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STRONGLY LOCALIZED ALMOST MASSLESS CLUSTERS OF CONSTITUENTS

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Abstract

Composite systems with the product of mass and size close to zero are discussed from the point of view of relativistic potential models and Nambu Jona-Lasinio-like models with coherent collective states. This product is smaller than 10^{-6} for quarks and leptons in constituent models. It is pointed out that in potential models beside the scalar part the existence of the vector part of an effective potential is essential. The collective states require an understanding of the old problem of the meaning and the evolution of an infinite medium, the Dirac sea.

1 Introduction

There are several indications which support the idea that quarks and leptons are composite objects. On the other hand, noncomposite theories, with superstrings for example, may explain the mass hierarchy of quarks and leptons as well.

If quarks and leptons are indeed clusters of constituents, then such clusters have very peculiar properties$^{(1-3)}$. While for example clusters such as nucleons in nuclei or quarks in nucleons have a ratio[1] between the size of the system and its de Broglie wavelength of

[1] the units in which $\hbar = c = 1$ are used

the order of magnitude of one or greater, this ratio in the case of quarks and leptons is smaller than 10^{-6}. Again, while the contribution of quarks to the anomalous magnetic moment of nucleons is essential (the Dirac magnetic moment is usually neglected in relativistic potential models), the contribution of the internal degrees of freedom to the magnetic moment of quarks and leptons is very small in comparison with the Dirac magnetic moment.

In spite of all these problems, the above mentioned pecularities cannot be used as an argument against the constituent models of quarks and leptons since they already appear in theories describing hadron physics: the pion, for example, has as a Goldstone boson in the chiral limit a product of its size and mass not only smaller than 10^{-6} but zero. Beside that, chiral models with massless Goldstone bosons, work with an infinite medium - the vacuum. The problem of the quantum vacuum with an infinite energy is an old and unsolved problem, also for theories with superstrings. In this contribution I comment on two type of models which are used to simulate (almost) massless strongly localized states of clusters of constituents: namelly potential models and Nambu Jona-Lasinio-like models. Although relativistic potential models have many weak points, they still can serve for a rough estimation of properties, even of strongly localized almost massless clusters of constituents. It turns out[3,4] that in this case the effective potential has to have, besides a scalar part (which is the case in bag models), also a vector part.

Nambu Jona-Lasinio[5]-like models which are built on an infinite medium require an understanding of the meaning and evolution of the vacuum in cosmology.

2 Potential models

When trying to generate the peculiar dynamics of the substructure of quarks and leptons, the postulation of a new interaction - the hypercolour interaction - is needed as well as the appropriate equations of motion. Since the bound states of relativistic particles are involved, interacting (very probably) through non-abelian gauge fields, both problems are nontrivial (unsolved even on the hadron level). We can simplify both at the same time by studying the system of independent particles in an effective potential which is able to localize particles strongly and yet assure the energy of particles is close to zero so that the product of the mass E of the system and its size $< r >$ is close to zero ($E < r > < 10^{-6}$ for example). A bag-like model cannot be used since the bag model has $E \cdot < r > \sim 2.0$.

We suppose that our Hamiltonian

$$H = \sum_{i=1}^{N} H_{(i)} = \sum_{i=1}^{N} [\vec{\alpha}_{(i)} \cdot \vec{P}_{(i)} + \beta_{(i)}(M_{(i)} + W(r_i)) + V(r_i)] \qquad (1)$$

has an effective potential with a vector $V(r_i)$ (a zero-th component of a four vector) and a scalar $W(r_i)$ part and that both are radially symmetric. We look for functions $W(r_i)$ and $V(r_i)$ such that the system has the above required properties. (One could add also tensor terms.) In this way we proceed similarly as in the hadron case: postulating the non-Abelian hypercolour interaction we expect that this interaction manifests in the "low" momenta region as an effective potential with a vector and a scalar part and that the effective potential in principle could be derived from the quantum theory of hypercolour dynamics in a similar way as the scalar potential on the hadron level(6) may be derived from QCD.

It turns out(3,4) that the appropriately chosen scalar and vector potentials indeed force particles in a cluster to localize strongly allowing the mass of the cluster to be close to zero. The choice of the effective potential is (of course) not unique(4) but the following condition seems essential: beside a strongly confining scalar part a strong vector part is essentially needed[2].

In Fig. 1 the effective potential with a scalar part (the dashed line represents two delta functions at R_1 and R_2 and an infinite wall at R_3) and a vector part (the full line represents $1/r$ potential with two step functions at R_1 and R_2) is presented which reproduces not only strongly localized, almost massless clusters but the experimental masses and (roughly) the electromagnetic properties of the electron, mion and tau(3) (See Table I).

