An Orthogonality Condition Model for 3-Cluster Systems Based on the Direct Use of Norm Kernel. Application to the 3α System

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An orthogonality condition model (OCM) for 2-cluster systems based on the direct use of norm kernel N

$$\sqrt{N}\,(T_{rel} + V^{(eff)}(R) - E_{rel})\sqrt{N}\,\chi = 0 \qquad (1)$$

was proposed by Saito et al.[1] and was applied successfully to the α-^{16}O system[2] and to the α-^{12}C system[3]. Here $\sqrt{N}$ eliminates the redundant components out of χ. $V^{(eff)}(R)$ is an effective potential between the clusters; the direct potential $V^D(R)$ is often used. Equation (1) is regarded to approximate the RGM equation $\langle \phi_1 \phi_2 | H - E | \mathcal{A}[\phi_1 \phi_2 \chi] \rangle = 0$.

For 3-cluster systems, as an approximation of RGM equation $\langle \phi_1 \phi_2 \phi_3 | H - E | \mathcal{A}[\phi_1 \phi_2 \phi_3 \chi] \rangle = 0$, Horiuchi[4] proposed an OCM

$$\Lambda\Big(\sum_{i=1}^{3} T_i - T_G + \frac{1}{2}\sum_{i\neq j}^{3} V_{ij}^{(eff)} - E_{rel}\Big)\Lambda\,\chi = 0, \qquad (2)$$

where $V_{ij}^{(eff)}$ (in Ref. 4, $V_{ij}^{(eff)} = V_{ij}^{D}$) is the effective potential between clusters i and j, and Λ is the projection operator on to the allowed states; the OCM Hamiltonian is diagonalized in a certain truncated space of the allowed H.O. basis functions.

Here we propose an extension of (1) to 3-cluster system by giving an OCM equation

$$\sqrt{N}\Big(\sum_{i=1}^{3} T_i - T_G + \frac{1}{2}\sum_{i\neq j}^{3} V_{ij}^{(eff)} - E_{rel}\Big)\sqrt{N}\,\chi = 0, \qquad (3)$$

where χ is to be normalized as $\langle \chi N \chi \rangle = 1$ for bound or quasi-bound states. N denotes the norm kernel of the 3-cluster RGM and $\sqrt{N}$ eliminates the forbidden states out of χ; it is not necessary to describe χ in terms of H.O. functions and to make χ be symmetric (antisymmetric) with respect to the same clusters (Nχ is symmetric (antisymmetric)). This type of 3-cluster OCM (3) is formally equivalent to (2) (except for the normalization) but will be suited to treat collisions and well-developed clustering states.

In practical calculations, we introduced ψ by $\chi = \sqrt{N}\psi$ and solve ψ. It is very convenient to transform (3) to the corresponding GCM framework by using the treatment similar to that of 3-cluster RGM of Ref. 5. In the present calculation of the 3α system, the same type of basis functions for expanding ψ is adopted as used in Ref. 5. Some of the OCM result of Ref. 4 were reproduced very closely by the present OCM with using 12 basis functions for each L^π; energies of $0_1^+,2_1^+,4_1^+,1_1^-,3_1^-,0_2^+$ and 2_2^+ are reproduced in the case of Coulomb interaction included and $V_{ij}^{(eff)}(R)=V_{ij}^D(R)$.

Fig. 2 shows energy (measured from the calculated 3α break-up threshold) of the lowest 0^+ state obtained by the present OCM and by the RGM of Ref. 5; $\beta=m\omega/\hbar=0.55$ fm^{-2} for all the cases. One sees that the role of the dynamical particle-exchange between clusters which is not taken in this OCM with the direct potential V_{ij}^D causes attractive effect considerably. The strong exchange effect by Schmidt-Wildermuth force in RGM makes 3α collapse; this is due to the fact that the force does not make α particle saturate properly. In Fig. 3, we give a comparison between the OCM(with V_{ij}^D) and RGM[5] in energies of several levels. State-dependence of the dynamical particle-exchange effect is observed. Further investigations of theis effect will supply the OCM study with proper effective potentials $V_{ij}^{(eff)}$ instead of V_{ij}^D.

