A Variational Method in RGM for Scattering between Complex Nuclei Based on the Use of GCM kernel

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A new treatment of the Kohn-Hulthên-Kato type variational method is Proposed in RGM by improving the method of Mito and Kamimura[1]. The present method is an extension of the work of Kawai, Kamimura, Mito and Takesako[2] to the microscopic case. This improvement enables us to adopt a variety of forms of the trial function based on the use of GCM kernel.

The RGM equation to be solved is

$$\langle \phi_A \phi_B \mid H - E \mid \mathcal{A}[\phi_A \phi_B \, u_L(R)/R \cdot Y_{LM}(\hat{R})] \rangle = 0 \qquad (1)$$

which leads to the integro-differential equation

$$\mathcal{L}_L \cdot u_L = 0 \; ; \quad \mathcal{L}_L(R,R') \equiv [T_L(R) + V^D(R) - E_{cm}]\,\delta(R,R') + \int_0^\infty dR' \, W_L(R,R') \qquad (2)$$

Notation in (1) and (2) is usual one. According to Ref. 2, the stationary expression (functional) of the S-matrix, S_{st}, is given by

$$S_{st} = \sum_{i=0}^{N} c_i s_i + \frac{i\mu}{\hbar^2 k}(u_t \, \mathcal{L}_L \, u_t),$$

$$(f \, \mathcal{L}_L \, g) \equiv \int_0^\infty \int_0^\infty f(R) \, \mathcal{L}_L(R,R') \, g(R') \, dR \, dR'.$$

Here $u_t(R)$ is a trial function of $u_L(R)$ and is described as

$$u_t(R) = \sum_{i=0}^{N} c_i u_i(R) \quad , \quad u_i(R) = \begin{cases} \alpha_i \, u_i^{(in)}(R) & (R < R_0) \\ u_L^{(-)}(k,R) - s_i \, u_L^{(+)}(k,R) & (R \geq R_0) \end{cases}$$

where $u_L^{(-)}$ and $u_L^{(+)}$ are incoming and outgoing wave functions that satisfy

$$[\, T_L + V^D(R) - E_{cm} \,] \, u_L^{(\pm)}(k,R) = 0 \, . \quad (R \geq R_0)$$

We have assumed that $W_L(R,R')=0$ for $R \geq R_0$ and for $R' \geq R_0$; this is the definition of R_0. The complex coefficients α_i and s_i are determined by the matching of $u_i(R)$ at R_0. Imposing the normarization condition

$$\sum_{i=0}^{N} c_i = 1 \qquad \left(c_0 = 1 - \sum_{i=1}^{N} c_i \right).$$

we confine c_0 (any one). From this condition we have

$$u_t = u_0 + \sum_{i=1}^{N} c_i \, (u_i - u_0)$$

$$\left(u_t = u_L^{(-)} - S_t\, u_L^{(+)} \quad \text{for } R \geq R_0 \text{ with } S_t = \sum_{i=0}^{N} c_i s_i \right).$$

It is to be noted that S_t itself is not a variational parameter.

The stationary condition $\partial S_{st}/\partial c_i = 0$ $(i=1\cdots N)$, gives

$$([u_i - u_0]\ \mathcal{L}_L\ u_t) = 0 \qquad (i = 1 \cdots N)\ ,$$

which leads to N linear equations for c_i's $(i=1\cdots N)$:

$$\sum_{j=1}^{N} \mathcal{L}_{ij}\, c_j = m_i\ , \qquad (i = 1 \cdots N)$$

$$\mathcal{L}_{ij} = K_{ij} - K_{i0} - K_{0j} + K_{00},\quad m_i = K_{00} - K_{i0}\ ,$$

$$K_{ij} \equiv (u_i\, \mathcal{L}\, u_j) = \alpha_i \alpha_j \left\{ (u_i^{(in)} \mathcal{L}_L\, u_j^{(in)}) - \int_{R_0}^{\infty} u_i^{(in)}(R) [T_L + V^D - E_{cm}]\, u_j^{(in)}(R)\, dR \right\}$$

As the trial form of $u_i^{(in)}(R)$, the followings are suited :

$$(i)\quad u_i^{(in)}(R)/R = 4\pi e^{-\lambda(R^2+S_i^2)} \mathcal{J}_L(2\lambda S_i R) = \int Y_{LM}^{*}(\hat{R})\, e^{-\lambda(\mathbf{R}-\mathbf{S})^2} \delta(S-S_i)/S^2 \times Y_{LM}(\hat{\mathbf{S}})\, d\mathbf{S}\, dR$$

$$(ii)\quad u_i^{(in)}(R)/R = R^L L_{N_i}^{(L+\frac{1}{2})}(2\lambda R^2) e^{-\lambda R^2} = \text{const}\times \int Y_{LM}^{*}(\hat{R})\, e^{-\lambda(\mathbf{R}-\mathbf{S})^2} \Phi_{N_i LM}(\lambda\lambda\mathbf{S})\, d\mathbf{S}\, dR$$

$$(iii)\quad u_i^{(in)}(R)/R = R^L e^{-\nu_i R^2} = \text{const}\times \int Y_{LM}^{*}(\hat{R})\, e^{-\lambda(\mathbf{R}-\mathbf{S})^2} \Phi_{0LM}(\lambda\nu_i \mathbf{S})\, d\mathbf{S}\, dR.$$

