TEST OF THE QUARK-LINE RULE IN $p\bar{p}$ ANNIHILATIONS

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The validity and consequences of the quark-line rule (QLR) are investigated in proton-antiproton annihilation into two and three mesons. Recent data on $p\bar{p} \to$ (two neutral pseudoscalar or vector mesons) are analysed to test the η-η' mixing scheme with the QLR. It is found that a value of the mixing angle θ_{PS} restricted to $-26^{o} < \theta_{PS} < -2.2^{o}$ is consistent with the QLR. The data on $p\bar{p} \to \pi^{o}\rho^{o}, \pi^{o}\omega, \eta\omega, \eta\rho^{o}$ and the dominance of the annihilation quark-line diagram (QLD) favours $\theta_{PS} \simeq -20^{o}$. Predictions of the QLR for $p\bar{p} \to M_1 M_2 M_2$ under the dominance of different QLD's are derived and their experimental test discussed.

Presented at the meeting "SNOPOVI I ČESTICE I", Haludovo, Malinska, Yugoslavia, 27-29 April 1989.

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CONTENTS

1. Introduction

New experimental facilities of LEAR at CERN and at Brookhaven and KEK[1], which make possible precise measurements of a number of specific mesonic channels into which the $p\bar{p}$ system annihilates, have initiated renewed theoretical interest in some of the fundamental questions related to the physics of the $p\bar{p}$ system, in particular the features following from the underlying quark-gluon dynamics at low energies.

In spite of a considerable increase of experimental information and improvement in extracting the branching ratios for $p\bar{p}$ annihilation at rest into mesons[2], the experimental data and theoretical understanding are at present far from being ideal, with several important ratios either not yet measured or only very poorly known. As a consequence of that and for lack of a reliable operational microscopic theory of quarks and gluons, there exist a number of alternative models[3,4] claiming to be able to explain some selection of the data with apparent disregard for the others. In the case of $p\bar{p}$ annihilation into mesons, many authors have attempted to describe the annihilation within the quark-model approach[4] by emphasizing the importance of either quark "rearrangement" or quark-antiquark ($q\bar{q}$) "annihilation", or combination of both types of diagrams (Figs. 1-5) with two possible elementary vertices ^3P_o (the pair-creation model) and $^3S_1$ (the one-gluon-exchange model) for basic $q\bar{q}$ annihilation.

For a given $q\bar{q}$ vertex and given nonrelativistic initial and final state wave functions, $p\bar{p}$ annihilation into mesons can in principle be evaluated, leading to model-dependent predictions when compared with data. At low energies, however, the non-perturbative effects become important and none of the models using ^3P_o or $^3S_1$ vertices for $q\bar{q}$ annihilation are thoroughly reliable, although the flux-tube model[6] prefers the ^3P_o vertex over the $^3S_1$ one.

It is therefore desirable to generate awareness of the fact that there are hierarchies in the reliability of QCD-based models.

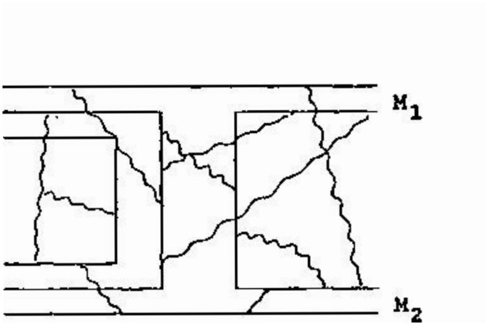

Fig. 1
Annihilation diagram for annihilations into two mesons M.

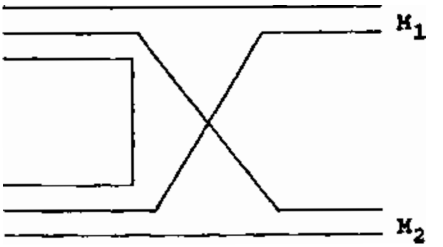

Fig. 2
Rearrangement diagram for annihilations into two mesons M.

