INELASTIC NUCLEON-NUCLEUS SCATTERING IN THE FRAMEWORK OF GREEN'S FUNCTIONS

M.F. Weigel*

Physics Department, University Munich, Garching, Germany

The aim of this talk is to present inelastic scattering in the language of Green's functions. The Green's function method has turned out to be a useful tool for investigating nuclear properties, and can deal in a transparent manner with effective quantities - for instance renormalized forces etc. - and permits the formulation of the phonon problem in a consistent many-body approach. Since phonons are also expected to play an essential role in the intermediate structure of the scattering process, it seems to be worthwhile to formulate scattering theory within the framework of Green's functions. This can be achieved by a method of Zhivopistev |1|, in which the S-matrix for general scattering processes is expressed by boundary values of Green's functions. The simplest process is nucleon scattering on odd nuclei, which amounts essentially to an extension of the RPA to the continuum. More interesting is inelastic nucleon scattering on even nuclei (closed shell), which we will consider in the following.

The exact wave function in the generalized shell-model representation is as follows: $|0\rangle$ ($|N\rangle$) denotes the exact ground (excited) state of the target; a_f^+, (a_i^+) is the creation operator in the final (initial) shell-model state f(i) with standing wave boundary conditions; $\delta_i(\delta_f)$ denotes the phase shift

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*For further details: M. Weigel, J. Phys. A7 (1974) 1731.

$$<\phi^{(-)}_{f,N}|\Omega^{(+)}_{i}> = e^{i(\delta_i+\delta_f)}\{\delta_{N,O}\delta_{fi} + \frac{1}{E_i-E_O+i\eta}<(H-E_f)a^{\dagger}_{f}|N>|\bar{\Omega}^{(+)}_{i}>$$

$$= e^{i(\delta_i+\delta_f)} i\eta_i \, g^{NO}_{fi}(\omega=\varepsilon_i + \frac{E_O-E_N}{2}) \ , \qquad (1)$$

with

$$|\Omega^{(+)}_{i}> = e^{i\delta_i} \frac{i\eta_i}{\varepsilon_i+E_O-H+i\eta_i} a^{(\dagger)}_{i}|O> = e^{i\delta_i}|\bar{\Omega}^{(+)}_{i}> \ , \qquad (2)$$

$$g^{NO}_{\rho\delta}(\omega) = <N|\{a_\rho(\omega-H + \frac{E_N-E_O}{2} + i\eta)^{-1}a^{\dagger}_{\delta}+a^{\dagger}_{\delta}(\omega+H-\frac{E_N+E_O}{2} - i\eta)^{-1}a_\rho\}|O> \qquad (3)$$

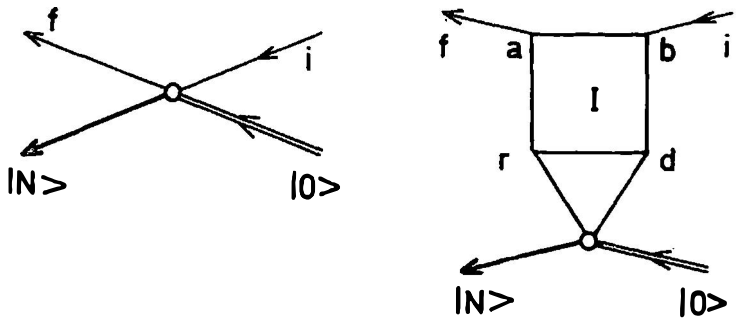
The asymmetric Green function can be projected out of the linear response function, and one obtains the following equation for g^{NO} ($g^{OO} = g$; I denotes the energy-dependent effective particle-hole force; $\sum$ means summation or integration)

$$g^{NO}_{fi}(\varepsilon_i + \frac{E_O-E_N}{2}) = \delta_{NO}\, g_{fi}(\varepsilon_i) - i(1-\delta_{N,O})$$

$$\cdot \sum_{\alpha\beta} i\, g_{f\alpha}(\varepsilon_i+E_O-E_N)\, g_{\beta i}(\varepsilon_i) \qquad (4)$$

$$\cdot \sum_{\delta\rho} \int d\omega\, I_{\alpha\delta\beta\rho}(\varepsilon_i + \frac{E_O-E_N}{2}, \omega; E_O-E_N)\, g^{NO}_{\rho\delta}(\omega)$$

