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$K \to \pi\pi\pi$ AND $K \to \pi\pi\gamma$ REEXAMINED IN A LARGE N LIMIT

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ABSTRACT:

An approach to the $K \to \pi\pi\pi$ and $K \to \pi\pi\gamma$ decays in the standard model based on the large N expansion is presented. The Dalitz plot for $K \to \pi\pi\pi$ is very well reproduced and CP violating parameter is found. The inner bremsstrahlung and direct emission parts of the amplitudes for $K \to \pi\pi\gamma$ are determined.

1. INTRODUCTION

At large distances the QCD coupling constant is large and the standard perturbative expansion can not be applied. In 1974 'tHooft has shown that QCD simplifies considerably when one assumes a large number of colors, N, instead of three[1,2,3]. He suggested that 1/N expansion could be used to obtain approximate solutions in QCD. Namely, in the large N limit the QCD - theory of quarks and gluons, becomes equivalent to a theory of weakly interacting mesons.

Their interactions can be described by the most general SU(3) x SU(3) chiral Lagrangian. The large N limit strongly constraints chiral corrections and one keeps only single flavor trace coupling so that the most general chiral Lagrangian becomes[4,5]:

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}[\partial_\mu U \partial_\mu U^+ + r(M(U+U^+)) - \frac{r}{\Lambda_0^2}(M(\partial^2 U + \partial^2 U^+)) +$$

$$+\frac{1}{\Lambda_1^2}(\partial_\mu U^+\partial_\mu U\,\partial_\nu U^+\partial_\nu U)-\frac{1}{\Lambda_2^2}(\partial_\mu U^+\partial_\nu U\,\partial_\mu U^+\partial_\nu U)]$$
$$-eN\frac{1}{48\pi^2}\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{Tr}[Q(\partial_\nu UU^{-1})(\partial_\alpha UU^{-1})(\partial_\beta UU^{-1})+$$
$$+Q(U^{-1}\partial_\nu U)(U^{-1}\partial_\alpha U)(U^{-1}\partial_\beta U)]A_\mu \quad (1.1)$$

The last term in (1.1) summarizes the effects of anomalies in current algebra and it is known as Wess-Zumino term[5]. These terms are suppressed by 1/N, but they must be taken into account for the determination of the direct emission part of the amplitude in $K\to\pi\pi\gamma$ decays.

The effective weak Hamiltonian in the large N limit can be separated into the sum of I = 1/2 and I = 3/2 pieces (neglecting CP violating parts)[4]:

$$H = g_0\left\{(Q_2-Q_1)+\frac{\omega}{\sqrt{2}}(2Q_1+Q_2)\right\} \quad (1.2)$$

where Q_1 and Q_2 are given by

$$Q_1=(\bar{s}d)_{V-A}(\bar{u}u)_{V-A}\;;\quad Q_2=(\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \quad (1.3)$$

The explicit calculations of the next-to-leading order terms in the large N show quite good agreement of theoretical and experimental values of g_0 and ω [1,2,3]. In our calculations we use g_0 = 8.8 10^{-6} GeV^{-2} and ω = 1/22.

2. $K\to\pi\pi\pi$ DECAYS

The Dalitz plot for the $K\to\pi\pi\pi$ is associated with the I = 1/2 and I = 3/2 weak operators[6]:

$$\sqrt{2}\,A(K^0\to\pi^+\pi^0\pi^-)\underset{\Delta I}{\equiv} a_I+b_IY+c_I\left(Y^2+\frac{X^2}{3}\right)+d_I\left(Y^2-\frac{X^2}{3}\right)$$
$$X=\frac{1}{m_\pi^2}(S_+-S_-)\;;\qquad S_i=(p_K-p_i)^2$$
$$Y=\frac{1}{m_\pi^2}(S_0-S)\;;\qquad S=\frac{1}{3}(S_++S_0+S_-) \quad (2.1)$$

A straightforward calculation of the direct and pole diagrams leads to the following results:

$$a_{\frac{1}{2}} = g_o \frac{m_K^2}{3}\left\{1 + 6\frac{m_\pi^2}{\Lambda_o^2} + \frac{4}{3}(m_K^2 - 3m_\pi^2)\left(\frac{1}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right\} = 0.91$$

$$b_{\frac{1}{2}} = g_o m_\pi^2 \left\{1 + \frac{2}{3}(m_K^2 + 3m_\pi^2)\left(\frac{1}{\Lambda_1^2} + \frac{2}{\Lambda_2^2}\right)\right\} = 0.26$$

$$c_{\frac{1}{2}} = g_o m_\pi^2 \left\{-m_\pi^2\left(\frac{1}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right\} = -0.0018$$

$$d_{\frac{1}{2}} = g_o m_\pi^2 \left\{-m_\pi^2\left(\frac{1}{\Lambda_1^2} + \frac{2}{\Lambda_2^2}\right)\right\} = -0.0082$$

$$a_{\frac{3}{2}} = -\sqrt{2}\,\omega\, a_{\frac{1}{2}} = -0.059 \tag{2.2}$$

$$b_{\frac{3}{2}} = \frac{5\sqrt{2}}{4} g_o \omega m_\pi^2 \left\{1 - \frac{18}{5}\frac{m_\pi^2}{\Lambda_o^2} + \frac{1}{15}(m_K^2 + 3m_\pi^2)\left(\frac{1}{\Lambda_1^2} + \frac{2}{\Lambda_2^2}\right)\right\} = 0.014$$