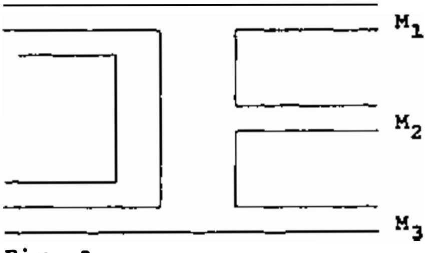

Fig. 3
Annihilation diagram for annihilations into three mesons M.

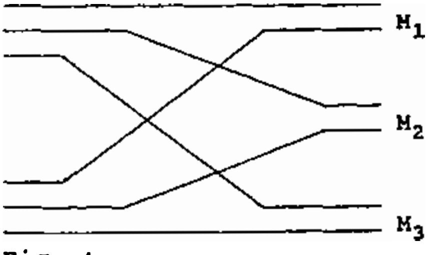

Fig. 4
Rearrangement diagram for annihilations into three mesons M.

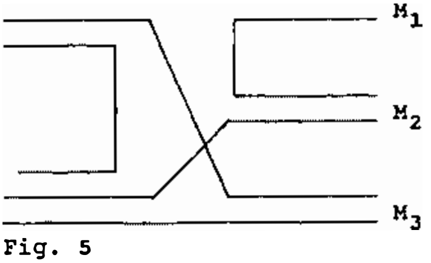

Fig. 5
Mixed, rearrangement and annihilation diagram for annihilations into two strange (M_1 and M_2) and one non-strange meson M_3.

2. The quark-line rule (QLR)

Consider, for example, the quark-line diagrams (QLD) shown in Figs. 1 and 2. These stand for all the diagrams that can be generated from them by adding gluon lines in an arbitrary way. Starting from nothing but QCD, it is impossible to decide if one of the two diagrams should dominate over the other and, if so, which of them. It is one of our main points that this question can be settled by analysing the data that can (and should!) be obtained in the near future. The tests to be discussed below are independent of the unknown gluon contributions to the diagram under consideration. They depend only on the quark flavour flux within the diagram, the so-called quark-line rule (QLR). This programme has already been outlined in Refs. 5-8.

Two main effective mechanisms in the quark-line picture of $p\bar{p}$ annihilation can be distinguished: annihilation and rearrangement (e.g. Figs. 3 and 4). As has been anticipated, we will analyse consequences of assuming that either of the two mechanisms dominates over the other; which of them dominates depends on the unknown gluon dynamics. From the outset, it is of course unlikely that the two different mechanisms should be equally important.

If the QLR is valid, an $s\bar{s}$ state can only be produced in $p\bar{p}$ annihilation together with strange particles. It is therefore very important to determine experimentally if indeed (and to what extent) the production of $s\bar{s}$ final states without associated production of strange particles is suppressed; namely, if all mesons in a given annihilation process are non-strange,

$$p\bar{p} \to (s\bar{s}) + M_1 + M_2 + \ldots M_n, \qquad (1)$$

i.e. they contain neither s nor $\bar{s}$ quarks, then the QLR demands that the $p\bar{p}$ production amplitude $T((s\bar{s})M_1 \ldots M_n)$ for the above process should vanish[9].

If we write the η and η' mesons as

$$\begin{aligned} \eta &= x((u\bar{u})_{PS}+(d\bar{d})_{PS})/\sqrt{2}+y(s\bar{s})_{PS}+\text{gluon contribution}, \\ \eta' &= -y((u\bar{u})_{PS}+(d\bar{d})_{PS})/\sqrt{2}+x(s\bar{s})_{PS}+\text{gluon contribution}, \end{aligned} \qquad (2)$$

and define K by

$$K \equiv (y/x)^2, \qquad (3)$$

then no gluon contribution to the pseudoscalar-meson states means $x^2+y^2 = 1$, and K is related to the pseudoscalar-meson mixing angle θ_{PS} by

$$K^{-1} = \tan(\theta_{ideal} - \theta_{PS}) = x^2/(1-x^2) \qquad (4a)$$

or

$$x^2 = 1/(1 + K), \qquad (4b)$$

with θ_{ideal} the ideal mixing angle $\theta_{ideal} \approx 35.3^o$.