In Feynman graphs we have the following structure:



(5)

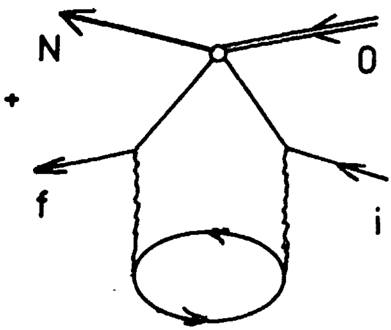
Since the limes $\eta_i \to 0$ can be performed easily (on the right-hand side of eq. (4))

$$e^{i(\delta_i+\delta_\beta)} i\eta_i g_{\beta i}(\varepsilon_i) = \langle O | a_\beta \frac{i\eta_i}{\varepsilon_i-(H-E_o)+i\eta_i} a_i^\dagger | O\rangle$$

$$\cdot\, e^{i(\delta_i+\delta_\beta)} = {}_\beta^{(+)}(\varepsilon_i) \qquad (6)$$

the main problem is the investigation of I. If one assumes an instantaneous effective particle-hole force, one of course obtains the standard DWBA result. We have investigated the structure of the irreducible vertex I for four approximations ($I = \delta\tilde{\Sigma}/\delta g$; $\tilde{\Sigma}$ denotes the self-energy (effective s.p. - potential; Γ reducible vertex):

a) Perturbation theory:



(one example) (7)

b) Ladder approximation ($\tilde{\Sigma} = -i\Gamma^{L}_{g}$; similar in structure to perturbation theory).

c) Linear approximation ($\frac{\delta\Gamma}{\delta g} = 0$).

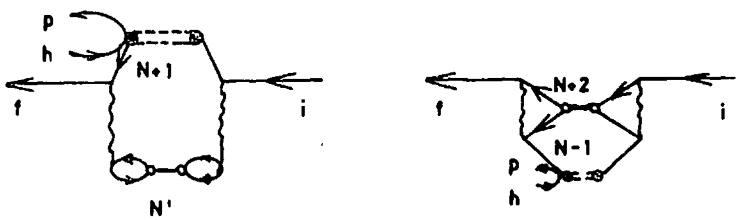
d) Neglect of effective three-body interactions.

The essential result is the following structure for I:

$$I(\varepsilon,\varepsilon';\Omega) = I^{(o)}(\varepsilon,\Omega) + \sum_{p} \frac{I^{(1)}(\varepsilon,\Omega)}{\varepsilon'-\omega_{p}+i\eta} + \frac{I^{(2)}(\varepsilon,\Omega)}{\varepsilon'-\omega_{p'}-i\eta} \quad , \qquad (8)$$

where ω and $\omega_{p'}$ are functions of ε and Ω. The residues $I^{(1),(2)}$ can be calculated from perturbation theory, ladder approximation and the p-p and p-h RPA-problem depending on the chosen approximate treatment of I.

In the next step one has to consider the structure of $g^{NO}_{\rho\delta}(\omega)$. The simplest approach is again perturbation theory. A further possibility is assuming a p-h-(phonon) structure of $|N\rangle$. One then has to consider, for instance, matrix elements of the form $\langle O|a^{\dagger}_{h}a_{p}a_{\rho}|N+1\rangle$, which can be determined by the 2p-1h RPA. Another possibility is to link g^{NO} to Migdal's theory |2|, since one can express g^{NO} by $\langle N\underline{+}1|a^{\dagger}_{\alpha}a_{\beta}|\overline{N\underline{+}1}\rangle$ etc. and the phonon structure of $|N\rangle$. So one has a direct connection to nuclear structure calculations or experimental information (transitions etc.), respectively. Typical decompositions of the scattering process are given in the next graphs



(9)

One hopes that it is sufficient to take into account the results of the nuclear-structure problem in the integral of eq. (4). In this way one would have established a theory of scattering directly connected to the nuclear-structure problem. The quantities (like forces etc.) used in this approach are determined to a large extent, since the same quantities ought to be used in different calculations.

REFERENCES

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|2| A.B. Migdal, Many-body description of nuclear structure and nuclear reactions (Academic Press, N.Y., 1965).