PAIRING IN GAPLESS SUPERCONDUCTORS

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The Hartree-Bogolubov method in its operator form , as formulated first by Herbut and Vujičić[1], is very suitable to study the pairing in superfluid systems. In this approach the normal state of the system is described by means of linear operators , as the density operator ϱ and the Hartree-Fock potential operator Γ, while the superfluid correlations are expressed in terms of anti-linear operators , as the pairing operator P_a , the correlation operator $t_a = (\varrho - \varrho^2)^{1/2} P_a$ and the pairing potential operator Δ_a. The case when P_a is fixed to be equal to the time reversal operator T_a , and all the operators entering into the theory commute , is discussed in the ref.[1] : this is the BCS case . More general , non-commutative case occurs in the gapless superconductors[2,3]. Having in mind systems which are not invariant to the time reversal /like a dirty superconductor in high magnetic field , or a superconducting alloy with a small percentage of magnetic impurities / we consider a system described by the Hamiltonian

$$H = H_0 + H_1 ,$$

where H_0 is given by the eq. / 1 / of the ref.[1] , and H_1 is due to the perturbation breaking the time reversal invariance. H_1 anti-commutes with T_a , $[H_1 , T_a]_+ = 0$. Basic dynamical eqs. derived in[1] are still valid ,

$$A_a = [h , t_a]_+ - [\Delta_a , \varrho - \tfrac{1}{2}]_+ = 0$$
$$B = [h , \varrho]_- - [\Delta_a , t_a]_- = 0 , \qquad /1/$$

but the one-particle hamiltonian contains a part h_1 coming from the external perturbation , $h = h_0 + h_1$.

In the case $P_a = T_a$ there are additional relations of commu tation

$$[h_1, P_a]_- \neq 0 \qquad [h_1, P_a]_+ = 0, \quad \text{but} \quad [h_1, t_a]_+ = 0$$

if and only if $[h_1, \varrho]_- = 0$. For each particular case /given H_1/ the choice $P_a = T_a$ would be consistent only if exists a basis in the one-particle space in which ϱ is dia gonal, and T_a canonical.

On the other hand, starting with the eqs./1/ one can derive Hartree-Bogolubov-de Gennes equations[3] for the quasiparticle excitations. With the help of the operator formalism it is easy to see when the perturbation expansion for the excitation energy

$$\mathcal{E}_n = h_n + \sum_{m \neq n} \frac{|\Delta_{mn}|^2}{h_m + h_n} \qquad /2/$$

converges. In the commutative case the eigen basis $|n\rangle$ of h is canonical for Δ_a / and for $P_a = T_a$/, so that only matrix elements between the time -reversed states $\Delta_{n\bar{n}}$ are different from zero.

The eq. /2/ becomes

$$\mathcal{E}_n = h_n + \frac{|\Delta_{n\bar{n}}|^2}{2 h_n}$$

and $\mathcal{E}_n$ diverges for $h_n \to 0$:there is a gap in the spectrum. In the general non-commutative case the basis $|n\rangle$ is not canonical for Δ_a and T_a : $\mathcal{E}_n$ converges and there is a gapless spectrum. One can also try to fix up $P_a = T_a$ before the variational procedure leading to /1/. One gets then a rel. between h, P_a, Δ_a and ϱ which is equivalent to /1/ only if $[\Delta_a, P_a]_- = 0$ and $[h, \varrho]_- = 0$.

1. F.Herbut and M. Vujičić, Phys. Rev. 172, No.4, 1968

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3. P.C. de Gennes, Superconductivity of Metals and Alloys Benj. Inc. /1966/
4. C. Bloch and A. Messijah, Nucl. Phys. 39, 1962.