Present evidence suggests that there is no noticeable gluon contribution to the η and η' mesons. We assume therefore that there is none. If that is so, it is obvious that neither η nor η' can be produced from the gluon sea of the $p\bar{p}$ system. For pseudoscalar $s\bar{s}$ pairs, the suppression of the reaction in Eq. (1) implies that η and η' mesons are mainly produced via their $(u\bar{u}+d\bar{d})$ content such that

$$y\cdot T(\eta,M_1,\ldots,M_n) = -x\cdot T(\eta',M_1,\ldots,M_n). \qquad (5)$$

The relation (5) is also valid when some of the M_1, $M_2,\ldots,M_n$ mesons coincide with η and/or η'. For the phase-space corrected cross sections $\bar{\sigma}$ (= cross section/phase space) Eq. (5) implies

$$\bar{\sigma}(\eta',M_1,\ldots,M_n) = K\bar{\sigma}(\eta,M_1,\ldots,M_n)$$

and (6)

$$2\bar{\sigma}(\eta',\eta',M_1,\ldots,M_n) = K\bar{\sigma}(\eta,\eta',M_1,\ldots,M_n) =$$
$$= 2K^2\bar{\sigma}(\eta,\eta,M_1,\ldots,M_n).$$

These relations are valid for any linear combination of $p\bar{p}$, $n\bar{n}$, $p\bar{\Delta}$, $n\bar{\Delta}$ or $\Delta\bar{\Delta}$ initial states; that is, the results are independent of initial-state interactions leading only to isospin mixing.

We emphasize once again that testing these relations is important. If one of them is violated, either the η-η' mixing is more complicated than presently thought of and/or the QLR is not valid.

3. Annihilation into two mesons

It is clear that more detailed predictions can be obtained if among several QLR-allowed diagrams certain ones dominate over the others. In the two-meson annihilation channels

$$p\bar{p} \to \eta\eta, \eta\eta', \eta'\eta', \pi^0\eta, \pi^0\eta' \text{ and } \pi^0\pi^0,$$

which at low energies proceed from the initial $p\bar{p}$ ^3P_J states, with $J = 0,2$, the amplitudes for $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$ production are, according to Eq. (5), proportional to each other. The same is true for $\pi^0\eta$ and $\pi^0\eta'$ amplitudes.

In the annihilation channels

$$p\bar{p} \to \eta\rho^0, \eta'\rho^0, \eta\omega, \eta'\omega, \pi^0\rho^0 \text{ and } \pi^0\omega,$$

which at low energies proceed from the initial $p\bar{p}$ $^3S_1$ states, the amplitudes ηV and $\eta' V$, with $V = \rho^0, \omega$, are also proportional to each other.

The final-state mesons we consider are mixtures of $q\bar{q}$ states such as (η and η' have been written out in the preceding section):

$$\begin{aligned}
(u\bar{u})_{PS} &= \frac{1}{\sqrt{2}}(\pi^0 + x\eta - y\eta'),\\
(d\bar{d})_{PS} &= \frac{1}{\sqrt{2}}(-\pi^0 + x\eta - y\eta'),\\
(s\bar{s})_{PS} &= y\eta + x\eta',\\
(u\bar{u})_V &= \frac{1}{\sqrt{2}}(\rho^0 + \omega),\\
(d\bar{d})_V &= \frac{1}{\sqrt{2}}(\omega - \rho^0),\\
(s\bar{s})_V &= -\phi.
\end{aligned} \tag{7}$$

In our approximation, the vector mesons ω and ϕ are ideally mixed.