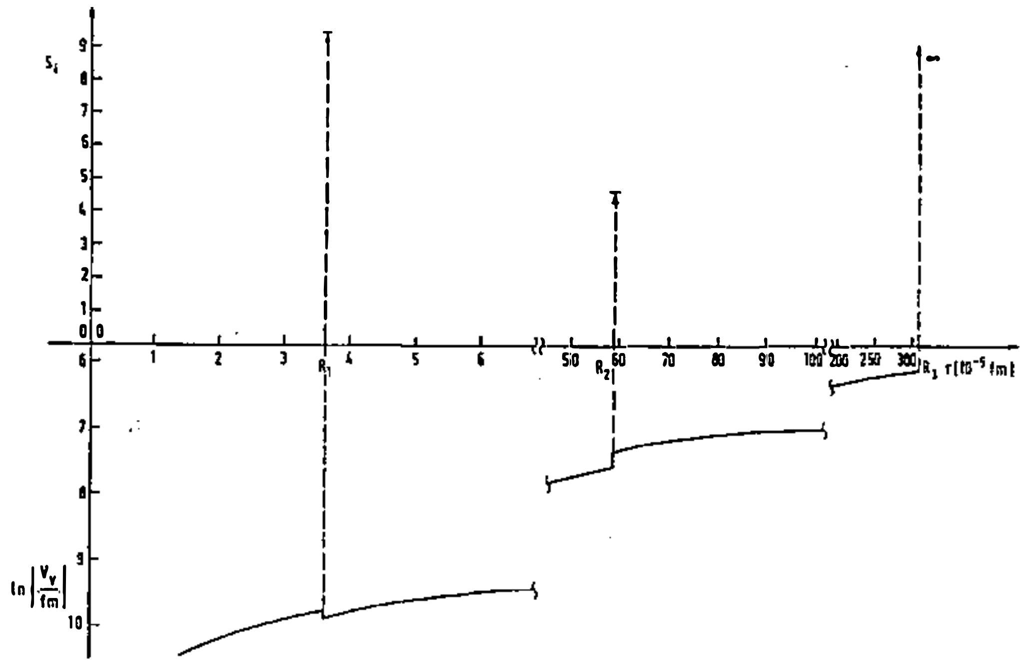
Table 1. The masses, the mean radii and the contribution of the internal structure to the magnetic moment of e, μ and $\tau^{(3)}$

	Mass	$< r >$ (fm)	μ (fm e_o)
e^-	0.51 MeV	$2.3 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$
μ^-	105 MeV	$3.5 \cdot 10^{-4}$	$1.6 \cdot 10^{-4}$
τ^-	1.8 GeV	$8.2 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$

[2]If both parts of the potential are step functions:

$$W = \left\{ \begin{array}{ll} 0, & r \leq R \\ \infty, & r > R \end{array} \right\}; \qquad V = \left\{ \begin{array}{ll} -a, & r \leq R \\ 0, & r > R \end{array} \right\}$$

then $< r > \sim R$ while $a \sim \frac{2.0}{R}$ ensures $E \sim 0$. More sophisticated potentials can do more(3).



The above mentioned findings suggest the following questions:

(i) If the vector part of the effective interaction at "low" momenta on the substructure level is so strong, is the corresponding vector part of an effective QCD interaction at r $\geq$ 1 fm then also non-negligible? Can it be calculated from the lattice QCD at momenta $\frac{1}{Q^2} \leq (1fm)^2$ along with the scalar effective interaction? Is it large enough at the QCD level to be distinguished from numerical errors? Are the tensor terms also large enough in this case to be calculated?

(ii) What makes the vector part of the effective interaction at "low" momenta in the case of hypercolour dynamics so strong? The group structure? The broken symmetries at different energy scales (bringing more parameters into the game)? Or is it an artificial effective term simulating the important degrees of freedom not included in this simple model space? Are the neglected tensor terms important as well?

3 Collective degrees of freedom in Nambu Jona-Lasinio-like models

In the literature[1,2] the problem of families (three or more) of quarks and leptons is usually treated not with potential models but rather with massless collective fermion states which appear due to some global symmetry group which is partly dynamically broken. Since the collective massless states are fermions, supersymmetry seems to be needed. The procedure is, besides the choice of global symmetry, pretly much like in the pion case when it is presented as a Goldstone boson, except that the difficulties with generating the appropriate masses of quarks and leptons are much more complicated and they even seem to be unsolvable. We shall therefore first discuss the pion as a Goldstone boson. Following Nambu and Jona-Lasinio[3] (5), the massless collective (coherent) states of quark and antiquark pairs appear within (and out of) the infinite vacuum - the Dirac sea - in a similar(7) way that superconductivity occurs due to the correlations among conducting electrons in crystals. Even the Lagrangian is of the same type in both cases, only the symmetry, which is spontaneously broken, is different (in the case of superconductivity it is the fermion number, in the case of the pion it is the chiral symmetry), and due to different broken symmetry different types of quasi-particles appear. In addition, the kinematics is nonrelativistic in the superconductivity case.