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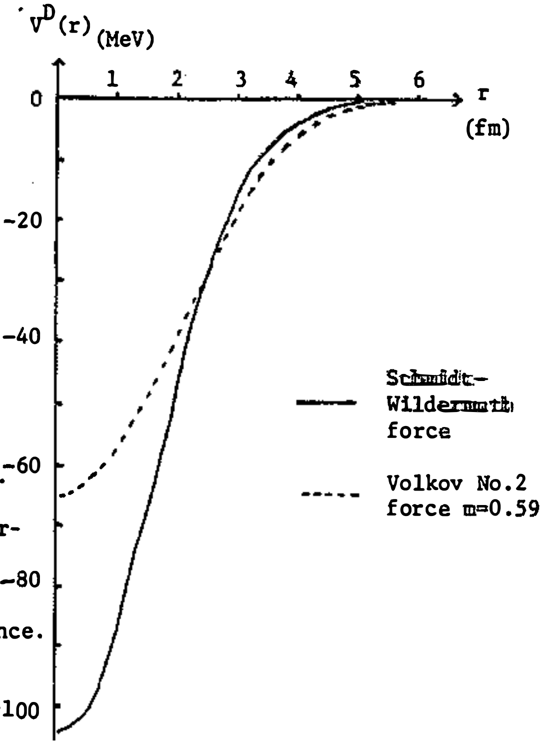

Fig. 1. Nuclear part of Direct potential (V_{ij}^D) between α and α. Since the energy spectrum of the OCM with Schmidt-Wildermuth force (SW) is like that of the RGM with Volkov No. 2 force (V2), the diference between V^D(V2) and V_D(SW), roughly speaking, could correspond to the difference between V^D(V2) and V^{eff}(V2) to be derived from RGM with V2.

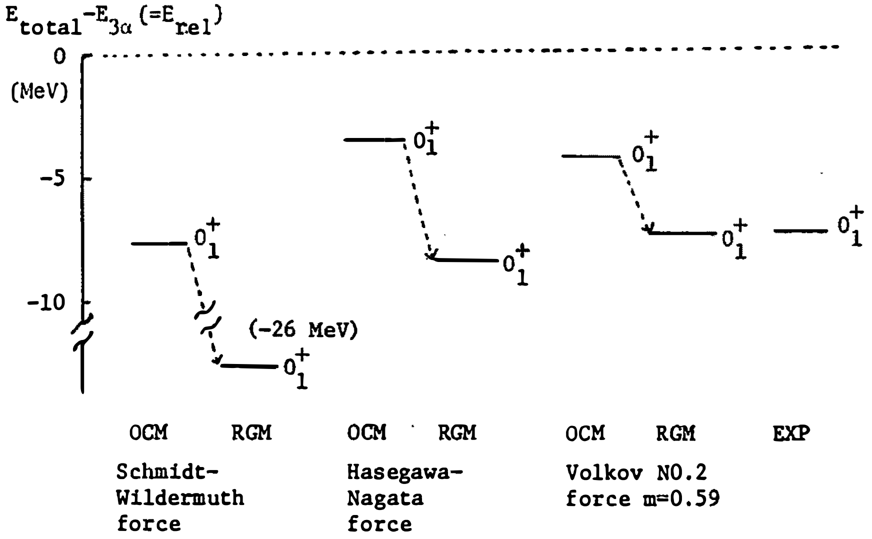

Fig. 2. The Lowest Level of 3α System.