According to the QLR, the interaction yields certain coherent mixtures of $q\bar{q}$ that are not influenced within the physical mesons. The production cross section of these is computed by taking the scalar product of the $q\bar{q}$ linear combination produced with the linear combination of the physical mesons, i.e. via mixing. It is the main implication of the QLR applied to QLD's that certain quark-state production amplitudes vanish. Since, under our assumptions, gluon components of mesons are not produced, we need not take them into account.

The proportionality of the amplitudes for η,η' production can be used to obtain an independent determination of K in terms of measurable branching ratios. We define K_M for the mesons $M = (\pi^0, \eta, \eta', \rho^0, \omega)$:

$$K_M = \left(\frac{q_{\eta M}}{q_{\eta' M}}\right)^{2\ell_f+1} \cdot \frac{B(p\bar{p} \to \eta' M)}{B(p\bar{p} \to \eta M)} \cdot S, \qquad (8)$$

where $q_{\eta M}$ is the meson c.m. momentum, ℓ_f is the orbital angular momentum between the two mesons in the final state and S is the statistical factor, i.e. $S = 1/2$ (2) for $M = \eta$ ($M = \eta'$). At low energies, $\ell_f = 0$ ($\ell_f = 1$) dominates for M, a pseudoscalar (vector) meson. The prediction $K_M = K$, independent of the particular meson M, is compared with present data in Table 1. We see that the values of K are still rather confusing; an average value is found to be

$$K_{av} = 1.0 \pm 0.7.$$

Expressing this result in terms of the mixing angle θ_{PS}, we obtain

$$-26^0 < \theta_{PS} < -2.2^0, \qquad (9)$$

which is to be compared with the theoretical predictions from the quadratic (linear) version of the Gell-Mann-Okubo mass formula, where K becomes approximately 1.0 (0.38) when θ_{PS} is -10^{o} (-23^{o}). Tables 3-5 give the values of K determined by assuming the dominance of the annihilation QLD (Fig. 1). We see that in this case the data favour $\theta_{PS} \simeq -20^{o}$.

However, the dominance of the rearrangement QLD (Fig. 2) leads only to the relations

$$\begin{aligned} x^2T(\pi^o\pi^o) + T(\eta\eta) &= 2xT(\pi^o\eta), \\ xT(\rho^o\pi^o) + T(\omega\eta) &= xT(\omega\pi^o) + T(\rho^o\eta), \\ T(\rho^o\rho^o) + T(\omega\omega) &= T(\rho^o\omega) + T(\omega\rho^o) \ , \end{aligned} \tag{10}$$

from which no meaningful experimental test is possible without additional assumptions.

For the annihilations $p\bar{p} \to \rho^o\rho^o$ and $\omega\omega$, the dominance of the annihilation QLD (Fig. 1) leads to a definite prediction that

$$\bar{\sigma}(\omega\omega) = \bar{\sigma}(\rho^o\rho^o). \tag{11}$$

Experimentally, the branching ratio $B(\rho^o\rho^o)$ is $(0.12\pm0.12)\%$ from Diaz et al.[10] and $(0.4\pm0.3)\%$ from Baltay et al.[11], while for $B(\omega\omega)$ it is $(1.4\pm0.6)\%$ from Bloch et al.[12]. We conclude that present data do not support the prediction $\bar{\sigma}(\omega\omega) = \bar{\sigma}(\rho^o\rho^o)$ obtained from the assumption that the annihilation QLD (Fig. 1) dominates.

The prediction of the rearrangement QLD (Fig. 2) is the triangle inequality[9]

$$\begin{aligned} \left|\sqrt{\sigma(\rho^o\rho^o)} - 2\sqrt{\sigma(\rho^o\omega)}\right|^2 &\leq \sigma(\omega\omega) \\ &\leq \left|\sqrt{\sigma(\rho^o\rho^o)} + 2\sqrt{\sigma(\rho^o\omega)}\right|^2. \end{aligned}$$

