ON THE EXISTENCE OF LOCAL LEVELS IN A HEISENBERG FERROMAGNET

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In his fundamental paper (1) Dyson was able to predict the existence of local levels in the Heisenberg ferromagnet arising from the kinematical interaction of spin waves. He claimed however that the interaction of spin waves at one specific lattice point can be relevant only in the short wave region and at high temperatures so that for the low temperature analysis, he proposed his boson representation of spin operators disregarding the above-mentioned kinematical effects.

Consequently one could expect that as a result of such an interaction, elementary excitations in the short wave region can arise giving exponentially small contributions to the low temperature magnetization.

The object of this paper is to analyse the adequacy of the mentioned picture of spin wave interaction, to prove the presence of local levels predicted by Dyson and to develop a procedure for the decoupling of the higher order Green's function which is more correct than the existing, usual ones.

The proposed procedure takes into account couplings among operators belonging not only to the same moment of time but also among operators belonging to different moments. Existing usual treatments in the quantum theory of magnetism that take into account only couplings of operators belonging to the same moments of time

are able to give the same results as the Dyson method of ideal spin waves but not other, additional levels.

We shall fix our attention to the Heisenberg ferromagnet with $S = 1/2$, so that the spin operators can be replaced by Pauli operators.

We are interested in the Green's function $\langle\langle P_{\vec{k}}(t) | P^{+}_{\vec{k}}(0) \rangle\rangle$ where $P_{\vec{k}}(t)$ are Fourier components of the Pauli operators. The Pauli operators shall be expressed by means of Bose operators (reference (2)) and an improoved decoupling of the Bose Green's functions will be applied

$$\langle\langle B_{\vec{k}}(t) | B^{+}_{\vec{k}-\vec{q}_1+\vec{q}_2}(0) B^{+}_{\vec{q}_1}(0) B_{\vec{q}_2}(0) \rangle\rangle = N^{(0)}_{\vec{q}_2} G_{\vec{k}}(t) \left(\delta_{\vec{q}_1,\vec{q}_2} + \delta_{\vec{q}_1,\vec{k}} \right) =$$

$$= \langle\langle B^{+}_{\vec{q}_2}(t) B_{\vec{q}_1}(t) B_{\vec{k}-\vec{q}_1+\vec{q}_2}(t) | B^{+}_{\vec{k}}(0) \rangle\rangle$$

$$\langle\langle B^{+}_{\vec{q}_2}(t) B_{\vec{q}_1}(t) B_{\vec{k}-\vec{q}_1+\vec{q}_2}(t) | B^{+}_{\vec{k}-\vec{q}_3+\vec{q}_4}(0) B^{+}_{\vec{q}_3}(0) B_{\vec{q}_4}(0) \rangle\rangle =$$

$$= D_{\vec{q}_2}(t) G_{\vec{q}_1}(t) \left\{ G_{\vec{q}_3}(t) \delta_{\vec{q}_3,\vec{k}-\vec{q}_1+\vec{q}_2} + G_{\vec{k}-\vec{q}_1+\vec{q}_2}(t) \delta_{\vec{q}_1,\vec{q}_3} \right\} \delta_{\vec{q}_2,\vec{q}_4} + O(C_0^2)$$

So we find the following three poles of the Green's function $\langle\langle B_{\vec{k}} | B^{+}_{\vec{k}} \rangle\rangle_E$:

$$\mathcal{E}_1 = \frac{1}{2}(J_0 - J_{\vec{k}}) + \frac{1}{N}\sum_{\vec{q}} (J_{\vec{k}} + J_{\vec{q}} - J_0 - J_{\vec{k}-\vec{q}}) N^{(0)}_{\vec{q}} \qquad \text{(a)}$$

$$\mathcal{E}_{2,3} = \frac{\hbar^2}{2m} \left(\frac{\mu_0^2}{6} \pm \sqrt{k^4 + \frac{11}{9} k^2 \mu_0^2 - \frac{31}{36} \mu_0^4} \right) \qquad \text{(b)}$$

where the intensity of the boundary wave vector of the first Brillouin zone is of the order $\mu_0 \approx 10^8\,\mathrm{cm}^{-1}$ and the lattice constant $a = (6\pi$

The energy $\mathcal{E}_1$ gives the well known low temperatures expansion for the magnetization and the result (b) is in full agreement with Dyson's prediction.The kinematical interaction of spin waves (this interaction is included in the representation (reference (2)),gives the additional energy levels in the system only in the short wave region (for $k \gtrsim 2^{-1/2}\mu_0$ i.e. of the order$(0.5 \div 1)10^8\,\mathrm{cm}^{-1}$and higher.These excitations have the gap $\Delta = \hbar^2\mu_0^2/12m \approx 0.1\,eV$ so that their contributions to the thermodynamical characteristics of the system are negligible at low temperatures, just as we expected by Dyson.

It has to be noticed,however,that the result (b) represents an expression which can be used to make only approximate estimations concerning the additional levels as in the derivation of this result the effective mass approximation was used among other usual simplifications.A more exact evaluation of energy levels that does not make use of the effective mass approximation,would lead to cumbersome numerical calculations.

References

(1) F.J.Dyson, Phys.Rev. 1o2, 1217 and 123o (1956).

(2) V.M.Agranovič and B.S.Tošić, Zh.eksper.teor.Fiz. 53, 149 (1967).