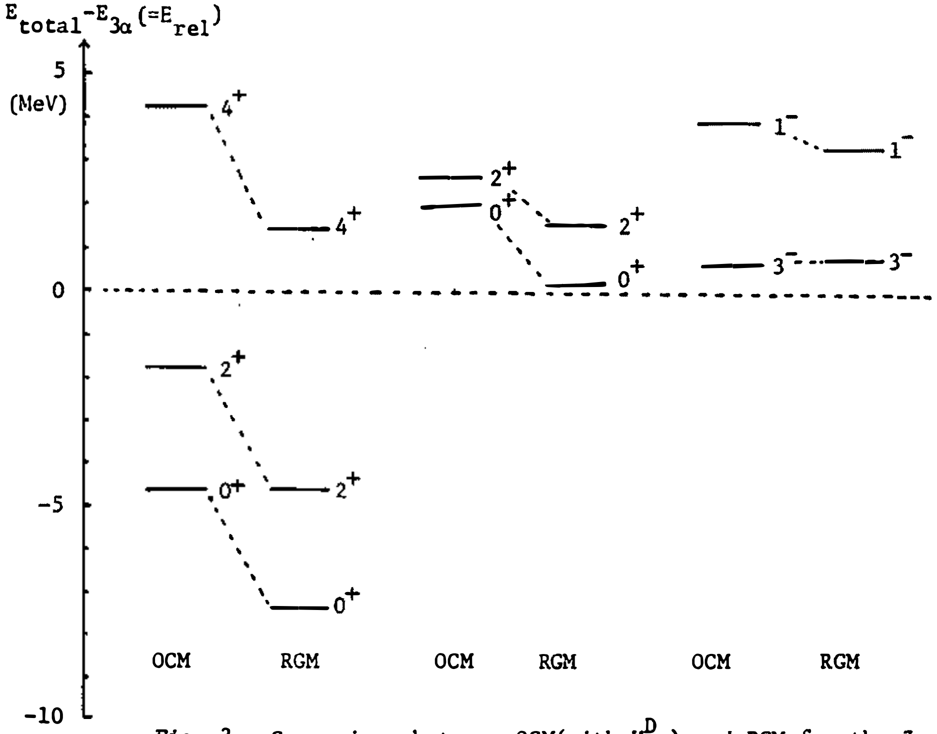

Fig. 3. Comparison between OCM(with V_{ij}^{D}) and RGM for the 3α in the energy spectrum of ^{12}C (Volkov No.2; m=0.59; β=0.55 fm^{-2}).

Resonating Group Method for the 3α System and the Structure of ^{12}C

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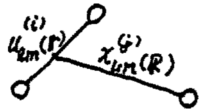
The RGM wave functions of the 3α system is assumed to be

$$\Psi_{JM}=\mathcal{A}[\phi_1\phi_2\phi_3\, U_{JM}],\quad U_{JM}(\boldsymbol{r},\boldsymbol{R})=\sum_{ij\ell L} C^{(ij)}_{\ell LJ}\,[u^{(i)}_{\ell}(\boldsymbol{r})\otimes\chi^{(j)}_{L}(\boldsymbol{R})]_{JM}$$

We adopt the following RGM basis functions with their GCM representation: It is very convenient to assume the form $u^{(i)}_{\ell m}(\boldsymbol{r})$ as



$$u^{(i)}_{\ell m}(\boldsymbol{r})=r^{\ell}e^{-\nu_i r^2}Y_{\ell m}(\hat{\boldsymbol{r}})=\mathrm{const}\times\int e^{-\lambda(\boldsymbol{r}-\boldsymbol{t})^2}\Phi_{0\ell m}(\lambda\nu_i\boldsymbol{t})\,d\boldsymbol{t} \qquad (1)$$

with $\lambda=\beta$, $\beta=m\omega/\hbar$ being the single-particle oscillator size parameter. The GC harmonic oscillator functions $\Phi_{NLM}(\lambda\nu\boldsymbol{t})$ are introduced in Ref. 1. As for $\chi^{(j)}_{LM}(\boldsymbol{R})$, the following three types are suited for practical calculations:

$$\text{(i)}\ \chi^{(j)}_{LM}(\boldsymbol{R})=R^{L}e^{-\mu_j R^2}Y_{LM}(\hat{\boldsymbol{R}})=\mathrm{const}\times\int e^{-\lambda'(\boldsymbol{R}-\boldsymbol{S})^2}\Phi_{0LM}(\lambda'\mu_j\boldsymbol{S})\,d\boldsymbol{S}.$$

$$\text{(ii)}\ \chi^{(j)}_{LM}(\boldsymbol{R})=R^{L}e^{-\lambda'R^2}L^{(L+1/2)}_{N_j}(2\lambda'R^2)Y_{LM}(\hat{\boldsymbol{R}})=\mathrm{const}\times\int e^{-\lambda'(\boldsymbol{R}-\boldsymbol{S})^2}\Phi_{N_jLM}(\lambda'\lambda'\boldsymbol{S})\,d\boldsymbol{S};$$

