll} + H^{ncoll}_{\widetilde{RPA}} + H^{coup})$. One can easily verify that the Hamiltonian $\tilde{H}$ has the same eigenvalues as the following one [8]:

$$\bar{H} = H^{coll} + H^{eff}(E)$$

where H^{eff} is an effective and energy dependent Hamiltonian involving only collective phonons. Its expression is:

$$H^{eff}(E) = PH^{coup}Q \frac{1}{E-(H^{coll}+\tilde{H}^{ncoll}_{RPA})} Q\tilde{H}^{coup}Q$$

Here P and Q stand for the projectors associated with the pure collective subspace and its complementary subspace, respectively.

In this way the initial eigenvalue problem is replaced by a nonlinear eigenvalue problem (formulated within the pure collective subspace) which is to be solved selfconsistently.

The concrete results which will be exposed later on correspond to a truncation of Q which neglects the projectors onto the states containing more than three phonons.

The Hamiltonian $\bar{H}$ has been used for the case of even-even isotopes of tin ^{114}Sn, ^{116}Sn, ^{118}Sn - where the following single neutron-states were involved: $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $h_{11/2}$, $d_{3/2}$.

We treated first the Hamiltonian H^{coll} alternatively by perturbation and diagonalization. In both cases we used the same parameters: i) the pairing strength G was fixed by fitting the mass difference of neighbouring odd-even nuclei, ii) for a fixed pair (X_2,X_3) of quadrupole and octupole strength parameters the pair of independent parameters $(a_{21}+a_{22}, a_{31}+a_{32})$ has been determined from the condition that the energy of ground state be minimum, iii) by varying the pairs (X_2,X_3) we calculated the energies of $2^+(E_{2^+})$ and $3^-(E_{3^-})$ in the second perturbation order. Then, we picked out the pair (X_2,X_3) which corresponds to the pair (E_{2^+}, E_{3^-}) reasonably close to experimental data |11|.

In the second perturbation order we calculated the energies of single-phonon states as well as of one quadrupole one octupole states. The wave functions have been estimated in the first perturbation order. The base of perturbation contains only states whose number of phonons does not exceed 3. Using the above perturbed states as well as the first order expansion associated with the multipole operators we evaluated the transitions: $B(E2;2^+\rightarrow0^+)$, $(E3;3^-\rightarrow0^+)$. Equating these values to the corresponding experimental values |12,13|, we obtained the effective charges $e_{eff}^{(2)}$ and $e_{eff}^{(3)}$, respectively. Using the above values of $e_{eff}^{(2)}$, we have calculated the quadrupole moments for the states 2^+ and 3^-. Also, using the perturbed state 3^{-1} and the octupole effective charge, the transition $B(E3;3^{-1}\rightarrow0^+)$ is completely determined.

TABLE 1

Energies of levels JP are given for the cases: a) E_{JP} are obtained in second order of perturbation applied to H^{coll}. b) Eigenvalues of H^{coll} predicted by diagonalization.
c) Solution of nonlinear eigenvalue problem $\overline{H}(E)\psi = E\psi$ (including the coupling with noncollective degrees of freedom).
For the three isotopes the RPA eigenfrequencies are: $\overset{o}{\omega}_2 = 1.14, 1.19, 1.40$; $\overset{o}{\omega}_3 = 2.0, 2.4, 2.5$. The parameters of canonical transformation are: $a_{21}+a_{22} = 0.86, 0.89, 0.91$; $a_{31}+a_{32} = 0.88, 0.92, 0.93$.