$$c_{\frac{3}{2}} = 2 g_o \omega m_\pi^2 \left\{m_\pi^2\left(\frac{1}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right\} = 0.00016$$

$$d_{\frac{3}{2}} = -\frac{1}{4} g_o \omega m_\pi^2 \left\{m_\pi^2\left(\frac{1}{\Lambda_1^2} + \frac{2}{\Lambda_2^2}\right)\right\} = -0.000093$$

where we have used experimental values for the $a_{\frac{1}{2}}$ and $b_{\frac{1}{2}}$ coefficients and fit for the scales $\Lambda_1 \approx 0.93$ GeV and $\Lambda_2 \approx 1.26$ GeV. The experimental results[6] are in very good agreement with our results.

The CP violation induced by $K \to \pi\pi\pi$ is defined by[4]:

$$\eta_{+0-} = \frac{A(K_S \to \pi^+\pi^0\pi^-)}{A(K_L \to \pi^+\pi^0\pi^-)} = \varepsilon + \varepsilon'_{+0-}$$

In the isospin symmetry limit, the inclusion of the next-to-leading operators does not modify Li-Wolfenstein relation[7]. This relation is mainly modified by $\pi^0 - \eta - \eta'$ correction:

$$\varepsilon'_{+0-} = -1.3\,\varepsilon'$$

where ε' is CP violating parameter in $K \to \pi\pi$[4].

3. $K \to \pi\pi\gamma$ DECAYS

The $K^+ \to \pi^+\pi^0\gamma$ and $K_L^0 \to \pi^+\pi^-\gamma$ decays exhibit two separate partial amplitudes: the inner bremsstrahlung and direct emission term. For the inner bremsstrahlung part of the amplitudes we determine:

$$A^{IB}(K^+ \to \pi^+\pi^0\gamma) = -e g_0 \frac{\omega}{\sqrt{2}} 3 f_\pi \left(1 + \frac{m_\pi^2}{\Lambda_0^2}\right)(m_K^2 - m_\pi^2)\left(\frac{\varepsilon \cdot p_+}{p_+ \cdot q} - \frac{\varepsilon \cdot k_+}{k_+ \cdot q}\right) \quad (3.1)$$

$$A^{IB}(K^0 \to \pi^+\pi^-\gamma) = -e g_0 \left(1 + \frac{2}{\omega}\right)\sqrt{2} f_\pi (m_K^2 - m_\pi^2)\left(1 + \frac{m_\pi^2}{\Lambda_0^2}\right)\left(\frac{\varepsilon \cdot p_+}{p_+ \cdot q} - \frac{\varepsilon \cdot p_-}{p_- \cdot q}\right) \quad (3.2)$$

The phase space integration over the experimentally allowed energy regions implies the following values for the branching ratios:

$$\begin{aligned} Br^{IB}(K^+ \to \pi^+\pi^0\gamma) &= 2.88 \cdot 10^{-4} \\ Br^{IB}(K_L^0 \to \pi^+\pi^-\gamma) &= 1.45 \cdot 10^{-5} \\ Br^{IB}(K_S^0 \to \pi^+\pi^-\gamma) &= 2.41 \cdot 10^{-3} \end{aligned} \quad (3.3)$$

The experimental values are taken from[8,9,10].
For the direct emission terms we obtain[11]:

$$A^{DE}(K^+ \to \pi^+\pi^0\gamma) = \frac{e}{2\pi^2 f} g_0 \left(1 + \frac{m_K^2 - m_\pi^2}{\Lambda_0^2}\right)\varepsilon^{\mu\nu\alpha\beta} k_\nu^+ q_\alpha p_\beta^+ \varepsilon_\mu$$

$$A^{DE}(K_L^0 \to \pi^+\pi^-\gamma) = \frac{e}{2\pi^2 f} g_0 \left(1 + \frac{m_K^2 - m_\pi^2}{\Lambda_0^2}\right)\varepsilon^{\mu\nu\alpha\beta} p_\nu^+ p_\alpha^- q_\beta \varepsilon_\mu \,*$$
$$\left\{ 1 + \frac{m_K^2}{m_K^2 - m_\eta^2}\frac{1}{3}(\cos\theta + 2\sqrt{2}\sin\theta)(\cos\theta - \sqrt{2}\sin\theta) \right.$$
$$\left. + \frac{m_K^2}{m_K^2 - m_{\eta'}^2}\frac{1}{3}(\sin\theta - 2\sqrt{2}\cos\theta)(\sin\theta + \sqrt{2}\cos\theta)\right\} \quad (3.4)$$

where θ is the mixing angle between η and η' mesons.
For the branching ratios we calculate:

$$Br^{DE}(K^+ \to \pi^+\pi^0\gamma) = 1.05 \cdot 10^{-5} \quad (3.5)$$

$$Br^{DE}(K^0_L \to \pi^+\pi^-\gamma) = 3.01\cdot 10^{-5}$$

where we have taken $\theta \approx -14^o$. The experimental values are from[12].

4. CONCLUSION

The Dalitz plot for $K\to\pi\pi\pi$ calculated within the chiral Lagrangian framework supplemented by large N limit reproduce fairly well the experimental results. The branching ratios for $K\to\pi\pi\gamma$ are also in agreement with experimental data.

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