$$\text{(iii)}\ \chi^{(j)}_{LM}(\boldsymbol{R})=\int e^{-\lambda'(\boldsymbol{R}-\boldsymbol{S})^2}\delta(S-S^{(j)})/S^2\cdot Y_{LM}(\hat{\boldsymbol{S}})\,d\boldsymbol{S}.$$

where $\lambda'=4\beta/3$. (The set (1) and (i) in the RG- ($\boldsymbol{r}$- and $\boldsymbol{R}$-) representation is the same as the RG treatment used in Ref. 2.)

The energy-matrix elements are then given by

$$\langle\mathcal{A}[\phi_1\phi_2\phi_3\{u^{(i)}_{\ell}\otimes\chi^{(j)}_{L}\}_{JM}]|H|\mathcal{A}[\phi_1\phi_2\phi_3\{u^{(i')}_{\ell'}\otimes\chi^{(j')}_{L'}\}_{JM}]\rangle$$
$$=\mathrm{const}\times\int[\phi^{(i)}_{\ell}(\boldsymbol{t})\otimes\Phi^{(j)}_{L}(\boldsymbol{S})]_{JM}H^{(GCM)}(\boldsymbol{t}\boldsymbol{S},\boldsymbol{t}'\boldsymbol{S}')\,[\phi^{(i')}_{\ell'}\otimes\Phi^{(j')}_{L'}]_{JM}\,d\boldsymbol{t}\,d\boldsymbol{S}\,d\boldsymbol{t}'\,d\boldsymbol{S}'$$

where $H^{(GCM)}(\boldsymbol{t}\boldsymbol{S},\boldsymbol{t}'\boldsymbol{S}')$ is the well known GCM kernel before projection; and similarly for the norm-matrix elements. Here $\Phi^{(j)}_{LM}(\boldsymbol{S})$ represents $\Phi_{0LM}(\lambda'\nu'\boldsymbol{S})$, $\Phi_{N_j LM}(\lambda'\nu'\boldsymbol{S})$ or $\delta(S-S^{(j)})/S^2\cdot Y_{LM}(\hat{\boldsymbol{S}})$; and $\phi^{(i)}_{\ell m}(\boldsymbol{t})=\Phi_{0\ell m}(\lambda\nu\boldsymbol{t})$.

In the present note, use is made of the basis functions of type (i) for $\chi^{(j)}_{LM}(\boldsymbol{R})$. It is to be stressed that due to the total antisymmetrization effect, an approximation of taking $\ell=0$ is found to be able to approximate very well the result with the full-space basis functions in the case of the low-lying states of ^{12}C; in energies of 0^+_1, 2^+_1, 4^+_1, 1^-_1, 3^-_1, 0^+_2 and 2^+_2 states this approximation gives very good agreement with the strong-coupling 3α GCM calculation[3](with Volkov No.1 force and β=0.503 fm^{-2}). Through the present calculation we put ℓ=0 and adopt four values of ν_i and also four values of μ_j; the total number of the basis functions is then 16 for each J^π (Only for $J=4^+$ states, we add L=0 (ℓ=4) configurations with keeping the number of the basis functions; this gives about 2-MeV gain).

Fig. 1 shows the energy spectrum in the case of Volkov No. 2 (V2) with m=0.59 and β=0.55 fm^{-2}, and Hasegawa-Nagata force[4] with β=0.56 fm^{-2}; the two-body Coulomb interaction is included exactly. Low-lying observed levels listed are reproduced fairly well both for the shell-model-like compact states (0_1^+, 2_1^+, 1_1^-, 3_1^-) and well-developed clustering states (0_2^+ and 2_2^+). In Fig. 2, for the investigation of the saturation property of the 3α system, we plot the energy spectrum (like Fig. 1) continuously versus the oscillator size parameter β in the case of V2. It is quite interesting that the size of the α-clusters in all the ^{12}C states considered here does not favour that of the α-clusters in the simple shell-model states ($\beta\sim$0.42 fm^{-2}), but keeps the same one ($\beta\sim$0.55 fm^{-2}) as in the free α particle(cf. $E_{3\alpha}(\beta)$). This property gives a support to approximate models in which 3α particles are assumed to be structureless, or the so-called orthogonality condition is taken based on the use of the size parameter of the free α particle.

