AN ANALYSIS OF THE PHASE TRANSITIONS FOR THE CLASSICAL COMPRESSIBLE HEISENBERG MODEL WITH THE ANISOTROPIC EXCHANGE INTERACTION

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Abstract: The method of the renormalization group technique (RNG) is used to analyse the effective Hamiltonian of a magnetic system, that is situated on the elastic cubic lattice, with an anisotropic exchange interaction. It is found that such a system exhibits the renormalized (according to the Fisher definition) second order phase transition if the spin anisotropy is relatively small, whereas the phase transition is of the first order kind in the case of a large anisotropy.

A finite compressibility of a lattice may change the nature of the phase transition of a magnetic system situated on it. Here we consider a magnetic system that would have the second order phase transition if the lattice were absosutely rigid. Changes of the nature of the phase transition depend very much on boundary conditions that are imposed upon the system[1].

We have studied the Larkin-Pikin model[2] adapted to include a spin anisotropy interaction in the case of the cubic lattice symmetry. The effect of the lattice compressibility consists in the appearance of a long-range four-spin interaction in the effective spin Hamiltonian. The effective spin Hamiltonian appears as a result of the decoupling of the spin and lattice variables in the original Hamiltonian*. The analysis of the lattice induced

*Details of the decoupling procedure with the corresponding boundary conditions, as well as details of the RNG transformations, will be published elsewhere.

long-range interaction has been accomplished within the RNG approach. Accordingly, the results will be presented in the RNG language.

The effective spin Hamiltonian, expressed in the Fourier transforms of the spin components, is given by

$$\mathcal{H}_{eff} = \frac{1}{2} \sum_i \int_{\vec{q}} (r^i+q^2)\sigma^i_{\vec{q}}\sigma^i_{-\vec{q}} + \sum_{i,j} u_4^{ij} \int_{\vec{q}_1}\int_{\vec{q}_2}\int_{\vec{q}_3} \times$$
$$\times\, \sigma^i_{\vec{q}_1}\sigma^i_{\vec{q}_2}\sigma^j_{\vec{q}_3}\sigma^j_{-\vec{q}_1-\vec{q}_2-\vec{q}_3} + \sum_{i,j} v_4^{ij} \int_{\vec{q}_1}\int_{\vec{q}_2} \sigma_{\vec{q}_1}\sigma_{-\vec{q}_1}\sigma_{\vec{q}_2}\sigma_{-\vec{q}_2} , \tag{1}$$

with the four-spin coupling constants

$$u_4^{ij} = \frac{u_o}{\frac{B}{2}+\mu\frac{d-1}{d}} \quad \frac{c_{11}}{2} - \frac{g_o^2(1+\Delta_i)(1+\Delta_j)}{4\,u_o} ,$$

$$v_4^{ij} = \frac{g_o^2(1+\Delta_i)(1+\Delta_j)}{\Omega(\frac{B}{2}+\mu\frac{d-1}{d})} , \tag{2}$$

where c_{11} is the elastic constant of the lattice, B and μ are the bulk and shear moduli, g_o is the spin-lattice coupling constant, and Δ is the spin anisotropy parameter. The volume and dimensionality of the system are denoted by Ω and d. It is assumed that spins have n components $(i,j = 1,2,\ldots,n)$.

We accept that $\Delta_i = -\Delta$ for the first m components of a spin $(m<n)$, while $\Delta_i = \Delta$ for the other components $(i = m+1, m+2,\ldots,n)$[3]. This assumption leads to the appearance of three isotropic interaction parameters A_u, B_u and C_u, instead of u_4^{ij}, whereas A_v, B_v and C_v appear instead of v_4^{ij}. With these interaction parameters we have found that the stable fixed points, in the case of a small

number of spin components $(n<4)$[4], are of the isotropic n-coupled kind, i.e. $A_u^* = B_u^* = C_u^* = u^*(n)$, $A_v^* = B_v^* = C_v^* = v^*(n)$. They are the Gaussian, Ising, Heisenberg (H) and new-Heisenberg (NH) like fixed points[5], which we have calculated to order ε^2 ($\varepsilon \equiv 4-d$).

Our analysis of the pertinent domains of the fixed points shows that the bear coupling constants, which violate the positivity of the four-spin interaction energy ($A_u>o$, $C_u>0$, $A_uC_u<B_u^2$), appear to be in the Gaussian and Ising fixed points domains, providing $\Delta>\Delta_o$, where $\Delta_o^2 = 4\ u_o(C_{11}/2 - g_o^2/4\ u_o)g_o^{-2}$. In the case $\Delta \geq \Delta_o$ and $B \to 0^+$, these constants are in the Heisenberg fixed point domain. These points correspond to the tricritical points in a space of the thermodynamic variables. However, when $\Delta \approx \Delta_o$ there appears the crossover from the Heisenberg to the new-Heisenberg fixed point controlled by the crossover exponent ϕ_v, for which we have found, to order ε^2, the same critical exponent ratio $\phi_v = \alpha/r$, established previously[5] to order ε.

Furthermore, our calculation of the critical exponents to order ε^2 at the Heisenberg and new-Heisenberg fixed points has vindicated Fisher's expressions[6] for the renormalized exponents. The Fisher renormalization turns out to be fulfilled for the spin anisotropy crossover exponent ϕ_Δ as well, i.e. $\phi_\Delta^{NH} = \phi_\Delta^H/(1-\alpha^H)$. Since ϕ_Δ^{NH} is found to be $\phi_\Delta^{NH}>1$, it means that the NH fixed point corresponts to a higher order critical point. The similar points have been found and analysed previously[7].

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