	E_{JP} \ JP	2^+	3^-	1^-	2^-	3^{-1}	4^-	5^-	Quintet ordering
^{114}Sn	a	1.284	2.738	3.857	3.802	4.427	4.638	3.953	$2^-,1^-,5^-,3^-,4^-$
	b	1.323	2.605	3.969	3.677	4.377	4.326	3.971	$2^-,1^-,5^-,4^-,3^-$
	c	1.320	2.601	3.941	3.782	4.439	4.227	3.953	$2^-,1^-,5^-,4^-,3^-$
^{116}Sn	a	1.184	2.986	3.779	3.959	4.655	4.876	4.008	$1^-,2^-,5^-,3^-,4^-$
	b	1.162	2.779	3.818	3.763	4.449	4.389	3.915	$2^-,1^-,5^-,4^-,3^-$
	c	1.153	2.765	3.781	3.842	4.452	4.312	3.886	$1^-,2^-,5^-,4^-,3^-$
^{118}Sn	a	1.347	2.856	3.405	4.355	4.667	4.933	3.870	$1^-,5^-,2^-,3^-,4^-$
	b	1.477	2.713	3.701	4.180	4.473	4.551	3.942	$1^-,5^-,2^-,3^-,4^-$
	c	1.456	2.681	3.656	4.205	4.435	4.503	3.898	$1^-,5^-,2^-,3^-,4^-$

TABLE 2

The quadrupole ($e^{(2)}_{eff}$) and octupole ($e^{(3)}_{eff}$) effective Charges, the quadrupole moments for the states 2^+ and 3^- and the ratio $B(E3;3^{-1}\rightarrow 0^+)/B(E3;3^-\rightarrow 0^+)$ predicted by the perturbation treatment.

	$e^{(2)}_{eff}$	$e^{(3)}_{eff}$	$\frac{B(E3;3^{-1}\rightarrow 0^+)}{B(E3;3^-\rightarrow 0^+)}$	$Q_2(2^+)$ (in eb)	$Q_2(3^-)$ (in eb)
^{114}Sn	0.493	1.442	0.365	0.166	-0.059
^{116}Sn	0.476	1.801	0.515	0.195	-0.050
^{118}Sn	0.497	1.670	0.548	0.142	-0.050

Alternatively, the eigenvalues of H^{coll} have been obtained by the diagonalization procedure. We used the basis defined by the states $[|N_2\nu_2\alpha_2I_2\rangle \times |N_3\nu_3\alpha_3I_3\rangle]_{IM}$, where the two components of the vector product constitute the irreducible representation for the groups SU(5) and SU(7), respectively, determined by the reduction schemes: $SU(5) \supset R(5) \supset R(3)$ and $SU(7) \supset R(7) \supset R(3)$. The above basis was truncated by the restrictions: $\{N_2 \leq 6,\ N_3 \leq 4,\ N_2+N_3 \leq 6\}$. In this way the dimensions of diagonalization spaces used for the states 2^+, 1^-, 2^-, 3^-, 4^-, 5^- are: 79, 41, 48, 74, 68, 75, respectively.

Further, the nonlinear eigenvalue problem of $\bar{H}$ was solved selfconsistently |10,14|.

The concrete results we have obtained by the above described algorithm, are listed in tables 1 and 2.

The sequence of energies predicted by the perturbation treatment is the same as that given in ref. 15 by linearizing the coupled equations of motion. The quintuplet ordering predicted by diagonalization and perturbation procedures are the same in the case of ^{118}Sn, but they are slightly different in the case of ^{114}Sn (the relative position of 3 and 4 is modified) and ^{116}Sn (the relative positions within the pairs (3,4) and (1,2) are modified). These modifications might be caused by the fact that we used different basis for the two procedures.

Comparing the results of rows b) and c) from table 1 one sees the influence of $H^{eff}(E)$ on the eigenvalues of H^{coll}. Including the coupling between collective and non-collective degrees of freedom, the relative position of quintuplet levels predicted by diagonalizing H^{coll} is not altered in the case of ^{114}Sn and ^{118}Sn; relative position of 1^- and 2^- is changed in the case of ^{116}Sn.

Concerning the quadrupole moments of 2^+, the results given in table 2 are very close to those given in ref. |16|.

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