TABLE 1

M	$B(\bar{n}M)\times10^3$	$B(nM)\times10^3$	K_M	Phase-space correction
π^0	0.5 ± 0.19[a]	0.46 ± 0.13[a]	1.3 ± 0.9	$\ell=0$
		0.133 ± 0.027[b]	4.6 ± 2.7	1.2
ρ^0	1.4 ± 0.8[e]	2.2 ± 1.7[f]	3.9 ± 5.3	
		9.6 ± 1.6[d]	0.9 ± 0.66	$\ell=1$
		6.43 ± 1.3[c]	1.34 ± 1	6.16
		5.3(+2.0-0.8)	1.6 ± 1.4	

References to Table 1:

a) M. Chiba et al., Phys. Lett. B202 (1988) 447 and Phys. Rev. D36 (1988) 2021
b) L. Adiels et al., CERN-EP/88-142
c) M. Foster et al., CERN-EP/88-142
d) KEK E68, M. Chiba et al., Proc. IV LEAR Workshop (1987)
e) P. Espigat et al., Nucl. Phys. B36 (1972) 93
f) C. Baltay et al., Phys. Rev. 145 (1966) 1103

TABLE 2

$M_1...M_n$	$B(\bar{n}M_1...M_n)\times10^3$	$B(nM_1...M_n)\times10^3$	K
$\pi^+\pi^-$	3.1 ± 0.6[a]	12 ± 3[b]	0.26 ± 0.12
		12.6 ± 1.3[a]	0.25 ± 0.07
		3.2 ± 0.4[c]	0.97 ± 0.31
$2\pi^+2\pi^-$	0.4 ± 0.1[d]	1.7 ± 0.7[d]	0.24 ± 0.16

References to Table 2:

a) M. Foster et al., Nucl. Phys. B8 (1968) 174
b) C. Baltay et al., Phys. Rev. 145 (1966) 1103
c) P. Espigat et al., Nucl. Phys. B36 (1972) 93
d) C. Ghesquiere, Symposium on $N\bar{N}$ Interactions, Liblice, CERN 74-18 (1974) 436

TABLE 3

$B(\pi^{o}\rho^{o})\times 10^{3}$	$B(\eta\omega)\times 10^{3}$	x^{2}	Phase-space correction
16±1 [a]	10.4±0.95 [b]	1.1±0.17	ℓ=1
14±2 [c]	4.4±1.4 [c]	0.47±0.18	1.6
average 15±1 used for x^{2}			

References to Table 3:

a) M. Chiba et al., Phys. Lett. B202 (1988) 447
b) L. Adiels et al., CERN-EP/88-142
c) KEK E68, M. Chiba et al., Proc. IV LEAR Workshop (1987)

TABLE 4

$B(\rho^{o}\eta)\times 10^{3}$	$B(\pi^{o}\omega)\times 10^{3}$	x^{2}	Phase-space correction
2.2±1.7 [a]	5.2±0.5 [d]	0.7±0.6	ℓ=1
9.6±1.6 [b]	4.9±3.6 [e]	3±0.8	1.6
5.3 (+2−0.8) [c]	average 5±0.5 used for x^{2}	1.7±0.6	

References to Table 4:

a) C. Baltay et al., Phys. Rev. 145 (1966) 1103
b) KEK E68, M. Chiba et al., Proc. IV LEAR Workshop (1987)
c) L. Adiels et al., CERN-EP/88-142
d) M. Chiba et al., Phys. Lett. B202 (1988) 447
e) LEAR PS 182, S. Carius, Ph. D. Thesis, Royal Inst. of Technology, Stockholm (1986)

TABLE 5

$B(\pi^0\pi^0)\times10^3$	$B(\eta\eta)\times10^3$	x^4	Phase-space correction
0.48 ± 0.1[a]	0.081 ± 0.031[c]	0.21 ± 0.12	
0.14 ± 0.03[b]		0.7 ± 0.4	$\ell=0$
0.206 ± 0.014[c]		0.48 ± 0.22	1.22
0.25 ± 0.03[d]		0.39 ± 0.2	

