EXCHANGE CONTRIBUTION TO LINDHARD' S DIELECTRIC FUNCTION

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The dielectric function of the degenerate electron gas interacting with the uniform background of the positive charge was first calculated by Lindhard[1], in the approximation which is equivalent to the RPA. Although Suhl and Werthamer[2] and Watabe[3] developed a general approach for improving the RPA with higher-order terms, explicit calculations of this type were performed only for small momentum transfers q and the limiting cases of high and low frequencies ω. In the present paper, these results are extended to arbitrary frequencies, excluding only the vicinity of the point at which the frequency is equal to the product of the momentum transfer and the Fermi velocity v_F.

In the second RPA, the dielectric function of the electron plasma may be written in the form

$$\varepsilon(q,\omega) = 1 + V_q \sum_{sk} \frac{N_{sk+q} - N_{sk}}{\hbar\omega + E_k - E_{k+q}}$$

$$+ V_q \sum_{skk'} V_{k-k'} \frac{(N_{sk+q} - N_{sk})(N_{sk'+q} - N_{sk'})}{\hbar\omega + E_k - E_{k+q}}$$

$$\left(\frac{1}{\hbar\omega + E_k - E_{k+q}} - \frac{1}{\hbar\omega + E_{k'} - E_{k'+q}}\right) . \qquad (1)$$

V_q is equal to $4\pi e^2/q^2$, E_k is the electron kinetic energy $\hbar^2 k^2/2m$ and N_{sk} is the occupation number in the state with spin s and wave vector $\vec{k}$.

We have evaluated the real part of the dielectric function in the approximation that q is small compared with the Thomas-Fermi wave number q_{TF}. The influence of the exchange contribution measures the quantity

$$\gamma = 3\left(\frac{\hbar\omega_p}{2mv_F^2}\right)^2 , \qquad (2)$$

ω_p being the electron plasma frequency

$$\omega_p^2 = \frac{4\pi Ne^2}{m} . \qquad (3)$$

Note that in our approach higher-order terms in the perturbation expansion are omitted. This requires that γ must be much smaller than unity.

In the table are listed numerical results for Lindhard's dielectric function $\varepsilon_L(q,\omega)$ and the corresponding exchange correction

$$\Delta\varepsilon(q,\omega) = \varepsilon(q,\omega) - \varepsilon_L(q,\omega) \qquad (4)$$

as a function of the ratio ω/qv_F for $q=0{,}01\ q_{TF}$ and for different values of γ.

TABLE

$\frac{\omega}{qv_F}$	$\varepsilon_L(q,\omega)$	$\Delta\varepsilon(q,\omega)$ $\gamma=0{,}02$	$\gamma=0{,}04$	$\gamma=0{,}06$	$\gamma=0{,}08$	$\gamma=0{,}1$
0	10 000	200	400	600	800	1000
0,1	9900	190	380	580	770	960
0,2	9600	170	330	500	670	840
0,3	9100	130	250	380	500	630
0,4	8300	62	120	190	250	310
0,5	7300	- 25	- 50	- 75	-100	-120
0,6	5800	-140	-290	-430	-580	-720
0,7	3900	-310	-620	-930	-1200	-1600
0,8	1200	-550	-1100	-1700	-2200	-2800
0,9	-3200	-910	-1800	-2700	-3600	-4500
1,1	-6700	- 95	- 190	- 280	- 380	- 470
1,2	-4400	-8,1	- 16	- 24	- 32	- 41
1,3	-3200	1,6	3,1	4,7	6,3	7,8
1,4	-2500	2,8	5,7	8,5	11	14
1,5	-2100	2,6	5,2	7,9	10	13
1,6	-1700	2,2	4,4	6,5	8,7	11
1,7	-1500	1,8	3,5	5,3	7,0	8,8
1,8	-1300	1,4	2,8	4,2	5,6	7,1
1,9	-1100	1,1	2,3	3,4	4,6	5,7
2,0	- 990	0,92	1,8	2,8	3,7	4,6
3,0	- 400	0,17	0,35	0,52	0,70	0,87
4,0	- 220	0,055	0,11	0,16	0,22	0,27
5,0	- 140	0,022	0,044	0,066	0,088	0,11

References

1) J. Lindhard, Kgl.Danske Videnskab.Selskab.Matt-fys. Medd. 28 (1954) 8;
2) H. Suhl and R.N. Werthamer, Phys.Rev. 122 (1961) 359;
3) M. Watabe, Progr.Theoret.Phys. 28 (1962) 265.