DIRECT-REACTION MECHANISMS IN THE $\alpha(p,d)^3$He REACTION

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1. Introduction

The resonating-group method (RGM) has been successfully used to explain the behavior of the five-nucleon system[1]. By incorporating into the calculation both the p + α channel and the d + ^{3}He channel, it was found that the calculated differential elastic and reaction cross sections were in good agreement with experimental results.

Since resonating-group calculations employ totally antisymmetric wave functions, all exchange effects are explicitly included. Thus it is possible[2] to use these calculations for a study of direct-reaction mechanisms. Previously, we have made such an investigation in the $\alpha(p,d)^3$He case by setting various parts of the coupling kernel equal to zero [3]. Here we shall make a further study of this reaction using the plane-wave Born approximation (PWBA), in the hope of obtaining a more transparent picture of the roles played by the various direct-reaction processes.

2. Brief description of the method of analysis

The formulation of the two-channel problem is given in ref. 1 and, hence, will not be further described here. It suffices to say that the transition from the p + α channel (channel g) to the d + ^{3}He channel (channel f) is effected by the coupling kernel K_{fg} (hereafter simply written as K) which has the form

$$K = K^a + K^b \tag{1}$$

with

$$K^a(\vec{R}_f', \vec{R}_g'') = (N_f N_g)^{-\frac{1}{2}} \langle \phi_\tau(123)\phi_d(45)\xi_f \delta(\vec{R}_f - \vec{R}_f') Z(\vec{R}_{cm}) \left| H - E_t \right| (1-P_{45}) \mathcal{A}_\alpha [\phi_\alpha(1234)\xi_g \delta(\vec{R}_g - \vec{R}_g'')] Z(\vec{R}_{cm}) \rangle \tag{2}$$

and

$$K^b(\vec{R}_f', \vec{R}_g'') = (N_f N_g)^{-\frac{1}{2}} \langle \phi_\tau(123)\, \phi_d(45)\, \xi_f\, \delta(\vec{R}_f - \vec{R}_f') Z(\vec{R}_{cm}) \left| H - E_t \right| (-P_{15} - P_{25} - P_{35}) \mathcal{A}_\alpha [\phi_\alpha(1234)\, \xi_g \delta(\vec{R}_g - \vec{R}_g'')] Z(\vec{R}_{cm}) \rangle \; , \tag{3}$$

where the notation is, except for minor modifications, the same as that of ref. 3. From eqs. (2) and (3), one can clearly see that K^a corresponds to a process in which the incident proton is contained in the outgoing deuteron, while K^b corresponds to a process in which the incident proton is contained in the outgoing ^{3}He. That is, K^a describes a one-nucleon pickup process, while K^b describes a two-nucleon pickup process.

To make the interpretation as clear as possible, we simplify the formulation of ref. 1 in the following way: (i) a single-Gaussian deuteron function ϕ_d yielding the same rms radius as the three-Gaussian function of ref. 1 is used, (ii) a simplified nucleon-nucleon potential with a Serber mixture is employed; this potential is described in a previous publication[4], and (iii) the Coulomb interaction is neglected.

By summing over spin and isospin coordinates, the kernel functions K^a and K^b become

$$K^a = K_T^a + K_E^a + K_{12}^a + K_{14}^a + K_{15}^a + K_{45}^a \tag{4}$$

and

$$K^b = K_T^b + K_E^b + K_{12}^b + K_{14}^b + K_{23}^b + K_{24}^b + K_{45}^b \ . \tag{5}$$

In these equations, the functions K_T^a, K_T^b, K_E^a, and K_E^b arise from the kinetic-energy operator T and the total energy E_t. The remaining functions come from the potential-energy operator; they have the forms

$$K_{ij}^a = C_{ij}^a \int [\psi_f(123;45)]^* \, v_{ij}\psi_g \,(1234;5)\, d\tau \tag{6}$$

and

$$K_{ij}^b = C_{ij}^b \int [\psi_f(123;45)]^* \, v_{ij} \, \psi_g \,(5234;1) d\tau \ , \tag{7}$$

where we have introduced the notation

$$\psi_f \,(123;45) = \phi_\tau(123)\ \phi_d(45)\ \delta(\vec{R}_f - \vec{R}_f')Z(\vec{R}_{cm}) \ , \tag{8}$$

$$\psi_g \,(1234;5) = \phi_\alpha(1234)\delta(\vec{R}_g - \vec{R}_g'')Z(\vec{R}_{cm}) \ , \tag{9}$$

and

$$\psi_g \,(5234;1) = P_{15}^r \ \psi_g(1234;5) \ . \tag{10}$$

Also, in eqs. (6) and (7), C_{ij}^a and C_{ij}^b are constant factors unimportant for our present consideration; v_{ij} is the form factor of the nucleon-nucleon potential given by

$$v_{ij} = \exp\,(-\kappa r_{ij}^2) \ , \tag{11}$$

and $d\tau$ signifies the integration over all spatial variables.