Obtained wave functions are examined by investigating electron-scattering form factors and E0, E2 and E3 transitions. The comparison between the present calculation and observations is given in Fig. 3 and Table I. Agreement is very good. Further inclusion of configurations other than the [4] symmetry ones will reduce the present values slightly.

Thus, in the study of ^{12}C, the 3α RGM is found to be very useful as well as the strong coupling GCM of Ref. 3. Detail investigations are being done on the role of Pauli principle and two-body interactions in the 3α system.

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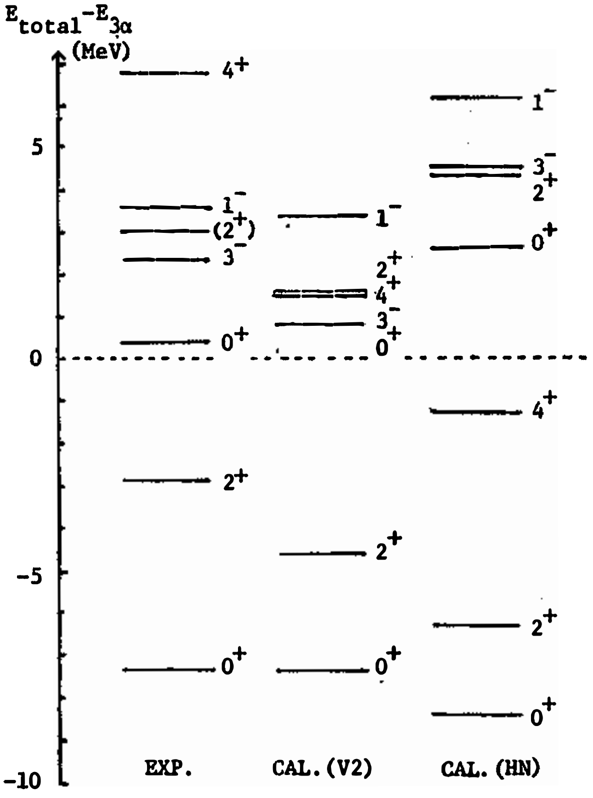

Fig. 1. Level structure of ^{12}C; the first L^{π} and the 0_2^+ and 2_2^+ states.

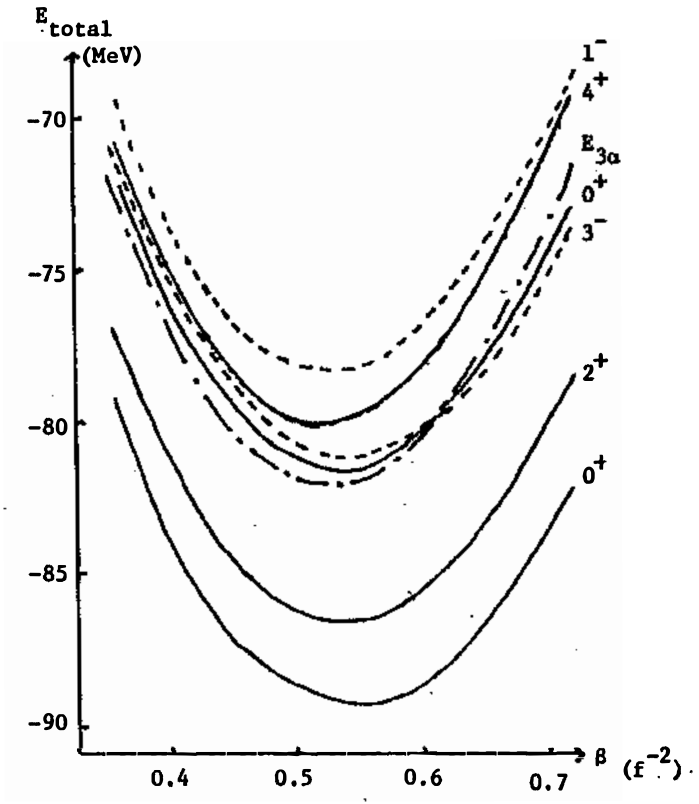

Fig. 2. Saturation of 3α system (V2). B.E.$(^{12}C)_{EXP}$=92.16 MeV.