References to Table 5:

a) S. Devons et al., Phys. Rev. Lett. 27 (1971) 1614
b) G. Bassompierre et al., Proc. 4th European Antiproton Symposium, Barr, Vol. 1 (1978)
c) L. Adiels et al., Z. Phys. C35 (1987) 15
d) M. Chiba et al., Phys. Lett. B202 (1988) 447

TABLE 6

p_{Lab}/GeV	$\sigma(\rho^0 2K_S)\times10^3$/mb	$\sigma(\omega 2K_S)\times10^3$/mb	R
1.2	51 ± 7[a]	46 ± 7.2[a]	1.1 ± 0.32
2.3	27 ± 12[b]	25 ± 9[b]	1.1 ± 0.9
2.75	4 ± 2[b]	13 ± 6[b]	0.3 ± 0.3

References to Table 6:

a) J. Duboc et al., Nucl. Phys. B46 (1972) 429
b) B.Y. Oh et al., Nucl. Phys. B63 (1973) 1

TABLE 7

p_{Lab}/GeV	$\sigma(\rho^0 K^+K^-)\times10^3$/mb	$\sigma(\omega K^+K^-)\times10^3$/mb	R
2.45-2.60	45 ± 15[a]	32 ± 17[a]	1.4 ± 1.2

Reference to Table 7:

a) B.Y. Oh et al., Nucl. Phys. B63 (1973) 1

Experimentally, $B(\rho^o\omega) = (2.26\pm0.23)\%$[13], $(0.7\pm0.3)\%$[14] and $(3.94\pm0.59)\%$[15] with an average value $B_{av}(\rho^o\omega) = 2.0\%$. On inserting the numerical values, we find that the prediction of the rearrangement QLD is

$$5.6\% < B(\omega\omega) < 10.7\%,$$

which is to be compared with $B_{exp}(\omega\omega) = (1.4\pm0.6)\%$.

The data concerning Eq. (11), however, allow us to give two different interpretations. First, we note that this relation is satisfied within two error bars by present data. Thus, the dominance of the odd part of the vector meson-meson production amplitude $T(\rho^o\omega) = -T(\omega\rho^o)$ (or, more importantly, the dominance of Fig. 1, as seen below) is not excluded. Second, the data suggest that $\sigma(\rho^o\rho^o)$ is zero compared with $\sigma(\omega\omega)$. Then, in Eq. (10), $T(\rho^o\rho^o)$ vanishes, so that now

$$T(\omega\omega) = T(\rho^o\omega) + T(\omega\rho^o) \qquad (12)$$

from the dominance of the rearrangement. According to this the maximal possible value $\sigma(\omega\omega)$ would be obtained if $T(\rho^o\omega)$ were symmetric, so that we must have $B(\omega\omega) \leq 2B(\rho^o\omega)$, in agreement with experiment. The equality $B(\omega\omega) = 2B(\rho^o\omega)$ follows if $T(\rho^o\omega)$ were symmetric indeed. This agrees with one (Ref. 14) and strongly disagrees with the other two experimental values for $B(\rho^o\omega)$ (Refs. 15 and 16).

4. Annihilation into three mesons

The methods developed in Sec. 3 are easily extended to the annihilation into three mesons. The number of unknown amplitudes, when written in the quark basic (Eq. (7)), becomes very large and no simple relation connecting them can be found without additional assumptions, except one between η and η'. Table 2 shows the sparse present data on the production of η and η', together with more than one further non-strange meson. Since the phase-space corrections are unknown and may be large at low energies, no firm conclusions follow from these data. Without phase-space corrections they favour a large negative value of θ_{PS}, whereas a phase-space correction factor of 3 would imply $\theta_{PS} \approx -14^{\circ}$.

