MICROSCOPIC AND PHENOMENOLOGICAL DESCRIPTION OF PARTICLE-PHONON COUPLING
THE ENERGY SPLITTINGS OF THE ^{209}Bi MULTIPLETS

D. Zawischa*
Institut für Kernphysik der KFA Jülich,
Jülich, Germany

1. MICROSCOPIC CALCULATION

Calculations of the particle-phonon energies in ^{209}Bi using the Migdal force have already been published |1|, but since then a set of force parameters giving a better fit to the spectra of the A=206 and A=210 nuclei has been determined |2|, so it was thought worth-while to repeat the calculations.

The method is given in ref. |1|. There is only a slight change in the particle-hole interaction including a spin orbit force: $f^{sl}_{in} = 0$, $f^{sl}_{ex} = -0.3$. The new parameters of the particle-particle force are

(a) proton-proton and neutron-neutron interaction:

$$(\alpha+\beta)_{in} = 0.05, \; (\alpha+\beta)_{ex} = -2.7, \; f^{sl}_{in} = 0, \; f^{sl}_{ex} = -1.5$$

(b) proton-neutron interaction:

$$(\alpha+\beta)_{in} = 0.05, \; (\alpha+\beta)_{ex} = -2.7, \; (\alpha-\beta)_{in} = -0.35,$$

$$(\alpha-\beta)_{ex} = -3.4, \; f^{sl}_{in,ex} = 0$$

with $C = 380$ MeV·fm^3, $R = 6.8$ fm, $a = 0.5$ fm, $k_F^2 = 2$ fm^{-2} as in |1|.

The results on the h9/2 x 3^- septuplet energies are shown in table 1. Major changes compared with the previous

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*On leave from Institut für Theoretische Physik der TU Hannover.

TABLE 1

Energy shifts of the h9/2 x 3^- septuplet in ^{209}Bi in keV. Column a gives the contribution of the "Compton scattering" graphs, columns b to e the contributions of the proton-proton, proton-neutron, proton-(proton)$^{-1}$, proton-(neutron)$^{-1}$ force, respectively, to the diagonal terms in the particle phonon interaction. Column f gives the additional shift due to the nondiagonal terms. The $13/2^+$ single particle state has been included explicitly in the diagonalization.

2j	a	b	c	d	e	f	E_{th}	E_{exp}
3	44	-100	-371	197	108	-15	-139	-120
5	-10	-50	-181	164	85	-28	-20	4
7	12	-46	-227	143	81	-14	-50	-29
9	-103	-26	-120	140	79	-11	-47	-49
11	-45	-34	-174	143	81	-12	-43	-16
13	-100	-26	-124	128	76	51	5	-14
15	184	-46	-274	158	96	-4	112	130

values have been obtained for the higher multiplets; the results on the h9/2 x 5^- and h9/2 x 4^- levels are given in fig. 1. The experimental data are from ref. |3|. We predict the lowest $1/2^+$ state at approximately 2.4 MeV where it has not yet been detected. One has to compare the remaining discrepancies to the absolute values of the energy shifts for each of the multiplets separately to appreciate the agreement with experiment which seems very satisfactory.

2. SEMIMICROSCOPIC CALCULATION

In contrast to the higher multiplets, the h9/2 x 3^- septuplet should be amenable to a phenomenological approach. Hamamoto |4| has calculated the "Compton scatter-

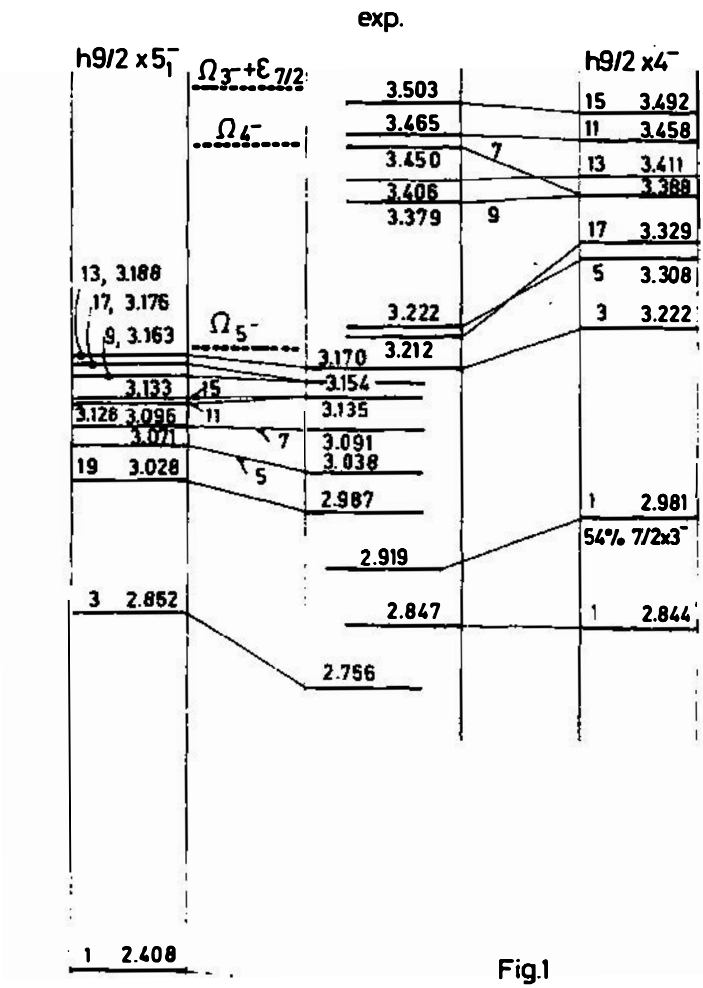

Fig.1

Fig.1. The h9/2x5⁻ and h9/2x4⁻ multiplets in ^{209}Bi. The levels are labeled by 2j, and the energies are given in MeV. The bars for the experimental levels in the middle are displaced to the left or right according to whether the main component in the wave function if h9/2x5⁻ or h9/2x4⁻. On the left and right the theoretical values are given. The unperturbed energies are indicated by dotted bars.