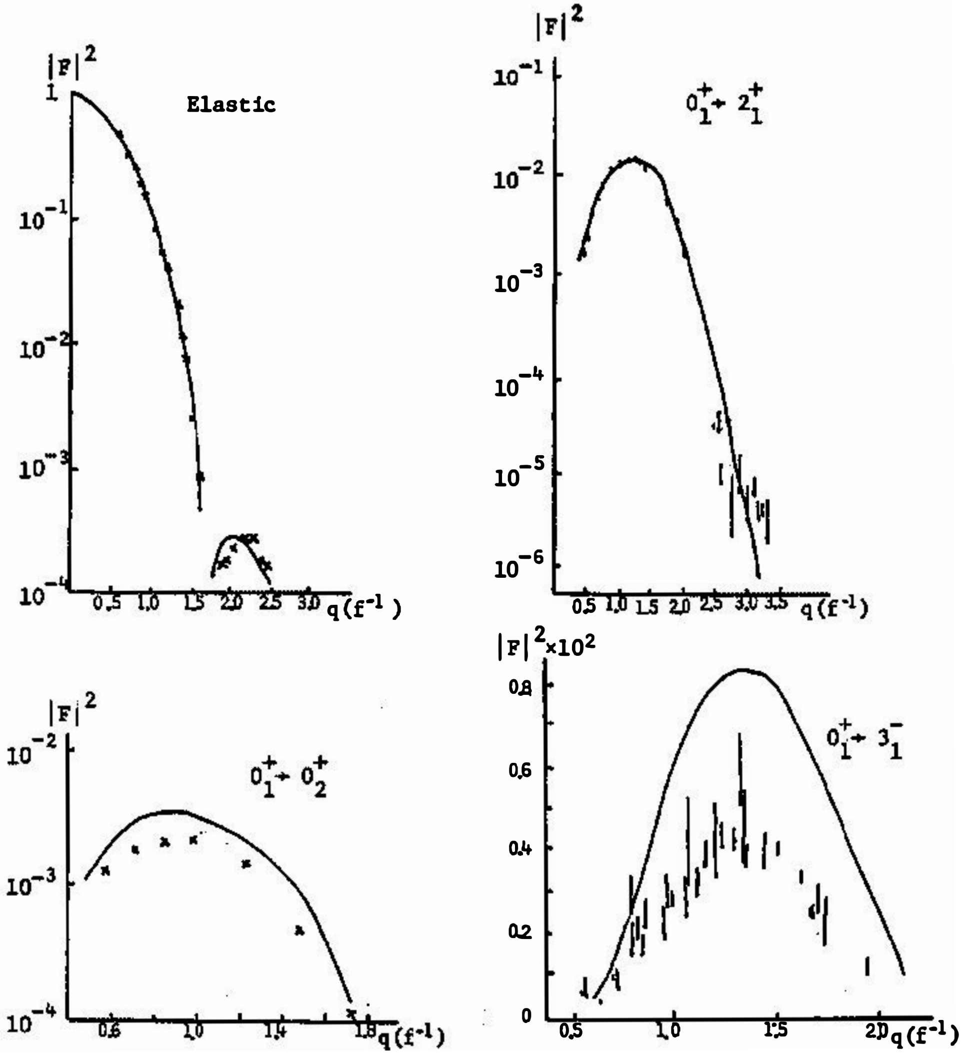

Fig. 3. Squared form factors for the elastic and inelastic electron scattering on ^{12}C. The crosses or bars denote the experimental values. The solid curves are the calculated results (V2); the correction caused by the finite size of proton is included.

Table I

	CAL (V2)	EXP
$B(E2;2_1^+\rightarrow 0_1^+)$	9.28 e^2f^4	8.45 e^2f^4
$B(E3;3_1^-\rightarrow 0_1^+)$	123.9 e^2f^6	111.9 e^2f^6
$M(0_2^+\rightarrow 0_1^+)$	6.7 f^2	5.7 f^2