In the quark case, one starts with a Dirac sea of massless noninteracting fermions, filling the sea from $E = -E_{min}(p_\mu p^\mu \leq \Lambda^2)$ to $E = 0$[4]. When the chiral invariant Hamiltonian is taken into account:

$$H = \sum_{i=1}^{N} \vec{\alpha}_i \cdot \vec{p}_i + \frac{g}{2} \sum_{i \neq j} \delta(\vec{x}_i - \vec{x}_j)[\gamma_{oi}\gamma_{oj}(1 - \gamma_{5i}\gamma_{5j})], \tag{2}$$

the Dirac sea shrinks in the mean field approximation if $g > 2\pi^2/\Lambda^2$ and the chiral symmetry of the vacuum state is broken, from $E = 0$ to $E = -m$. The true ground state of the above Hamiltonian is of course not the Dirac sea of the independent massive fermions, but the correlated many particle state. Analyzing the new vacuum state(5,7) one finds that quarks obtain their masses due to the coherent state of (massless) quark-antiquark pairs around them. One can estimate the size of such a coherent cloud, which has the quantum numbers of a pion. It turns out(7) that the size is finite for any finite m and Λ/m, while $\lim_{\Lambda/m \to \infty} < r^2 > \cdot m^2 = \lim_{\Lambda/m \to \infty} ln(\Lambda/m)/(2(\Lambda/m)^2) = 0$. The product of the mass of the pion and its mean radius is in the chiral limit ($m_\pi = 0$) equal to zero, so that it fulfills the same condition required for the substructure of quarks and leptons: $E \cdot < r >< 10^{-6}$. Without going into details, we may guess the (well known) similar scenario for the case of the substructure of quarks and leptons: the Dirac sea of massless preons (constituents of quarks and leptons) will due to the mean field hypercolour interaction, manifested in the Hamiltonian to some extend similar to Hamiltonian(2), correlate into coherent states of massless fermions, representing quarks

[3] He wrote down the Lagrangian for nucleons, thirty years later the same was used for quarks.

[4] Since the theory is not renormalizable, we let fermions occupy the states only up to $p_\mu p^\mu \leq \Lambda^2$.

and leptons. The size will again be finite. The main point in such theories is that an infinite (may be even finite) vacuum with infinite (or very large) energy is essentially needed to generate such massless states. In the case that the vacuum is finite, than the mass of coherent collective states (Goldstone fermions or Goldstone bosons) are non-zero (as rotational states in nuclei would have zero energy only if infinitely many nucleons would contribute to the collective states). Questions appear to which we don't know the answers: How can such a vacuum be built into the cosmological models since it would, due to its infinite energy, cause an infinite cosmological constant? If such a vacuum is not an artifact of quantum theories, how did it evolve with the evolution of our Universe? What are the characteristics of the vacuum now? One would guess that due to the correlations among constituents, the vacuum evolved in a similar way as the matter in the Universe.

4 Discussion and conclusion

We discussed how potential models may describe strongly localized almost massless clusters of constituents provided that the effective potential besides the confining scalar part also has a strong vector part. To try to understand where the vector part comes from, we suggest looking for the strength of the vector part in the QCD case by performing careful calculations on the lattice for the momenta $\frac{1}{Q^2} \leq$ (1 fm).

Using the Goldstone boson or Goldstone fermion mechanism, one can generate states with $E \cdot \, < r > \sim 0$, but the question of the reality of an (almost) infinite medium - a vacuum - occurs, since the (almost) infinite medium is essentially needed in such theories. It is the open question how such a vacuum evolves in the evolution of the Universe due to the correlations among constituents in the vacuum, how it looks now and why it does not influence the cosmological constant. It seems to me that the superstring theories confront the same problem. We must admit that the problem of the vacuum also occurs in potential models when the vacuum polarization and the interaction among constituents is taken into account. If forgeting on the problem of the vacuum the suggestion can be made to take into account collective and single particle degrees of freedom at the same time. This may simplify the problem which with the Goldstone fermions seems unsolvable.

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