THE πNN VERTEX

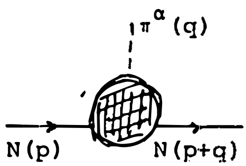
N. Zovko
Ruđer Bošković Institute, 41001 Zagreb,
P.O.B. 1016, Croatia, Yugoslavia

The problem of determining the πNN vertex with the pion off its mass shell has been open for about 30 years without a reliable solution in sight. There exist a few model calculations for zero-mass pions or very close to this limit: And what is important, these models are designed only for the space-like region. The agreement with data is rather poor.

There is no information on the structure of the πNN vertex in the time-like region where the form factor develops an imaginary part. The analytical S-matrix approach is blocked here by the fact that the closest singularity corresponds to the inelastic amplitudes out of the physical domain.

In what follows we present a model determination of the πNN form factor in the space-like region. The result is in remarkable agreement with existing data. For the time-like region, a method is proposed to overcome the difficulty in dealing with inelastic amplitudes of pions off their mass shell. The essence is in the observation that the $N\bar{N}$ pair is a pion too, but very far off its mass shell. The inelastic $\pi N \to \pi\pi N$ problem is in this way reduced to elastic $\pi\pi$ scattering where the celebrated Veneziano string amplitude may be used safely. Going off the pion mass shell is then done by the analytic continuation.

We define the πNN structure in the usual way:



$$\equiv \langle N(p+q)|j_\alpha(0)|N(p)\rangle$$

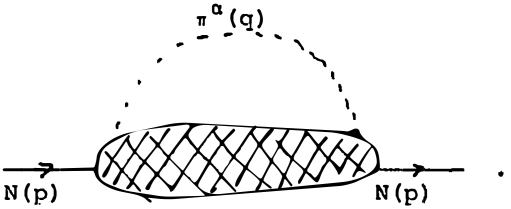
$$= iG_{\pi NN}(q^2)\bar{u}(p+q)\gamma_5\tau_\alpha u(p), \qquad (1)$$

where $p^2 = (p+q)^2 = -m^2$, while the pion is off its mass shell with the constraint $q^2 = -2pq$. The pionic form factor of the nucleon, $G_{\pi NN}(q^2)$, is so defined that on the pion mass shell it becomes the renormalized pion-nucleon coupling constant, $G_{\pi NN}(-m^2) = g_{\pi NN}$. In what follows we omit the subscripts and simply write G.

For zero-mass pions, the Goldberger-Treiman relation fixes G(0), which holds roughly in the whole soft-pion region. Using the parametrization of the N-N scattering potential with single-boson exchanges, or applying the same model to the electroproduction data, one obtains the fit by using an effective pole

$$G(q^2) = \frac{\lambda^2 - m_\pi^2}{\lambda^2 + q^2}, \qquad \lambda^2_{exp} \gtrsim 1\ \mathrm{GeV}^2 . \qquad (2)$$

The effective mass λ^2, evaluated within a variety of models$^{1-3)}$, varies from 0.36 GeV^2 to 0.81 GeV^2, and obviously underestimates the experimental value (2). What we propose here is to evaluate the dominant contribution to the nucleon strong self-energy part represented by the diagram



(3)

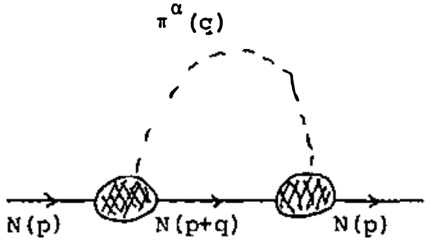
Using standard rules we obtain a strong-interaction Cottingham formula

$$\Sigma = i \int \frac{d^4q}{(2\pi)^4} \frac{\delta_{\alpha\beta}}{q^2-m_\pi^2+i\varepsilon} T^{\alpha\beta}_{\pi N}(p,q) \ , \tag{4}$$

where T represents the forward scattering amplitude of virtual pions on nucleons, and is given by the standard expression

$$T^{\alpha\beta}_{\pi N}(p,q) = i \int d^4x \, e^{iqx} \theta(-x_o) \langle p| [j_\alpha(x/2), j_\beta(-x/2)] |p\rangle . \tag{5}$$

