EQUIVALENCE BETWEEN DEEP AND SHALLOW NUCLEUS-NUCLEUS POTENTIALS FROM SUPERSYMMETRY

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The deep or shallow nature of nucleus-nucleus potentials is a controversial question for a long time. Deep potentials receive a confirmation from microscopic models : their additional bound states simulate the forbidden states which are a consequence of the Pauli principle. Shallow potentials are physically more satisfactory since all their bound states can be interpreted as molecular states of the unified nucleus.

Supersymmetric quantum mechanics[1] offers a simple technique for constructing shallow potentials which provide the same phase shifts as deep potentials[2]. Let us consider a potential V(r) which includes the Coulomb and centrifugal terms of a given partial wave ℓ . The corresponding hamiltonian H can be factorized (for $\hbar^2/2\mu = 1$) as

$$H = -d^2/dr^2 + V = A^+A^- + E^0 \qquad (1)$$

where A^+ is the adjoint of

$$A^- = -d/dr + (\ln \phi^0)' \qquad (2)$$

and where E^0 and ϕ^0 are the ground-state energy and wave function of H. The supersymmetric partner

$$H_1 = A^-A^+ + E^0 \qquad (3)$$

has the same bound spectrum as H except for the ground state which is suppressed. However H and H_1 provide different phase shifts. Let us now factorize H_1 as in (1) with the same factorization energy E^0. Since E^0 is not an eigenvalue of H_1, ϕ^0 is replaced in (2) by an unphysical solution of $H_1\phi = E^0\phi$ which is bounded at small r-values. Then the supersymmetric partner H_2 of H_1 has the same bound spectrum as H_1 but the same phase shifts as H. Indeed the potential V_2 corresponding to H_2 can be written as[2]

$$V_2 = V - 2 \left(\ln \int_0^r \phi^0(t)^2\, dt\right)'' \qquad (4)$$

Obviously, this potential does not differ from V for large r-values as soon as the bound-state wave function ϕ^0 becomes negligible so that both potentials are phase equivalent. Moreover they lead to the same bound spectrum except for the ground state of H.

Since ϕ^0 behaves as $r^{\ell+1}$ for small r-values, the potential V_2 behaves as

$$V_2 \approx V + 2(2\ell + 3)\, r^{-2} \approx (\ell + 2)(\ell + 3)\, r^{-2} \qquad (5)$$

and is therefore singular. This singularity is in fact imposed by the Levinson theorem and by its generalizations[3]: indeed two regular potentials with different numbers of bound states could not provide equivalent phase shifts at all energies.

Iterating the process described above allows one to suppress a given number N of bound states from a deep potential and make it shallow. The resulting potential is singular

$$V_{2N} \simeq (\ell + 2N)(\ell + 2N + 1)\, r^{-2} \qquad (6)$$

near r = 0, but is phase equivalent to V.

In order to illustrate this technique, we apply it to the simple α + α potential of Buck et al[4]. The succession of potentials V to V4 for the $\ell = 0$ partial wave are shown in Fig.1. The phase equivalent potentials V, V_2 and V_4 which differ only by their numbers (respectively 2, 1 and 0) of bound states (horizontal bars) are represented by full lines. They present very close asymptotic behaviours. The intermediate potentials V_1 and V_3 (dashed lines) have respectively the same bound spectrum as V_2 and V_4 but are not phase equivalent to V. Their asymptotic behaviour is markedly different. In Fig.2, the shallow potentials corresponding to the 0, 2, and 4 partial waves of the potential of Buck et al (full lines) are compared with the α + α potentials of Ali and Bodmer[5] (dashed lines). Both potentials exhibit a close resemblance except for the fact that the potentials of Ref.5 are not singular.

1. C.V.Sukumar, J. Phys. A 18 (1985) 2937
2. D. Baye, to be published
3. P. Swan, Nucl. Phys. 46 (1963) 669
4. B. Buck, H. Friedrich and C. Wheatley, Nucl. Phys. A275 (1977) 246
5. S. Ali and A.R. Bodmer, Nucl. Phys. 80 (1966) 99

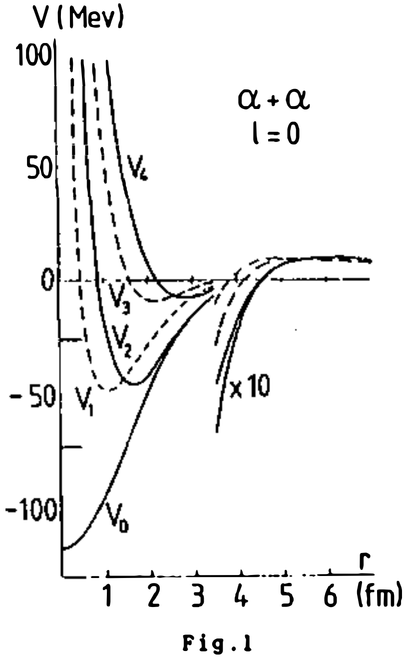

Fig.1

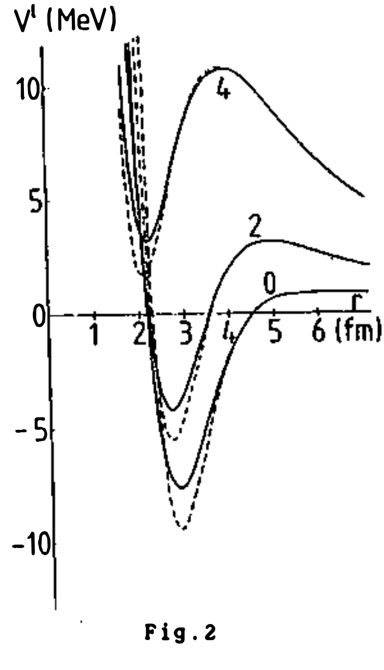

Fig.2