Next, we compute the PWBA reaction amplitude for each of the terms in K^a and K^b. For example, the Born amplitude corresponding to K_{ij}^a is

$$B_{ij}^a = -\frac{\mu_f}{2\pi\hbar^2} \int \exp(-i\vec{k}_f\cdot\vec{R}_f')K_{ij}^a \,(\vec{R}_f',\vec{R}_g'')\ \exp(i\vec{k}_g\cdot\vec{R}_g'')d\vec{R}_f' d\vec{R}_g'' \ , \tag{12}$$

where $\vec{k}_g$ and $\vec{k}_f$ are propagation vectors in channels g and f, respectively. By carrying out the spatial integrations in eqs. (6) and (7), analytic expressions for the various amplitudes can be obtained. The differential reaction cross section is then given by

$$\frac{d\sigma}{d\Omega} = \frac{v_f}{v_g} \ |B^a + B^b\,|^2 \ , \tag{13}$$

where v_g and v_f are, respectively, the relative velocities of the clusters at large separation in the incident and reaction channels.

3. Results

The validity of the PWBA at relatively high energies can be tested by making a comparison between the differential reaction cross section computed by solving the RGM coupled integrodifferential equations and that computed by using the PWBA. This comparison is shown in fig. 1 at an energy of E_g = 130 MeV (the corresponding value of E_f is 104.88 MeV). From this figure it is seen that, even though the over-all agreement is only fair, the Born result does reproduce the essential features, i.e., the occurrence of large peaks at forward and backward directions and a sharp minimum at around 100°. Thus, we feel that a study using the PWBA should be useful in yielding a qualitative understanding of the importance of various direct-reaction processes.

From the explicit expressions for the Born amplitudes, one finds that all terms contained in B^a (i.e., B^a_T, B^a_E, and B^a_{ij}) are peaked in the forward direction, while all terms contained in B^b (i. e., B^b_T, B^b_E, and B^b_{ij}) are peaked in the backward direction. This is shown in fig. 2, where the solid curves represent the amplitudes B^a and B^b at E_g = 130 MeV. From this figure one sees that, at such a relatively high energy, the cross-section behavior in the forward angular region can be accounted for by a one-nucleon pickup process, while the cross-section behavior in the backward angular region can be accounted for by a two-nucleon pickup process. In fact, it is evident that there is effectively no interference between these two processes. We should point out, however, that this lack of interference is strictly a high-energy phenomenon; already at a lower energy of E_g = 70 MeV, our calculation shows that interference effects become fairly important in the intermediate angular region.

A detailed examination shows that, even though many individual terms in B^a and B^b have appreciable magnitudes, the following approximate relations hold at relatively high energies:

$$B^a_{45} \approx B^a , \quad B^b_{12} \approx B^b . \tag{14}$$

This is shown in fig. 2, where the dashed curves represent the amplitudes B^a_{45} and B^b_{12} at E_g = 130 MeV. At lower energies of E_g = 100 and 70 MeV, these relations are still reasonably valid; for example, at $\theta = 0^o$ the ratios B^a_{45}/B^a are equal to 0.83 and 0.80 at these two energies, while at $\theta = 180^o$ the ratios B^b_{12}/B^b are equal to 1.09 and 1.12. Thus, referring to eqs. (6) and (7), one sees that, for an approximate description of the two pickup processes, it is only necessary to consider the interaction between the incident nucleon and the nucleons which are removed from the target (note that the spatial integrals involved in K^b_{12} and K^b_{13} yield the same result). This seems to be intuitively reasonable, but does require

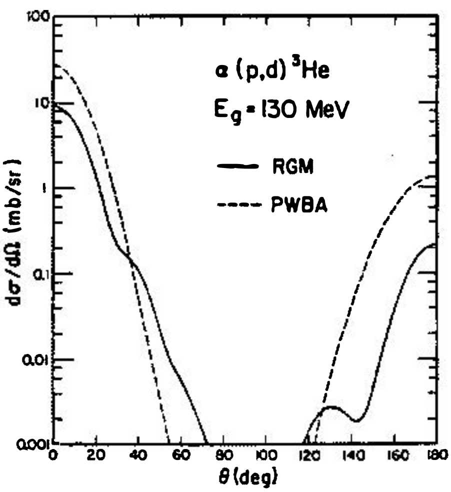

Fig. 1: Comparison of RGM and PWBA results.

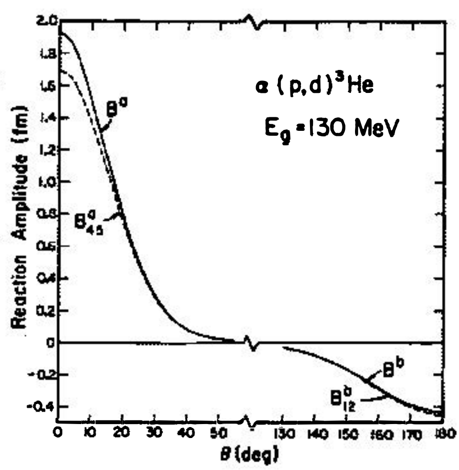

Fig. 2: Born amplitudes at E_g = 130 MeV.

to be further examined. At present, we are carrying out such an examination, with the major aim being to investigate the possibility of cancellations among the various Born amplitudes due to the presence of redundant solutions in our resonating-group formulation.

4. Conclusion

In this investigation, we have made a PWBA study of the $\alpha(p,d)^3He$ reaction. The result shows that, at relatively high energies, the cross-section behavior at forward angles can be explained as arising from a one-nucleon pickup process, while that at backward angles can be explained as arising from a two-nucleon pickup process.

In addition, it is found that, for an approximate description of these pickup processes, one needs only to consider the interaction between the incident nucleon and the nucleons removed from the target. This is an important finding, because if it should turn out to be generally true, then antisymmetrized direct-reaction calculations can be simplified to a considerable extent.

References

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