EXCITATION ENERGIES OF NON-ROTATIONAL STATES IN ODD-Z RARE EARTH NUCLEI

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In describing low lying states in deformed nuclei wave functions and eigenvalues to the nuclear hamiltonian are usually calculated in an average nuclear potential of the Woods-Saxon or the Nilsson type. Furthermore, pairing correlations, rotation-particle coupling and quasiparticle-phonon interaction have been included.

In the following is described a detailed comparison between two sets of excitation energy calculations for non-rotational states in odd-Z rare earth nuclei.

A. In a publication by F.A. Gareev et al. |1|, calculations performed using an axially symmetric Woods-Saxon potential with inclusion of hexadecapole deformation are presented. Pairing and quasiparticle-phonon interaction has been taken into account.

B. In a second calculation |2| the total nuclear potential energy surface is calculated as a function of quadrupole and hexadecapole deformation (1). Calculations are performed for the ground state and excited states.

$$E_{pot}(\varepsilon_2\varepsilon_4) = E_{LD} + \sum_{N,Z} E_{SP} - \Big(\sum_{N,Z} E_{SP}\Big)_{average} + \\ + E_{pair}(N,Z) + E_{odd} \qquad (1)$$

The single particle energies are found using a modified harmonic oscillator potential with κ and μ parameters as given in ref. 3. The pairing energy is included with the pairing gap equal to the experimental odd-even mass difference.

The rotational contribution to the band head energy is calculated from the general energy formula

$$E_{rot}(I,K) = \frac{\hbar^2}{2\mathfrak{J}} \Big[I(I+1) - 2K^2 + E_{RPC}(I,K) + \\ + \langle I,K,\alpha|\underline{j}^2|IK\alpha\rangle\Big] \tag{2}$$

Neglecting off diagonal elements in E_{RPC} and elements of the form $\langle K,\alpha|\underline{j}^2|K,\alpha'\rangle$ we get the zero point rotational energy:

$$E_{ZPR}(K,\alpha) = \frac{\hbar^2}{2\mathfrak{J}} \Big\{K - \delta_{K,1/2}\, a + 0.5\Big[\sum_{\nu}(\langle K,\alpha|j_-|K+1,\nu\rangle P_{K,K+1})^2 + \\ + \sum_{\nu}(\langle K-1,\nu|j_-|K,\alpha\rangle P_{K,K-1})^2\Big]\Big\} \tag{3}$$

where a is the decoupling parameter and $P_{K,K+1}$ is the usual pairing factor used in Coriolis coupling calculations.

The average agreement between calculated and experimental excitation energies are of approximately the same quality in the two studies. In most cases the trend in observed energies through a series of isotopes of a given element is reproduced. A typical example is shown in fig. 1, where experimental and calculated excitation energies are shown for all known non-rotational states in Lu isotopes.

The calculation performed with the harmonic oscillator potential and with inclusion of zero point rotational energy shows in the case of the Ho isotopes a strong compression of the low lying states. This is due to the large differences in zero-point rotational energy. For the