Other interesting relations involving higher powers of K are

$$\begin{aligned} \bar{\sigma}(M_1M_2\eta') &= K\bar{\sigma}(M_1M_2\eta), \quad 2\bar{\sigma}(M\eta'\eta') = K\bar{\sigma}(M\eta\eta') = \\ &= 2K^2\bar{\sigma}(M\eta\eta), \\ 3\bar{\sigma}(\eta'\eta'\eta') &= K\bar{\sigma}(\eta\eta'\eta') = K^2\bar{\sigma}(\eta\eta\eta') \\ &= 3K^3\bar{\sigma}(\eta\eta\eta), \end{aligned} \tag{13}$$

with M, M_1 and M_2 denoting any meson out of π^{0}, ρ^{0} or ω. More relations between different amplitudes may be derived if one of the QLD dominates over the others (see Figs. 1-5).

If[17] the rearrangement (Fig. 4) dominates, we obtain from the QLR and the $p\bar{p}$ initial state that $T((u\bar{u})(u\bar{u})(u\bar{u})) = T((u\bar{u})d\bar{d})(d\bar{d})) = T((d\bar{d}),(d\bar{d}),(d\bar{d})) = 0$ for any space-time quantum numbers and any permutation of the individual $q\bar{q}$. This predicts

$$\bar{\sigma}(\rho^0\rho^0\rho^0) = \bar{\sigma}(\omega\omega\omega),\ x^2\bar{\sigma}(\rho^0\rho^0\pi^0) = \bar{\sigma}(\omega\omega\eta),$$
$$x^4\bar{\sigma}(\rho^0\pi^0\pi^0) = \bar{\sigma}(\omega\eta\eta) \text{ and } x^6\bar{\sigma}(\pi^0\pi^0\pi^0) = \bar{\sigma}(\eta\eta\eta). \quad (14)$$

Since the K meson cannot be directly produced by the rearrangement QLD (Fig. 4) owing to the absence of s and $\bar{s}$ quarks in the initial $p\bar{p}$ state, its production should be suppressed.

If, on the other hand, the annihilation (Fig. 3) dominates, the amplitude vanishes if (for any space-time meson quantum numbers) any two of the three meson states M_1, M_2, M_3 are $q\bar{q}$ and $Q\bar{Q}$ with $Q \neq q$. As always in the case of annihilation, this remains true for any initial-state interaction that only results in isospin mixing. Thus, we predict

$$3\bar{\sigma}(\omega\omega\omega) = \bar{\sigma}(\rho^0\rho^0\omega),\ \bar{\sigma}(\omega\omega\eta) = \bar{\sigma}(\rho^0\rho^0\eta),$$
$$2^-(\omega\eta\omega) = x^2\bar{\sigma}(\rho^0\pi^0\omega),\ 2\bar{\sigma}(\omega\eta\eta) = x^2\bar{\sigma}(\rho^0\pi^0\eta),$$
$$\bar{\sigma}(\eta\eta\omega) = x^4\bar{\sigma}(\pi^0\pi^0\omega),\ 3\bar{\sigma}(\eta\eta\eta) = x^4\bar{\sigma}(\pi^0\pi^0\eta), \quad (15)$$
$$3\bar{\sigma}(\rho^0\rho^0\rho^0) = \bar{\sigma}(\omega\omega\rho^0),\ \bar{\sigma}(\rho^0\rho^0\pi^0) = \bar{\sigma}(\omega\omega\pi^0),$$
$$2x^2\bar{\sigma}(\rho^0\pi^0\rho^0) = \bar{\sigma}(\omega\eta\rho^0),\ 2x^2\bar{\sigma}(\rho^0\pi^0\pi^0) = \bar{\sigma}(\omega\eta\pi^0),$$
$$x^4\bar{\sigma}(\pi^0\pi^0\rho^0) = \bar{\sigma}(\eta\eta\rho^0) \text{ and } 3x^4\bar{\sigma}(\pi^0\pi^0\pi^0) = \bar{\sigma}(\eta\eta\pi^0).$$