Here $\lambda = N_A N_B \beta / 2(N_A+N_B)$, $\beta = m\omega/\hbar$ the single-particle H.O. size parameter; N_A and N_B are the mass numbers of the colliding nuclei A and B. The harmonic oscillator GC functions $\Phi_{NLM}(\lambda\nu\mathbf{S})$ are introduced in Ref. 3. The matrix elements $(u_i^{(in)} \mathcal{L}_L u_j^{(in)})$ are then given, in terms of GCM kernels, by

$$(u_i^{(in)} \mathcal{L}_L\, u_j^{(in)}) = \text{const}\times \int \phi_i(\mathbf{S}) [H^{(GCM)}(\mathbf{S},\mathbf{S}') - E\cdot N^{(GCM)}(\mathbf{S},\mathbf{S}')]\, \phi_j(\mathbf{S}')\, d\mathbf{S}\, d\mathbf{S}' \qquad (3)$$

where $\phi_i(\mathbf{S})$ represents $\delta(S-S_i)/S^2 \cdot Y_{LM}(\hat{\mathbf{S}})$, $\Phi_{N\,LM}(\lambda\lambda\mathbf{S})$ or $\Phi_{0LM}(\lambda\nu_i\mathbf{S})$. Even in the case of (ii) or (iii), the integral (3) is given in a simple form[3]. The additional integrals appearing in K_{ij} is easy to calculate.

Approximate S-matrix is given by S_{st} and S_t. The stationary value S_{st} is more accurate than the stationary point S_t; the error of S_{st} (S_t) is the second (first) order with respect to the error contained in the trial function. The check of the quantities $|S_{st}|-1$, $|S_t|-1$ and $|S_{st}-S_t|$ is a very good tool to examine how accurate the calculation is; this is an advantage of the Kohn-Hulthen-Kato type variational method (it is to be stressed that in complicated calculations such as RGM and GCM, how to examine the accuracy of the calculation is of great importance).

The present method is applied to the α-α scattering. Prameters are all the same as in Ref. 1; we adopt Volkov No. 1 force with m=0.56 and $\beta=m\omega/\hbar=0.535\ \mathrm{fm}^{-2}$. Here use is made of the type (i) for the trial form of $u_i^{(in)}(R)$. We set the GC mesh points S_i at a common interval ΔS. The matching radius R_0 is taken as $R_0=S_N$ (the outermost of S_i's) outside which we can regard $V^D(R)=Z_A Z_B e^2/R$. Let us denote the discretization points S_i's simply in the form $S_i[N,\ \Delta S;\ S_1 \sim S_N]$.

Table I shows calculated result of δ_{st}, δ_t, $|S_{st}|$ and $|S_t|$ for L=0; δ is defined by $S=|S|e^{2i\delta}$. The smallness of the quantities $|S_{st}|-1$, $|S_t|-1$ and $\delta_{st}-\delta_t$ assures high accuracy of the calculation; the present result agrees very well with another independent RGM result (the third-line phase shift)[3),1)]. Even the case of only six discretization points S_i gives quite satisfactory accuracy up to high energies. It was found that the present method works better than the method of Ref. 1.

Extension of the present method to the coupled-channel 3-cluster RGM and OCM is under investigation with the work of Refs. 2, 4 and 5 combined.

References;

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Table I

Illustration of accuracy of the present method for the α–α scattering. For each E_{cm}, the first line desplays δ_{st} and $|S_{st}|$, while the second line (in parenthesis) δ_t and $|S_t|$; the third line (in N=6) exhibits the phase shift obtained by another RGM. The phase shifts are given in degrees between 0° and 180°. Use is made of three kinds of the trial functions with the GC discretization points S_i[N=6, 0.55 fm; 2.5 fm∿5.25 fm], S_i[N=8, 0.5 fm; 2.5 fm∿6.0 fm] and S_i[N=10, 0.45 fm; 2.0 fm∿6.05fm].

E_{cm} (MeV)	N=6	(L=0)	N=8	(L=0)	N=10	(L=0)
	150.21	1.0000	150.30	1.0000	150.30	1.0000
1.0	(150.22	1.0000)	(150.33	0.9999)	(150.31	1.0000)
	150.14					
	63.19	1.0001	63.57	1.0000	63.60	1.0000
4.0	(63.47	1.0017)	(63.68	1.0045)	(63.63	1.0012)
	63.77					
	169.44	1.0001	170.99	1.0000	171.03	1.0000
10.0	(169.59	1.0121)	(170.99	0.9994)	(171.03	0.9996)
	171.15					
	111.14	1.0004	111.50	1.0000	111.50	1.0000
20.0	(111.42	0.9652)	(111.60	1.0001)	(111.51	1.0001)
	111.44					
	47.08	1.0001	47.79	1.0000	47.82	1.0000
40.0	(46.87	0.9829)	(47.87	0.9957)	(47.86	0.9978)
	47.71					
	9.39	1.0015	9.70	1.0003	9.84	1.0000
60.0	(8.48	1.0462)	(9.58	0.9680)	(9.85	1.0054)
	9.70					