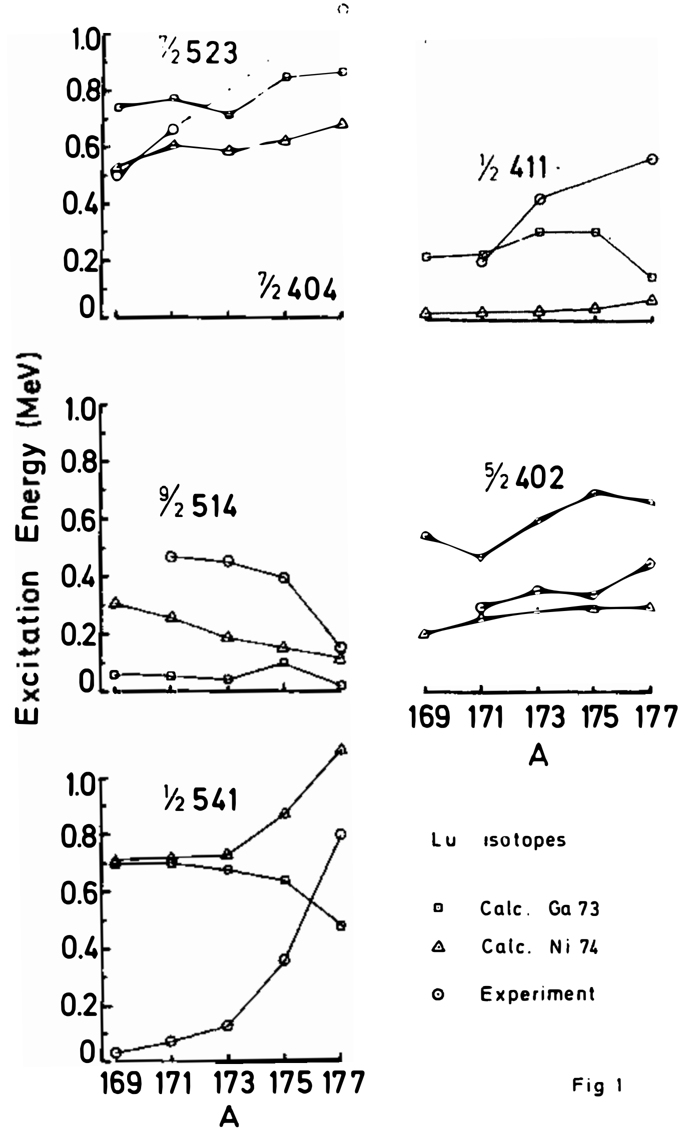

Fig 1

ground state $(7/2^-[523])$ we have $E_{ZPR} \simeq 22 \frac{\hbar^2}{2\mathcal{J}}$, whereas $E_{ZPR} < 11 \frac{\hbar^2}{2\mathcal{J}}$ for the lowest lying states $(5/2^+[413]$, $3/2^+[411]$, $1/2^+[411]$ and $7/2^+[404])$.

In neither of the two calculations it has been possible to reproduce the excitation of the $1/2^-[541]$ state.

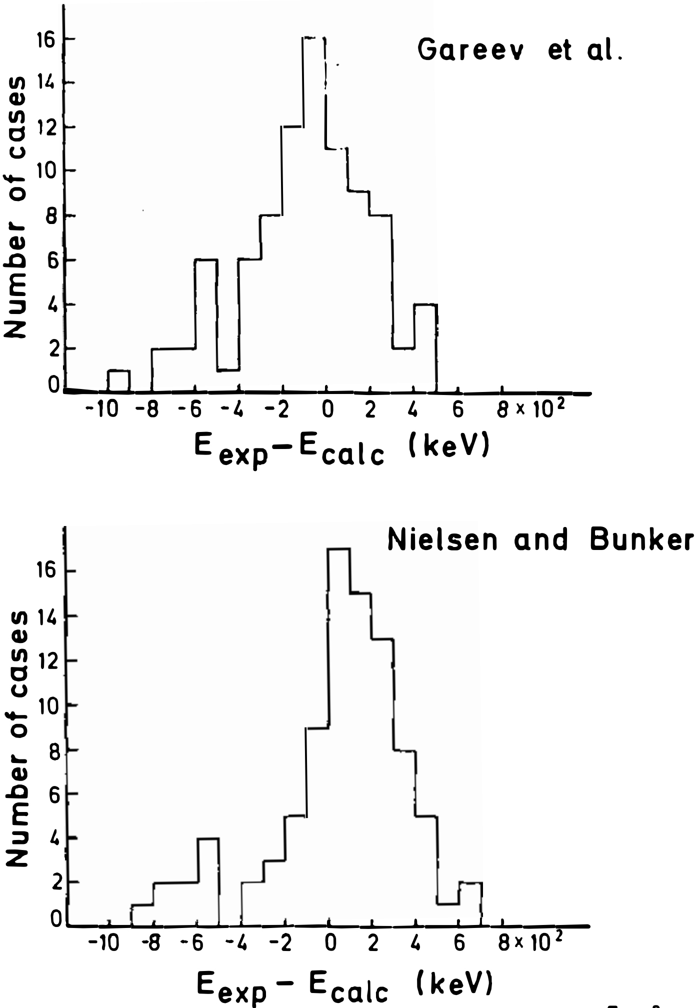

Fig. 2

A comparison of experimental and calculated energies is shown in table 1. In the case of the Lu isotopes the calculations performed with the Woods-Saxon potential give a wrong trend in the excitation energies (see fig. 1). This might be due to the way in which the (β_2, β_4) parameters are chosen.

TABLE 1

Excitation energies of the $1/2^-[541]$ state

Isotope	Experiment (keV)	Calc. |2| (keV)	Calc. |1| (keV)
^{159}Ho	-	998	980
^{161}Ho	423.8	938	1000
^{163}Ho	471.2	979	-
^{165}Ho	680	1119	-
^{165}Tm	182.2	908	980
^{167}Tm	171.7	987	950
^{169}Tm	341.9	1073	900
^{171}Tm	-	1203	940
^{169}Lu	29.0	707	700
^{171}Lu	71.7	717	700
^{173}Lu	128.2	727	680
^{175}Lu	358.2	872	640
^{177}Lu	∿800	1094	480
^{177}Ta	216.6	788	300
^{179}Ta	750.3	996	750
^{181}Ta	-	1213	750
^{181}Re	432.5	830	150
^{183}Re	702	1035	600
^{185}Re	1045	1169	620

On the basis of a detailed comparison between the excitation energies of the two calculations it is not possible to arrive at a definite conclusion concerning the applicability of the Woods-Saxon and the Nilsson model. Since the states considered are of predominantly single particle character the effect of including quasi-particle-phonon interaction is not expected to show any significant improvement of the agreement between experimental and calculated excitation energies. Fig. 2 shows in the form of histograms the agreement obtained in the two calculations.

REFERENCES

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