The K-meson production is, however, allowed by the annihilation QLD (Fig. 3). From $T(K^+K^-(d\bar{d})) =$ $= T(K^0\bar{K}^0(u\bar{u})) = 0$, where K and $q\bar{q}$ can have any space-time quantum numbers, it follows that

$$\sigma(K^+K^-\rho^0) = \sigma(K^+K^-\omega) \text{ and } \sigma(K^0\bar{K}^0\rho^0) = \sigma(K^0\bar{K}^0\omega), \qquad (16)$$

which is valid for any initial-state interaction that leads only to isospin mixing. We should note that with M_1 and M_3 K-mesons, the M_2 in Fig. 3 may be an $s\bar{s}$ state. Thus, the η meson may also be produced together with $K\bar{K}$ through its $s\bar{s}$ component. For these processes it follows that

$$\begin{aligned} xT(K^+K^-\eta) + yT(K^+K^-\eta') &= T(K^+K^-\pi^0), \\ xT(K^0\bar{K}^0\eta) + yT(K^0\bar{K}^0\eta') &= -T(K^0\bar{K}^0\pi^0). \end{aligned} \qquad (17)$$

The K mesons may also be produced through the mixed diagram in Fig. 5, which introduces only the minimal number of annihilations. Since, in this case, $T(K^+K^-(s\bar{s})) = T(K^0\bar{K}^0(s\bar{s})) = T(K^0\bar{K}^0(d\bar{d})) = 0$, the consequences are

$$\begin{aligned} y^2\bar{\sigma}(K^0\bar{K}^0\pi^0) &= \bar{\sigma}(K^0\bar{K}^0\eta') = K\bar{\sigma}(K^0\bar{K}^0\eta), \\ \sigma(K^+K^-\eta') &= K\bar{\sigma}(K^+K^-\eta) \text{ and } \bar{\sigma}(K^0\bar{K}^0\rho^0)=\bar{\sigma}(K^0\bar{K}^0\omega) \end{aligned} \qquad (18)$$

only for the annihilation from the $p\bar{p}$ initial state. It also follows that for any initial state that can be reached via isospin mixing, $\bar{\sigma}(K^+K^-\phi)$ and $\bar{\sigma}(K^0\bar{K}^0\phi)$ should be small compared with the corresponding cross section. We should also note that $\sigma(K^0\bar{K}^0\rho^0) = \sigma(K^0\bar{K}^0\omega)$ follows from both diagrams for KKM production. Without change, K can be replaced by K^* everywhere in the above relations.

At present, experimental data on three-meson production in $p\bar{p}$ annihilation are sparse and several branching ratios important from the QLR stand-point have not yet been measured. Thus, most of the QLR-predicted relations (Eqs. (13-18)) will have to wait for their verification in future experiments.

Since it is expected that the QLR will be better satisfied at higher energies, it is interesting to observe that existing data on $K\bar{K}M$ production in the GeV region support the QLR prediction that $\bar{\sigma}(K\bar{K}\rho^{o}) = \bar{\sigma}(K\bar{K}\omega)$ (see Table 6).

5. Conclusions

We have discussed in detail some of the consequences of the QLR for annihilation of $p\bar{p}$ into two or three mesons. These depend only on the quark flavour flux within the diagram to be tested, i.e. are independent of gluon contributions (which are essentially unknown). The importance of comparing η with η' production has been emphasized for testing the QLR. With present data, only a modest experimental estimate of K is possible (Tables 1-5). Expressing the results in terms of θ_{PS}, we have found that this mixing angle can only have values restricted to

$$-26^{o} < \theta_{PS} < -2.2^{o}.$$

The dominance of the annihilation QLD (Fig. 1), however, implies that $\theta_{PS} \simeq -20^{o}$, in agreement with

with recent[11] estimates of the mixing angle.

Among the relations which test the two-meson production mechanisms those involving $\sigma(\rho^o\rho^o)$, $\sigma(\rho^o\omega)$ and $\sigma(\omega\omega)$ seem to be difficult for the QLR if the experimental data remain as they are. Experimental results with small errors in these channels are therefore eagerly awaited. For the annihilation channels $p\bar{p} \to$ three mesons, the QLR predicts many relations. However, only some of them can be tested at present.

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