CROSSOVER AND SCALING AT LOW TEMPERATURES
IN THE GINZBURG-LANDAU MODEL

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Much of the current work on phase transitions is based upon the d-dimensional Ginzburg-Landau (G-L) functional

$$H[\phi(x)] = a\phi^2(x) + b\phi^4(x) + c[\nabla\phi(x)]^2 , \qquad (1)$$

where $\phi(x)$ is the classical n-component order parameter. For $d > 2$ the G-L model has been treated by the renormalization group [1], which justified the concept of scaling and universality. For $d < 2$ the solutions exist on three isolated lines $d = 0$ [2], $d = 1$ [2,3,4], and $n = \infty$ [5].

Treating here d and n continuosly we present a simple argument [6] which extends the scaling concept to the interior of the region $T_c=0$ (see fig.1) and gives the corresponding critical behaviour. It applies neither to the borderline given by $d = 2$ [7] and $d = n$ [8], nor to $d = 0$. Due to the fact that the ultraviolet divergencies do not exist for $d < 2$ the cutoff associated with (1) can be dismissed. Then the simple scale transformation suffices to derive the homogeneity relations for the thermodynamic functions. These relations describe the critical behaviour around $T_c = 0$.

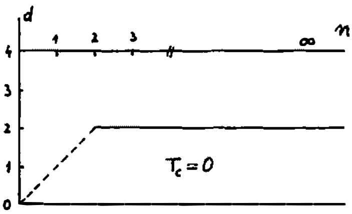

fig. 1

Let as sketch the procedure on the example of the correlation function. We write Γ as a function of parameters appearing in eq. (1)

$$\Gamma(x) = \frac{1}{Z}\int \mathcal{D}\phi(x)\, \exp\left\{\int H[\phi(x)]\, d^d x\right\} = \Gamma\left(x, \frac{a}{T}, \frac{b}{T}, \frac{c}{T}\right) . \qquad (2)$$

Then the scale change $x' = \frac{x}{s}$ and the field redefinition $\phi(x) = \sqrt{\alpha}\,\psi(x')$ give the homogeneity relation

$$\Gamma\left(x,\frac{a}{T},\frac{b}{T},\frac{c}{T}\right) = \alpha\,\Gamma\left(\frac{x}{s},\alpha s^{d}\frac{a}{T},\alpha^{2}s^{d}\frac{b}{T},\alpha s^{d-2}\frac{c}{T}\right)$$

$$\forall \alpha, s \,. \qquad (3)$$

From (3) we derive

$$\Gamma = -\frac{a}{b}\,\Gamma\left[\frac{x}{\xi_0}\left(\frac{T}{T_b}\right)^{\frac{1}{2-d}}, -\left(\frac{T_b}{T}\right)^{\frac{2}{2-d}}, \left(\frac{T_b}{T}\right)^{\frac{2}{2-d}}, 1\right] \qquad (4)$$

where $\xi_0 = \sqrt{-\frac{c}{a}}$ and $T_b = \xi_0^{d}\,\frac{a^2}{b}$ appear as the caracteristic parameters of the system. Integrating separately over phase and amplitude, we can apply the saddle point method in this latter. Therefore the amplitude fluctuations turn out to be unimportant (6,9) for the critical behaviour. This reduces the correlation function to the homogeneous form

$$\Gamma = \frac{-a}{2b}\, f\left[\frac{x}{\xi_0}\left(\frac{T_b}{T}\right)^{\frac{1}{2-d}}\right] \qquad (5)$$

Therefrom we obtain the critical exponents $\nu = \frac{1}{2-d}$, $\eta = 1/\nu$. The same procedure applied to the free energy density gives the remaining critical exponents and scaling relations

$$\gamma = \alpha_G + 2\Delta - 2 \qquad \alpha = \alpha_G - 1 = \frac{-d}{2-d}$$

$$2 - \alpha_G = 2\nu \qquad \Delta = \frac{2}{2-d} \qquad (6)$$

$$\gamma = \frac{2}{2-d}$$

They do not depend on n and agree with d = 1 (2,3,4) and $n = \infty$ $d < 2$ (5) results.

The described procedure can be also applied to the lattice anisotropy crossover $d \to d'$ for the cases where the lower dimension coresponds to the $T_c = 0$ phase transition. Following the usual procedure (10) we find the crossover and critical temperature shift exponents

$$\phi = \psi = \gamma(d') = \frac{2}{2-d} \quad . \tag{7}$$

References

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