Now we turn to the framework of the analytic S-matrix and evaluate T in the generalized Born approximation and put it into (4). This leads to the replacement of the general diagram (3) by the diagram



, (6)

which manifestly contains our πNN vertex. It is essential here that we work in the S-matrix framework, so that the intermediate nucleon is on the mass shell. The result which follows by use of standard rules is

$$\Sigma = -3i\,\frac{g^2_{\pi N}}{4\pi}\int d^4q\,\frac{\bar{u}(p)\gamma_\mu q^\mu u(p)G^2(q^2)}{(q^2-m_\pi^2)\left[(p-q)^2-m^2\right]}\ . \qquad (7)$$

The use of Feynman's method of symmetric integration ($q_0 \to -iq_0$) ensures that we always have a space-like q^2 in the integrand, namely,

$$-q^2 = -(q_0^2-\vec{q}^{\,2}) \to -(-q_0^2-\vec{q}^{\,2}) = q_0^2+\vec{q}^{\,2} \geq 0\ . \qquad (8)$$

However, for $-q^2 \geq 0$, G is always a real function, and we may safely use the effective-pole parametrization (2) since all singularities are on the semiaxis $-q^2 \leq 0$. Moreover, this is the way of determining the form factor in the regions $-q^2 \geq 0$ and $|q^2| \lesssim 1$ GeV^2. We also note that the pionic propagator in (7) strongly weights the soft-pion region of integration.

After evaluating (7), we obtain

$$\Sigma = \frac{3m}{8\pi}\,\frac{g_{\pi N}^2}{4\pi}\,\frac{\lambda^2}{m^2}\left(1 - \frac{\lambda^2}{2m^2}\ln\frac{\lambda^2}{m^2} - \frac{\lambda^2}{m^2}\,\frac{2m^2-\lambda^2}{4m^2-\lambda^2}\,Y\,\mathrm{arc\,tg}\,Y\right)$$

$$+ \mathcal{O}(m_\pi^2)\,, \qquad (9)$$

where

$$Y = \left(\frac{4m^2}{\lambda^2} - 1\right)^{1/2} .$$

The very assumption is now that in this way the nucleon acquires most of its strong mass, namely, that $\Sigma \gtrsim m$. What follows is a sum rule of the type

$$f(\lambda, g_{\pi N}^2) = 0 \rightarrow \lambda = \lambda(g_{\pi N}^2) \ . \qquad (10)$$

The experimental value $g_{\pi N}^2/4\pi = 14.4$ corresponds to the effective mass $\lambda^2 = 1.28$ GeV^2.

In the time-like region there is no information about $G(q^2)$. Dispersion theory fails here for the reasons mentioned in the introduction. The chief idea of the present model is to evaluate the leading unitarity diagram in the imaginary part,

Im () =  + ... (11)

We observe that when looking from above downward, we may understand the $N\bar{N}$ pair as an effective pion very far from its mass shell ($m_{\pi^*} \gtrsim 2$ GeV!). In this way both vertices become four-pion vertices with only two external pions out of the mass shell. Intermediate pions are physical pions by the very fact that in unitarity diagrams the intermediate set of states is always on the mass shell. So, in essence, the evaluation of the imaginary part will include the product of two pion-pion elastic scattering amplitudes, each of them containing one pion off the mass shell.

This seems again to be an unsurmountable difficulty. However, the idea is to use the Veneziano string amplitude and continue it analytically. The reason for optimism in this model is that the Veneziano ansatz$^{4)}$ works efficiently in all physical problems containing only pions (spin zero!). The work on this approach is in progress.

References

1) D. Schütte and A. Tillmans, Phys. Lett. B206 (1988)1;

2) N. Kaiser, U.-G. Meissner and W. Weise, Phys. Lett. B198 (1987) 319;

3) T.D. Cohen, Phys. Rev. D34 (1986) 2187;

4) C. Lovelace, Phys. Lett. 28B (1968) 264.