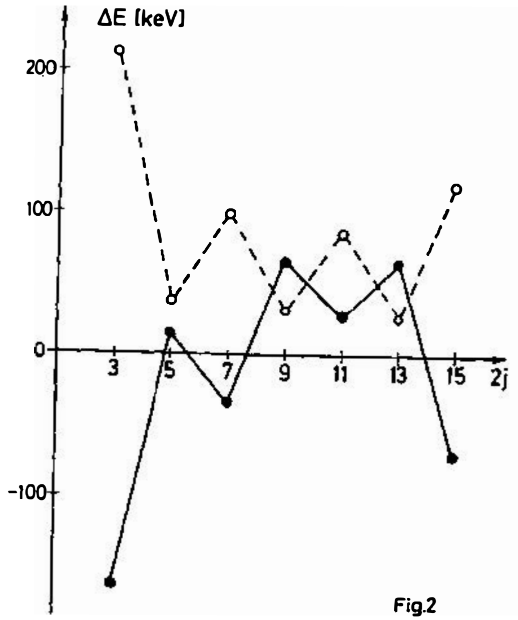

Fig.2. The points connected by the solid line give the difference $E_{exp}-E_{th}$ as obtained in the conventional weak coupling collective model. The open dots connected by the dashed line give the corresponding calculated energy shifts if one assumes that the potential vibrates with the mass but is otherwise unchanged.

ing" graphs (with photons replaced by phonons) and argued that the discrepancies just show the limitations of the collective model used. On the other hand, the systematics of the deviations between theory and experiment suggest that the missing term, corresponding to the change of the selfconsistent potential due to the vibration, should be obtainlable in a simple model too (fig. 2, solid line). Such a model has been found and a short account has already been given |5|.

We first assume that the single particle potential is fixed to the vibrating density. The average of the potential change is then

$$\delta V_{\lambda\mu}(r,\theta,\phi) = \int (|u_1(Q_{\lambda\mu})|^2-|u_o(Q_{\lambda\mu})|^2)V(r-\Delta R(Q_{\lambda\mu}))dQ_{\lambda\mu} \quad (1)$$

TABLE 2

Phenomenological calculation of the septuplet energy shifts. Column a: Contribution of the "Compton graphs" with Woods Saxon s.p. well, column b: the same with modified well which is 2.9 MeV deeper near the edge than at the centre, column c: energy shift obtained from eq. (5) with $\rho_m = 0.81\ \rho_{in}$, $R_\rho = 6.8$ fm, $a_\rho = 0.45$ fm, $V_{in} = -55$ MeV. The values have been shifted by +52 keV to yield the average zero. ΔE_{th} is the sum of the entries in b and c.

2j	a	b	c	ΔE_{th}	ΔE_{exp}
3	41	8	8 -111	-103	-120
5	-11	-8	33	25	4
7	4	-31	-18	-49	-29
9	-113	-79	40	-41	-49
11	-40	-20	-7	-27	-16
13	-79	-80	44	-36	-14
15	200	160	-34	126	130

u_o and u_1 being harmonic oscillator wave functions, V is the nuclear potential, and

$$\Delta R(Q_{\lambda\mu}) = R_o\ Y_{\lambda\mu}(\theta,\ \phi=0)\sqrt{\frac{h}{B\omega}}\ Q_{\lambda\mu} \quad (2)$$

(The factor R_o means that the oscillation is uniform over the diffuseness; one can use r or r^2/R_o instead which makes little difference). The expectation value of this between single particle states then gives $\delta E_{\lambda\mu, j_p j_m}$. Solving the system of equations

$$\sum_j |\langle j_p m_p \lambda\mu | jm\rangle|^2\ \delta E_j = \delta E_{\lambda\mu, j_p m_p} \quad (3)$$

one obtains the energy shifts E_j in question.

The open dots in fig. 2 show the result. Aside from the average value which is not near zero, the dependence of the shifts δE on j shows the wrong sign. Let us analyze this in detail. In the regions of large oscillation amplitude, the diffuseness of the potential is increased. This leads to a flattening of the well where the h9/2 wave function is large and a deepening more outside where the wave function is small. Thus the repulsive effects dominate in the oscillating regions which, as fig. 2 shows, is wrong. To account for the change in the selfconsistent potential, a model must supply additional attraction in the regions of large oscillation amplitude or repulsion near the nodal lines.

As a simplest but perhaps not too realistic model to yield this behaviour it was assumed that the potential depends on the local average density in the form

$$V = V_m + \frac{1}{2}(\rho - \rho_m)V' + \ldots \qquad (4)$$

The parameters are linked by the requirement that $V(r\to\infty) = 0$ and that the single particle energies are reasonable. The change of the potential is now

$$\delta V_{\lambda\mu} = \int \left(|u_1(Q_{\lambda\mu})|^2 - |u_o(Q_{\lambda\mu})|^2\right)\rho_o \left|1+\exp\frac{r-R_\rho-\Delta R(Q_{\lambda\mu})}{a_\rho}\right|^{-1} \cdot \frac{dV}{d\rho}\, dQ_{\lambda\mu} \qquad (5)$$

Table 2 gives the results.

Of course, fancier as well as more realistic models can be devised. One could think of the single particle potential being dependent on the relative velocity of the particle and the moving fraction of the density. This and other possibilities are being investigated.

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